Collective Learning about Systematic Risk

Yongjin Kim and Kai Li [1]

Abstract

We present an investment-based asset pricing model in which firms' exposure to systematic risk is uncertain. Beliefs about this parameter are updated from collective observations of firms' peers, causing an endogenous shift in the discount rate, so firms' real decisions and the market valuation should respond accordingly. We empirically show that the mean of beliefs about the risk exposure, formed through this collective learning, negatively predicts investment-capital ratio and market-to-book ratio and positively predicts the implied cost of capital. Besides, the precision of the parameter beliefs lowers the cost of capital and, in turn, raises the capital investment, consistent with the model prediction. In contrast, an alternative risk-estimate from firms' individual history is only insignificantly connected to the observables, revealing the collective nature of the learning.

JEL Codes: E2, E3, G12

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1. Introduction

Systematic risk is arguably one of the most fundamental quantities in finance. Various decisions ranging from portfolio choice to corporate investment often require the measurement of the risk exposure for determining the discount rate. Despite its importance, however, identifying the parameter has been elusive in practice because usual estimates tend to suffer from substantial measurement errors (Fama and French (1997); Levi and Welch (2016)). The imprecision is critical to the extent that these estimates and the resulting cost of capital often deliver puzzling predictions; for example, the cost of capital estimated through factor models appear to relate positively to firms' investment (Frank and Shen (2016)) and negatively to default risk (Chava and Purnanandam (2010)), in stark contrast to respective theories.

Facing this substantial uncertainty about the parameter, decision-makers would engage in learning about it over a nontrivial time. We propose that this learning takes place collectively. Specifically, beliefs about a particular firm's systematic risk are continuously updated upon new observations of its industry peers, which likely have similar risk exposures as often posited in the literature (for example, Brealey et al. (2016) Chapter 9). This collective form of learning has a theoretical appeal: it enables decision-makers to learn from a richer dataset than if they relied solely on firms' individual observations. If employed, this learning would, in turn, cause endogenous shifts in the discount rate. Conforming to this intuition, we find empirically that firms' real decisions and market valuations respond in a concerted way to the systematic-risk beliefs that evolve through collective learning. By contrast, the individual-learning estimate appears to be irrelevant in this regard.

To elucidate the connection between the parameter beliefs and firm observables, we build a collective-learning framework in the context of the investment-based asset pricing model. The new feature of the model is that firms' exposure to systematic risk – covariance between firm-level productivity and innovations in the stochastic discount factor – is unknown at the beginning. Instead, decision-makers gradually revise their beliefs about this parameter upon observing the newly realized productivity and macroeconomic shocks. Interestingly, this continuous update causes endogenous shifts in the discount rate as well as the risk estimate, although the true parameter is constant.

Our model provides unique predictions regarding how the belief distribution affects real decisions and market valuations. First, as the mean of beliefs about systematic risk is revised upward (downward), both the investment-capital ratio and the market-to-book ratio should decrease (increase). At the same time, the cost of capital should increase (decrease). Intuitively, perceiving a greater exposure to the systematic risk, investors require a higher return on firms' assets and thus evaluate lower the present value of future cash flows from new and existing capital.

Beyond the mean, we expect the precision of parameter beliefs as well to shape the decisions. In the model, uncertainty about the risk exposure creates additional risks that emerge from a comovement between the macroeconomic shocks and the risk-estimate revisions. Improvement in the precision reduces this particular risk, thereby decreasing the cost of capital. Consequently, both investment-capital ratio and market-to-book ratio should respond positively to the precision.

In addition, this learning results in a distinctive prediction on how investment relates to cash flows (productivity). That is, investment should respond negatively to the fraction of cash-flow growth for which the systematic component account. This unique pattern is caused by a learning-induced connection between the systematic fraction and risk-estimate revisions, not obtainable within the conventional understanding of the investment-cash flow association. For example, if cash-flow impact is mainly through alleviating financial constraints as in Fazzari et al. (1988), the level of cash-flow growth would matter, but not the composition of the growth.

The key finding of this study is that all of these learning-induced regularities hold empirically. Capital investment responds negatively to the mean of beliefs about the systematic risk with a t-statistic of -5.20. Economically, a one-standard-deviation rise in the mean leads to a 9.5% fall in the investment (e.g., the annual investment-capital ratio changes from 0.217 to 0.196). Simultaneously, we find that firms invest more when the beliefs become more precise, the association being significant at the 1% level. A one-standard-deviation increase in the precision raises the investment by 7.2%. Moreover, the investment-cash flow link further corroborates the learning impact; firms indeed reduce investment when the systematic component constitutes a larger fraction of productivity growth. All of these findings are robust to alternative systems of industry classifications, which we rely on to identify each firm's peers in collective learning. Irrespective of whether we use SIC, four-digit NAICS, or Hoberg and Phillips (2016)'s text-based industry classification, most of the associations continue to be significant at the 1% level.

Consistently, we find that the market-to-book ratio also responds to the evolution of the parameter beliefs. The market evaluates firms lower when a new signal indicates that the systematic risk is higher than previously thought. Also, the market value increases with the precision of the parameter beliefs. These established patterns in investment and valuation strongly support the time-series predictions of the parameter learning.

Next, we take a step toward testing the parameter beliefs' predictability in a cross-section of the cost of capital. We expect that, in every snapshot in time, a difference in systematic-risk beliefs across firms should create a cross-sectional dispersion in the required returns. Using the implied measure of the cost of capital from accounting information, as suggested by Hou et al. (2012), we find that the required return relates positively to the mean of beliefs; in Fama-MacBeth regression, an increase in the mean of risk exposure by one standard deviation raises the annualized cost of capital by 0.9%. Besides, we confirm that the precision of parameter beliefs plays an equally crucial role in determining firms' risk profiles. A one-standard-deviation improvement in the precision reduces the cost of capital by 0.7%, revealing that the market indeed penalizes the ambiguity about risk exposure.

We try further to define the extent to which the exact form of learning is collective. One may consider an alternative form: individual learning in which each firm's systematic risk is identified from its history only without reference to peers' observations. This idea is worth considering because the industry classifications might be only loosely defined, so even peer firms in the same industry might have different business profiles and risk exposures. In such a case, focusing on each firm's own history could better measure the risk exposure, so we conduct this alternative measurement accordingly.

The empirical analysis, however, shows that this individual-learning estimate is only insignificantly connected to all observables considered – investment, valuation ratio, and cost of capital. This finding highlights that the dataset's size, which differentiates between collective and individual learning, is critical in this context. This stark distinction arises because firms' typical characteristics render observations' signal-to-noise ratio remarkably low. Specifically, in firms' productivity growth, the idiosyncratic shock (noise) outweighs the systematic component (signal) in magnitude; in the calibrated model, the volatility of the former is approximately 30 times as large as the volatility of the latter. This substantial noise hampers identifying the risk exposure, especially when relying on a few observations that the individual learning offers. As a result, imprecise beliefs obtained from individual history turn out to be irrelevant for explaining the firm observables.

In a robustness test, we confirm that our main findings hold even when true exposure to systematic risk is dynamic. Suppose that the true exposure changes over time, contrary to our baseline assumption. In this case, the learning-based estimate, derived from the constant-risk assumption, might misleadingly capture variations in true risk characteristics. To address this concern, we explicitly model the true risk-exposure as an autoregressive process and estimate the belief distribution through the Kalman filter. In this setting, we focus on the ambiguity about the unconditional mean of systematic risk, of which true value is constant by nature. Our empirical analysis establishes that the belief distribution for this particular parameter predicts firm observables in a similar manner to the baseline findings. In essence, learning about the long-run mean of risk exposure continues to shape the decisions in this alternative consideration.

Literature Review This study builds on a growing body of literature that studies the implications of parameter learning with respect to asset pricing or corporate decisions. Pastor and Veronesi (2009) provides a comprehensive review of these learning models. Prior studies have considered uncertainty regarding the mean productivity (Pastor and Veronesi (2003); Alti (2003)), return-to-scale parameters in production function (Johnson (2007)), or consumption volatility (Weitzman (2007)). In particular, our intuition that learning about a constant paramter from growing observations endogenously changes the discount rate is along the lines of Weitzman (2007).

Distinct from these prior studies, however, we focus on implications of unknown exposure to systematic risk. This focus is similar to Ai et al. (2018), who document crucial impact of risk-exposure ambiguity on the term structure of equity returns. We complement this study by elaborating upon the learning mechanism, in which decision-makers learn from the history of realized output instead of noisy independent signals in the prior study. This setting generates a path dependence of the firm observables, which we use to show the empirical presence of the learning. The idea of learning from peers is similar to Foucault and Fresard (2014), who document that a firm's investment responds to its peers' Tobin's q. Our evidence further testifies to the importance of collective learning, showing that it predicts much wider range of firm observables.

This study is also related to the literature that examines the link from consumption risk to stock returns, including Lettau and Ludvigson (2001), Bansal et al. (2005), Da (2009), and Boguth and Kuehn (2013). These studies clearly document that a cross-sectional dispersion in expected returns is caused by cash flows' or returns' covariance with the macroeconomic shocks affecting consumption. Besides the consumption beta, we further reveal that the belief precision of this parameter is priced in the cross-section of return, highlighting the parameter uncertainty and subsequent learning. Moreover, we expand the scope of the analysis and document the response of corporate investments and valuation ratios to the consumption beta.

More broadly, our work is also related to dynamic investment models that investigate the implications of firms' optimal decisions on asset returns. Prior studies, including Berk et al. (1999), Gomes et al. (2003), Carlson et al. (2004), Zhang (2005), and Kuehn and Schmid (2014), show that peculiar patterns observed in stock and bond returns arise as a result of corporate investment policy. We complement the literature by establishing new regularities in investment and return, which the parameter learning causes.

The remainder of this paper is organized as follows. In section 2, we describe the theoretical model. In section 3, we explain the calibration and put forward testable predictions from the model. In section 4, we provide empirical tests and discusses the main findings. We conclude this paper with section 5.

2. Model

We consider peer firms that belong to one industry $\mathbb{I}$. These firms are heterogeneous ex-post but have a common characteristic: identical exposure to systematic risk. To define systematic risk in a tractable way, we specify a consumption-based stochastic discount factor. The representative agent has recursive preferences over exogenous consumption. Given the stochastic discount factor, each firm makes optimal investment decisions with reference to all up-to-date information. Our model only offers partial equilibrium since we do not connect the sum of production outputs in the economy back to aggregate consumption.

2.1. Stochastic Discount Factor

Preferences of the representative agent are recursive as in Epstein and Zin (1989). The preferences are characterized by the standard parameters, including the rate of time preference β, the elasticity of intertemporal substitution ψ, and the coefficient of relative risk aversion γ.

We assume that consumption growth conditional on time t is normally distributed as in Kuehn and Schmid (2014):

$$\ln\left(\frac{C_{t+1}}{C_t}\right) = g + \mu_c(\omega_t) + \sigma_c(\omega_t)\eta_{t+1} \tag{1}$$

where η_{t+1} is standard normal innovation, and the mean $\mu_c(\omega_t)$ and the volatility $\sigma_c(\omega_t)$ of the growth depend on the state of the economy ω_t. The economic state shifts over time following a Markov chain with transition matrix P.

The stochastic discount factor is

$$M_{t,t+1} = \beta^\theta \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \left(\frac{S_{t+1}+1}{S_t}\right)^{-(1-\theta)} \tag{2}$$

where S_t denotes the wealth-consumption ratio and $\theta = \frac{1-\gamma}{1-1/\psi}$. The wealth-consumption ratio is determined solely by the state of the economy, so $S_t = S(\omega_t)$. This ratio can be solved through the Euler equation described in Appendix A.

2.2. Firm's Production and Investment

Consider firm i that employs a production technology with decreasing return-to-scale. Its output in time t is

$$A_{i,t} K_{i,t}^{\alpha} \tag{3}$$

for which $A_{i,t}$ is the productivity shock, $K_{i,t}$ is the capital stock, and $0 < \alpha < 1$ is the capital share of the production. The capital stock accumulates according to

$$K_{i,t+1} = I_{i,t} + (1 - \delta) K_{i,t} \tag{4}$$

where $I_{i,t}$ is investment and δ is the constant depreciation rate. We assume that capital installments are not frictionless as in the literature, so the firm incurs convex adjustment costs in addition to purchasing costs. The adjustment costs are $\phi \left(I_{i,t} / K_{i,t} \right)^2 K_{i,t}$ where ϕ is a positive constant.

The firm's productivity growth is stochastic and correlated with consumption growth. Due to the tight connection between the consumption shock and the stochastic discount factor, this correlation engenders the firm's exposure to the systematic risk. Specifically, the productivity growth is

$$\ln \left(\frac{A_{i,t+1}}{A_{i,t}} \right) = g + \mu_c(\omega_t) - \frac{(b^{\mathbb{I}})^2 \sigma_c(\omega_t)^2}{2} + b^{\mathbb{I}} \sigma_c(\omega_t) \eta_{t+1} + \nu \epsilon_{i,t+1} \tag{5}$$

where $\epsilon_{i,t+1}$ is a firm-specific standard normal innovation, and ν controls the magnitude of this idiosyncratic shock.

The parameter $b^{\mathbb{I}}$, which is identical for all industry constituents, controls the productivity's exposure to the consumption shock η_{t+1}. An increase in $b^{\mathbb{I}}$ would amplify the covariance between productivity and consumption, thereby increasing the systematic risk. Meanwhile, the change in $b^{\mathbb{I}}$ would leave the the mean of future productivity unchanged. In particular, the third term in the right hand side, $(b^{\mathbb{I}})^2 \sigma_c(\omega_t)^2/2$, adjusts for the Jensen's inequality effect; otherwise, without this adjustment, a rise in $b^{\mathbb{I}}$ would increase the average of future productivity. In sum, the specification of equation (5) means that a rise in $b^{\mathbb{I}}$ leads to the mean-preserving spread of future productivity that the agent penalizes.

Importantly, we assume that the parameter $b^{\mathbb{I}}$ is unobservable for the agent and must be estimated from realized productivity. Learning about $b^{\mathbb{I}}$ is non-trivial because the productivity is also subject to the unobservable idiosyncratic shock, which the agent cannot distinguish from the systematic component. In the next section, we describe a framework in which the agent forms beliefs about the parameter using the up-to-date history of firms' peers.

2.3. Collective Learning About Systematic Risk

The industry $\mathbb{I}$'s risk exposure $b^{\mathbb{I}}$ is unknown at the beginning. The agent starts with prior beliefs about the parameter that are normally distributed with mean $m^{\mathbb{I}}_{b,0}$ and standard deviation $\sigma^{\mathbb{I}}_{b,0}$. Since then, the agent receives new information – the realized productivity of every industry constituent and consumption growth – and accordingly update the beliefs. Notice that in learning about a target firm's systematic risk, its peers' productivity is also informative due to identical risk exposures among industry constituents. Recognizing this, the agent refers to collective observations of the industry in updating the beliefs.

To formulate the learning process, we let $\mathbb{A}_t$ denote the $(n \times 1)$ vector of the time-t productivity growth for n constituents. In addition, $\mathbb{C}_t$ denotes the $(n \times 1)$ vector with all elements equal to the time-t consumption growth. Specifically,

$$\mathbb{A}_t = \left[\ln\left(\frac{A_{1,t}}{A_{1,t-1}}\right) \quad \cdots \quad \ln\left(\frac{A_{n,t}}{A_{n,t-1}}\right) \right]^T, \quad \mathbb{C}_t = \left[\ln\left(\frac{C_t}{C_{t-1}}\right) \quad \cdots \quad \ln\left(\frac{C_t}{C_{t-1}}\right) \right]^T. \tag{6}$$

Conditional on the observations of productivity and consumption, the parameter beliefs are revised according to Bayes' law. This learning mechanism induces a recursive structure of the posterior distribution $p()$:

$$p\left(b^{\mathbb{I}} | \mathbb{A}_1, \ldots, \mathbb{A}_t, \mathbb{C}_1, \ldots, \mathbb{C}_t\right) \propto p\left(\mathbb{A}_t | b^{\mathbb{I}}, \mathbb{C}_t\right) \times p\left(b^{\mathbb{I}} | \mathbb{A}_1, \ldots, \mathbb{A}_{t-1}, \mathbb{C}_1, \ldots, \mathbb{C}_{t-1}\right), \tag{7}$$

where we use the fact that $\mathbb{A}_t$ depends only on the current consumption growth and the risk exposure. Next, to obtain the posterior distribution in a tractable form, we linearize equation (5) with respect to $b^{\mathbb{I}}$ through the Taylor expansion. This approximation makes the sampling distribution $p\left(\mathbb{A}_t|b^{\mathbb{I}}, \mathbb{C}_t\right)$ normal, and therefore the posterior distribution stays normal like the prior. As a result, the mean $m^{\mathbb{I}}_{b,t}$ and the standard deviation $\sigma^{\mathbb{I}}_{b,t}$ of beliefs are updated recursively,

$$m^{\mathbb{I}}_{b,t} = \frac{\frac{1}{(\sigma^{\mathbb{I}}_{b,t-1})^2} \times m^{\mathbb{I}}_{b,t-1} + \frac{\overline{\mathbb{C}}_t^T \overline{\mathbb{C}}_t}{\nu^2} \times \widehat{b_t}}{\frac{1}{(\sigma^{\mathbb{I}}_{b,t-1})^2} + \frac{\overline{\mathbb{C}}_t^T \overline{\mathbb{C}}_t}{\nu^2}}$$

$$\frac{1}{(\sigma^{\mathbb{I}}_{b,t})^2} = \frac{1}{(\sigma^{\mathbb{I}}_{b,t-1})^2} + \frac{\overline{\mathbb{C}}_t^T \overline{\mathbb{C}}_t}{\nu^2}, \tag{8}$$

where $\overline{\mathbb{C}}_t$ is the column vector with all elements equal to a demeaned growth in productivity, and $\widehat{b_t}$ is a sample estimate of b using only time-t observations. Specifically, $\widehat{b_t}$ is $\left(\overline{\mathbb{C}}_t^T \overline{\mathbb{C}}_t\right)^{-1} \overline{\mathbb{C}}_t^T \overline{\mathbb{A}}_t$, where $\overline{\mathbb{A}}_t$ is a demeaned growth in productivity. The derivation and detailed expressions are provided in Appendix B.

Intuitively, having more observations over time improves the precision of beliefs, as measured by $1/\sigma^{\mathbb{I}}_{b,t}$. The posterior mean $m^{\mathbb{I}}_{b,t}$ is a weighted average of the prior mean and the sample estimate, with the weights being determined by parameter uncertainty $\left(\sigma^{\mathbb{I}}_{b,t-1}\right)^2$ and signal variance $\left(\nu^2/\overline{\mathbb{C}}_t^T \overline{\mathbb{C}}_t\right)$. As a result, this revision is more sensitive to new observations when the agent is more uncertain about the parameter, high $\sigma^{\mathbb{I}}_{b,t-1}$.

2.4. Valuation

Suppose that the financial markets are frictionless. Firms' financing choices are then irrelevant to firm value, so we assume for simplicity's sake that firms are entirely financed with equity. In this setting, firm i chooses investment to maximize its market value:

$$V_{i,t} = \max_{I_{i,t}} \left\{ A_{i,t} K^{\alpha}_{i,t} - I_{i,t} - \phi \left(\frac{I_{i,t}}{K_{i,t}} \right)^2 K_{i,t} + \mathbb{E}_t \left[M_{t,t+1} V_{i,t+1} \right] \right\}. \tag{9}$$

The state variables in this firm's problem are the aggregate state of the economy ω_t, the productivity shock $A_{i,t}$, capital stock $K_{i,t}$ and the distribution of posterior beliefs about the systematic risk: the mean $m^{\mathbb{I}}_{b,t}$ and the standard error $\sigma^{\mathbb{I}}_{b,t}$. These state variables evolve according to the law of motion described by equations (4), (5) and (8).

A distinctive feature of this problem is that firm i's investment and valuation are influenced by the history of its peers' realized productivity, which shapes the beliefs about the systematic risk. Instead of keeping track of the entire history, however, it is sufficient to refer to the industry-wide statistics, $m^{\mathbb{I}}_{b,t}$ and $\sigma^{\mathbb{I}}_{b,t}$, for firm i to make the optimal decisions.

As in the q-theory literature, one of the major determinants of the investment policy is the marginal value of capital. To analyze the capital value, let us define the "ex-dividend" value of unit capital as $P_{i,t} \equiv E_t \left[M_{t,t+1} \frac{\partial V_{i,t+1}}{\partial K_{i,t+1}} \right]$. The following proposition characterizes the capital value.

Proposition 1: *The ex-dividend value of firm i's unit capital satisfies the recursive equation*

$$P_{i,t} = \mathbb{E}_t \left[M_{t,t+1} \left(\alpha A_{i,t+1} K^{\alpha-1}_{i,t+1} + \phi \left(\frac{I_{i,t+1}}{K_{i,t+1}} \right)^2 + (1-\delta) P_{i,t+1} \right) \right]. \tag{10}$$

The proof is provided in Appendix C. Intuitively, the marginal value of capital consists of the present value of production output in the next period and the continuation value of capital after the production. Using this recursive structure, we numerically solve the capital value.

In addition, the capital value helps us to identify the firm's return. The realized gross return from time t to $t+1$ on capital investment is

$$R_{i,t+1} = \frac{\alpha A_{i,t+1} K^{\alpha-1}_{i,t+1} + \phi \left(\frac{I_{i,t+1}}{K_{i,t+1}} \right)^2 + (1-\delta) P_{i,t+1}}{P_{i,t}}. \tag{11}$$

This capital return is then equal to the stock return for the all-equity-financed firm (Cochrane (1991) and Liu et al. (2009)).

3. Asset Pricing Implications

This section presents our model predictions that parameter uncertainty and subsequent learning about systematic risk cause unique patterns in firms' investment, valuation, and cost of capital. Considering that the predictions may depend on model parameters, we first calibrate the theoretical model so that it matches relevant moments of the empirical data. After putting forward these model-implied predictions, we test them empirically in the following section.

3.1. Calibration

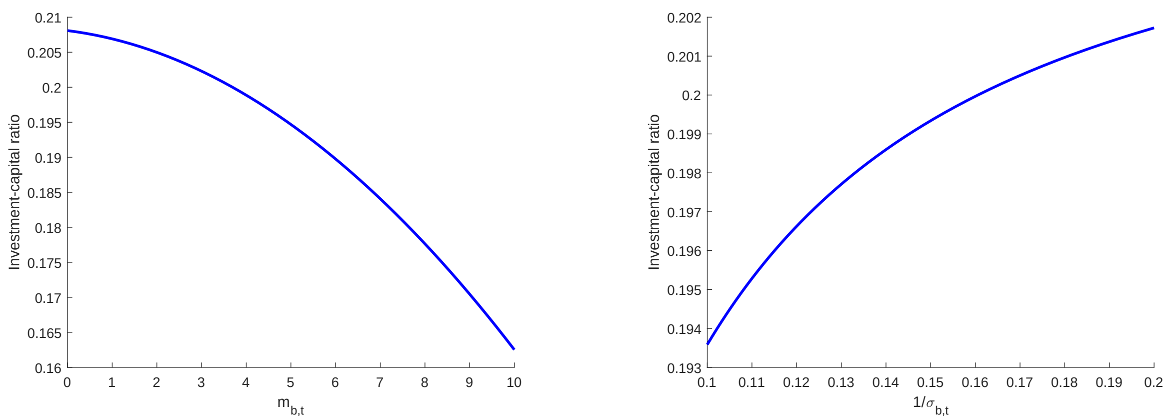
Table 1 presents the calibration results. We first calibrate parameters characterizing the stochastic discount factor, following the procedure described by Kuehn and Schmid (2014). The preference parameters are set within the range of common values used in the literature; the rate of time preference is 0.996, the elasticity of intertemporal substitution is 2, and the coefficient of relative risk aversion is 10. Next, in specifying the Markov chain of the consumption process, we assume that the process consists of five different states of the economy with different conditional means and volatilities of consumption growth. We calibrate these parameters and the transition matrix such that the Markov chain approximates the continuous-state consumption process of Bansal and Yaron (2004). Under these parameter choices, the simulated moments for consumption dynamics and risk-free returns align with the empirical counterparts.

With respect to firm-level parameters, we set the capital share of production to be 0.65, based on evidence by Cooper and Ejarque (2003). The capital depreciation rate is 3% per quarter as in Cooley and Prescott (1995). We set the productivity's exposure to the systematic risk to be 1.98. This particular level is the average of the risk exposure across industries from our estimation.[2] Note that this true value of the exposure is not observable by firms in our model, but crucial for generating the data that serves as a source of the parameter learning. The coefficient for investment-adjustment costs is chosen to match the empirical average of investment-capital ratio.

[2]We regard the most up-to-date estimate as the true risk exposure for each industry. This approximation is based on the theoretical property that the posterior estimate converges to the true parameter as the number of observations becomes infinitely large.

Figure 1: Beliefs about Systematic Risk and Firms' Investment

Panel A. $m^{\mathbb{I}}_{b,t}$ and Investment   Panel B. $1/\sigma^{\mathbb{I}}_{b,t}$ and Investment



This figure presents the investment-capital ratio for different moments of the belief distribution: the posterior mean $m^{\mathbb{I}}_{b,t}$ of the systematic risk and the precision of beliefs $1/\sigma^{\mathbb{I}}_{b,t}$.

Finally, we choose the volatility of idiosyncratic shock to productivity to match the standard deviation of the investment-capital ratio.

3.2. Testable Model Predictions

Having calibrated the model, we now put forward testable predictions. The first two corollaries describe the learning-induced regularities in firms' investment.

Corollary 1: *A firm's investment-capital ratio decreases with $m^{\mathbb{I}}_{b,t}$, the posterior mean of the systematic risk for the industry to which the firm belongs.*

Corollary 2: *A firm's investment-capital ratio increases with $1/\sigma^{\mathbb{I}}_{b,t}$, the precision of beliefs about the systematic risk for the industry to which the firm belongs.*

These model predictions are depicted in Figure 1. The intuition behind our results is as follows. A higher value of $m^{\mathbb{I}}_{b,t}$ means that future production outputs from new capital have, on average, a larger exposure to systematic risk, such that the marginal value of capital falls. Firms, therefore, invest less in equilibrium, as shown in Corollary 1 and Panel A of the figure.

Corollary 2 concerns the investment response to the precision of beliefs. As illustrated in Panel B of the figure, a greater uncertainty $\sigma^{\mathbb{I}}_{b,t}$, or lower precision of the parameter beliefs, discourages the investment. Interestingly, we find that this negative association emerges from the additional risks that the parameter uncertainty creates. Specifically, the updating mechanism in equation (8) implies that revisions in the risk estimate correlate to consumption shock as described in the following lemma.

Lemma 1: *The time-t covariance between the change in the risk estimate, $m^{\mathbb{I}}_{b,t+1} - m^{\mathbb{I}}_{b,t}$, and consumption shock, $\sigma_c(\omega_t)\eta_{t+1}$, is $-(m^{\mathbb{I}}_{b,t})^2\sigma_c(\omega_t)^2 E_t\left[\mathbb{K}_{t+1}\right]$. $\mathbb{K}_{t+1}$ is the Kalman gain and its time-t expected value increases with $\sigma^{\mathbb{I}}_{b,t}$.*

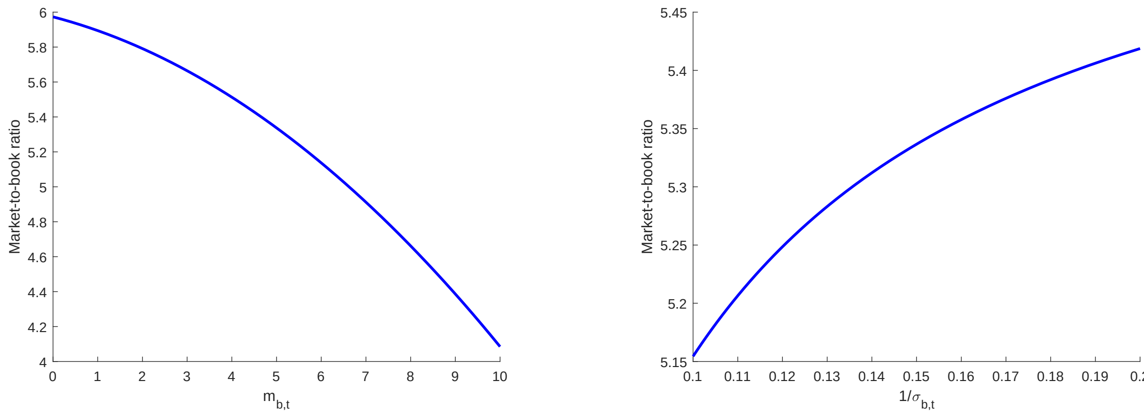
The proof is provided in Appendix D. The negative covariance in the lemma means that if $\sigma_c(\omega_t)\eta_{t+1} > 0$, the new risk estimate $m^{\mathbb{I}}_{b,t+1}$ tends to be revised lower than previously thought, thereby increasing the marginal value of capital. Furthermore, this positive covariance between consumption shock and capital value, or the systematic risk, is amplified by the parameter uncertainty $\sigma^{\mathbb{I}}_{b,t}$, according to the lemma. The rise in the systematic risk then discourages firms from investing.

This learning mechanism also generates a unique pattern in the investment-cash flow association. The literature has noted that cash flows may influence the investment for the following reasons. On the one hand, a lack of cash flows tends to constrain firms financially (Fazzari et al. (1988) Hoshi et al. (1991)). Alternatively, when the production technology has decreasing returns to scale, cash flows becomes informative about the marginal value of capital beyond what average q suggests (Cooper and Ejarque (2003)). Despite the logical difference, both propositions suggest that growth in cash flows encourages investment.

Distinct from the conventional understanding, our framework engenders a unique association. When the level of cash-flow growth is controlled for, the composition of the growth should predict investment because of its informativeness of firms' systematic risk. In particular, the investment should respond negatively to the fraction of cash-flow growth for which the systematic component accounts.

Figure 2: Beliefs about Systematic Risk and Firms' Valuation

Panel A. $m^{\mathbb{I}}_{b,t}$ and Market-to-Book Ratio　　Panel B. $1/\sigma^{\mathbb{I}}_{b,t}$ and Market-to-Book Ratio



This figure presents the market-to-book ratio for different moments of the belief distribution: the posterior mean $m^{\mathbb{I}}_{b,t}$ of the systematic risk and the precision of beliefs $1/\sigma^{\mathbb{I}}_{b,t}$.

To elaborate upon this prediction, we revisit equation (8), which formulates the updating of the systematic risk. Recall that $\overline{\mathbb{A}}_t$ is the demeaned growth in productivity and $\overline{\mathbb{C}}_t$ is the demeaned growth in consumption. Through element-wise dividing $\overline{\mathbb{C}}_t$ by $\overline{\mathbb{A}}_t$, we obtain a $(n \times 1)$ vector for industry $\mathbb{I}$, denoting it by $\mathbb{F}^{\mathbb{I}}_t$. In this vector, the ith element is firm i's fraction of the systematic component in productivity growth. The updating equation can then be rewritten as:

$$m^{\mathbb{I}}_{b,t} - m^{\mathbb{I}}_{b,t-1} = \frac{\left(\sigma^{\mathbb{I}}_{b,t}\right)^2}{\nu^2}\left(\overline{\mathbb{C}}_t^T\overline{\mathbb{A}}_t - m_{b,t-1}\overline{\mathbb{C}}_t^T\overline{\mathbb{C}}_t\right) \approx \frac{\left(\sigma^{\mathbb{I}}_{b,t}\right)^2}{\nu^2} n \left(\overline{\mathbb{A}}_t^T\overline{\mathbb{A}}_t\right)\frac{\mathbb{1}_n^T\mathbb{F}^{\mathbb{I}}_t}{n} \tag{12}$$

where $\mathbb{1}_n$ is a $(n \times 1)$ vector with all elements equal to one. The last approximation is based on the fact that the fraction of the systematic component is usually much smaller than one because the idiosyncratic component in cash-flow growth far outweighs the systematic part in the absolute size.[3]

According to equation (12), when an industry sees the systematic component accounting for a larger fraction in the cash-flow growth, high $\mathbb{1}_n^T\mathbb{F}^{\mathbb{I}}_t/n$, firms revise their respective risk estimates

[3]In the data, 10th percentile of the ratio of the systematic component to the productivity growth is -0.133 and 90th percentile is 0.130.

upward. This will, in turn, cause firms to reduce investment, as stated in the following corollary.

Corollary 3: *A firm's investment-capital ratio decreases with* $\mathbb{1}_n^T \mathbb{F}_t^{\mathbb{I}}/n$*, the industry* $\mathbb{I}$*'s average of the fraction of cash-flow growth for which the systematic component accounts.*

Notice that this association is exclusively caused by parameter learning. To elucidate this, let us suppose that the risk-exposure parameter is unambiguously known in the conventional frameworks for the investment-cash flow link, such as financial constraints or decreasing-returns-to scale production. In this case, the parameter uncertainty $\sigma_{b,t}^{\mathbb{I}}$ would be zero in equation (12), so the systematic fraction would not change the risk estimate. Hence, the composition of productivity growth would have no impact on investment.

Next, we turn to the implications for valuation. Applying the rationale in Corollaries 1 and 2 to the context of valuing the existing capital stock, we put forward the following hypotheses regarding the market-to-book ratio.

Corollary 4: *A firm's market-to-book ratio decreases with* $m_{b,t}^{\mathbb{I}}$*.*

Corollary 5: *A firm's market-to-book ratio increases with* $1/\sigma_{b,t}^{\mathbb{I}}$*.*

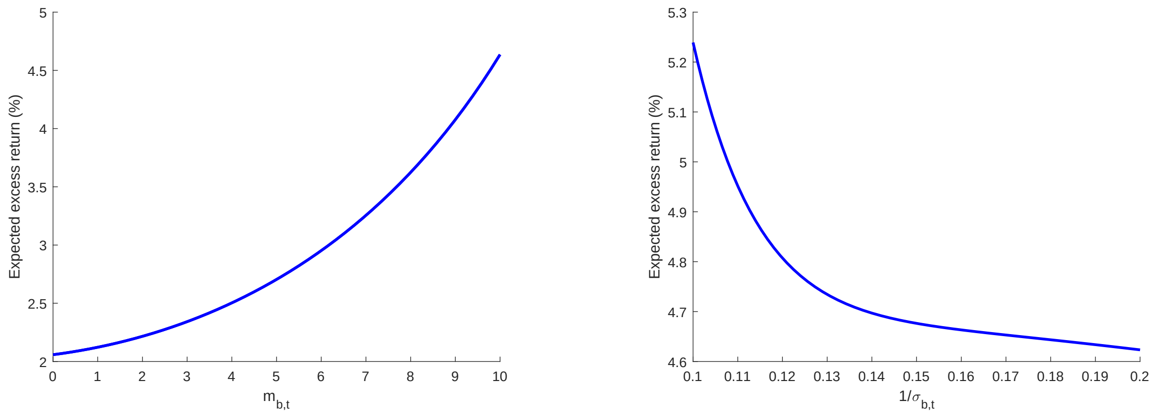
Figure 2 illustrates these predictions. Notably, the positive relationship in Corollary 5 between the market-to-book ratio and the precision of beliefs is distinct from Pastor and Veronesi (2003)'s prediction that the valuation ratio increases with uncertainty concerning a firm character – specifically, its mean profitability. In this prior study, a rise in the uncertainty does not change the firms' systematic risk. Instead, it increases the mean of future cash flows through Jensen's inequality. By contrast, our model predicts that uncertainty about the risk exposure amplifies the systematic risk, as stated in Lemma 1. Consequently, the market evaluates firms lower than otherwise.

The last dimension that we consider as evidence of the learning impact is the expected return, or cost of capital. Because the risk-exposure estimate is updated through the realized productivity that is unique for each industry, we conjecture that the parameter beliefs vary substantially across industries; our estimates confirm this as shown in Figure 5. The heterogeneity will naturally lead to a cross-sectional difference in the expected returns.

Figure 3: Beliefs about Systematic Risk and Expected Return

Panel A. $m^{\mathbb{I}}_{b,t}$ and Expected Return

Panel B. $1/\sigma^{\mathbb{I}}_{b,t}$ and Expected Return



This figure presents the expected return on stock in excess of the risk-free rate for different moments of the belief distribution: the posterior mean $m^{\mathbb{I}}_{b,t}$ of the systematic risk and the precision of beliefs $1/\sigma^{\mathbb{I}}_{b,t}$.

Corollary 6: *The expected return on stock in excess of the risk-free rate increases with* $m^{\mathbb{I}}_{b,t}$.

Corollary 7: *The expected return on stock in excess of the risk-free rate decreases with* $1/\sigma^{\mathbb{I}}_{b,t}$.

In Figure 3, we describe these model predictions. The return dependence on $m^{\mathbb{I}}_{b,t}$ in Corollary 6 is intuitive. When perceiving a greater average of systematic-risk exposure, market participants require a higher return on firms' assets.

With respect to Corollary 7, the negative impact on the expected return obtains by the same logic through which the precision of beliefs influence investment and valuation. Specifically, the correlation between risk-estimate revision and consumption shock in Lemma 1 eventually enlarges the capital return's systematic risk. As an illustration, if $\sigma_c(\omega_t)\eta_{t+1} > 0$, the risk estimate is likely to decrease, $m^{\mathbb{I}}_{b,t+1} - m^{\mathbb{I}}_{b,t} < 0$, thus generating a positive return on capital, all else being equal. This positive covariance between the return and the consumption increases with the uncertainty about the systematic risk $\sigma^{\mathbb{I}}_{b,t}$. Therefore, a larger uncertainty about the parameter causes market participants to require a higher return.

4. Empirical Analysis

4.1. Data

Our data consist of annual observations for non-financial and non-utilities firms on Compustat and CRSP for years 1952-2017. We choose the annual frequency to minimize a seasonality impact on corporate investment in our analysis. Among our firm-year observations, we exclude data points with negative or missing sales revenue or total assets, which results in a total of 121,279 observations.

Firm-level variables are measured in standard ways in the literature. Firm size is defined as the natural log of total assets (AT). The investment-capital ratio is capital expenditures (CAPX) divided by the beginning-of-period capital stock (PPENT). We measure productivity as the calibrated model implies; the productivity is operating profits (OIBDP) divided by the capital stock raised to the power of 0.65. The book leverage ratio is the sum of debt in current liabilities (DLC) and long-term debt (DLTT) divided by AT. Cash flow is operating profits (OIBDP) divided by the beginning-of-period AT. For Tobin's q and the market-to-book ratio, we follow the measurement of Erickson et al. (2014). The numerator of Tobin's q is DLTT plus DLC plus market equity minus current assets (ACT), in which the market equity is the product of common shares outstanding (CSHO) and stock price (PRCC). The denominator of Tobin's q is gross capital stock (PPEGT). The numerator of the market-to-book ratio is AT plus the market equity minus book common equity (CEQ) minus deferred taxes (TXCB). The denominator of the market-to-book ratio is AT. For return on equity, we follow the measurement of Pastor and Veronesi (2003). Earnings are income before extraordinary items available to shareholders (IBCOM), plus deferred taxes from income statements (TXDI), plus investment tax credits (ITCI). Book equity value is stockholders' equity (SEQ), plus deferred taxes and investment tax credit from balance sheets (TXDITC), minus the book value of the preferred stock (PSTKRV). A firm's age is measured by the log of the number of years since the firm's stock price first appeared on CRSP.

We measure firms' financial constraints as in Whited and Wu (2006). Each firm's cash flows, dividend, leverage ratio, total assets, and sales growth are aggregated to generate the composite index (hereafter, referred to as the WW-index).

Our analysis requires an industry classification. Following the standard in the literature, we identify industries using either four-digit SIC or four-digit NAICS codes. In addition, we employ the text-based classification recently developed by Hoberg and Phillips (2010). This classification is based on product similarity among firms that is measured through a text-based analysis of 10-K filings. This text-based network industry classification system (hereafter, referred to as TNIC) is obtained from the Hoberg-Phillips Data Library.

Once industries are defined, we calculate the Herfindahl index to measure the level of competition among industry constituents. To obtain the index, we first calculate the market share of each constituent using sales revenue (SALE) and sum the squared shares across the constituents. The final index is the reciprocal of the sum, so a higher value indicates a higher level of competition.

We estimate the time-t beliefs about industry $\mathbb{I}$'s systematic risk through the Bayesian learning, as formulated in equation (8). As to the prior distribution before the first observation, we arbitrarily set $m^{\mathbb{I}}_{b,0}$ to be 1 and $\sigma^{\mathbb{I}}_{b,0}$ to be 20. This particular choice represents decision-makers' ambiguous beliefs about the parameter at the beginning of the learning process. As a robustness test, we alternatively choose different values, 0.5 to 2 for $m^{\mathbb{I}}_{b,0}$, and 3 to 20 for $\sigma^{\mathbb{I}}_{b,0}$, and we still find that the main empirical findings continue to hold. After setting the prior distribution, we plug the realized productivity for every industry constituent into the updating equation and revise the industry's risk exposure every year.

We also consider an alternative estimation of firms' systematic risk that uses each firm's own history only for reasons we discuss in section 4.3.4. This estimation does not utilize the peers' observations, and thus it leads to an alternative posterior distribution of b. Its posterior mean is denoted by $m^{i}_{b,t}$ and the precision is $1/\sigma^{i}_{b,t}$.

These estimations require the time-series of the consumption shock. As the aggregate state of the economy shifts according to the Markov chain, we first need to identify the economic state for each time. For the identification, we use the quarterly time-series of consumption on non-durables and services from the Federal Research Economic Data. Applying the Bayesian filtering technique as in Cappe et al. (2011) to these observations, we obtain the probability distribution of consumption shock for each quarter. By aggregating into annual frequency and computing the average, we are finally able to estimate the annual time series of consumption shock.

To determine each firm's cost of capital, we follow the methodology suggested by Hou et al. (2012), which uses the information implied by accounting data. This implied cost of capital ($\text{ICC}_{i,t}$) is the particular discount rate that makes the present value of expected future cash flows equal to the firm's market value. Following the prior study, we forecast future cash flows from a cross-sectional model that relates firms' earnings to other observables.[4]

In addition, we measure firms' exposures to risk factors identified by Fama and French (2015). To determine each firm's time-t risk exposures, we regress the firms' monthly return up to time t onto the time-series of the risk factors from Kenneth French's website. This expanding-window estimation is to make the factor measurement compatible to that of $m^{\mathbb{I}}_{b,t}$.

4.2. A Look at Estimated Beliefs about Systematic Risk

Prior to the empirical tests, we first present the estimated beliefs about systematic risk. In Figure 4, we plot the time-series of the posterior distribution of the beliefs for these selected industries: Distilled and Blended Liquors (SIC 2085) and Retail - Variety Stores (SIC 5331). We choose these two industries as examples because the latest estimates of their risk exposures are very close; it is 2.55 for the former and 2.80 for the latter. Despite the similarity in these recent values, their historical paths of the parameter updates are strikingly different, especially in the early years of the industries. For example, the risk-exposure estimate for Liquor fluctuated around zero in the first ten years after an initial surge, whereas the estimate of Retail increased gradually

[4]The cross-sectional model relates earnings (cash flows) to other variables through the following regression specification:

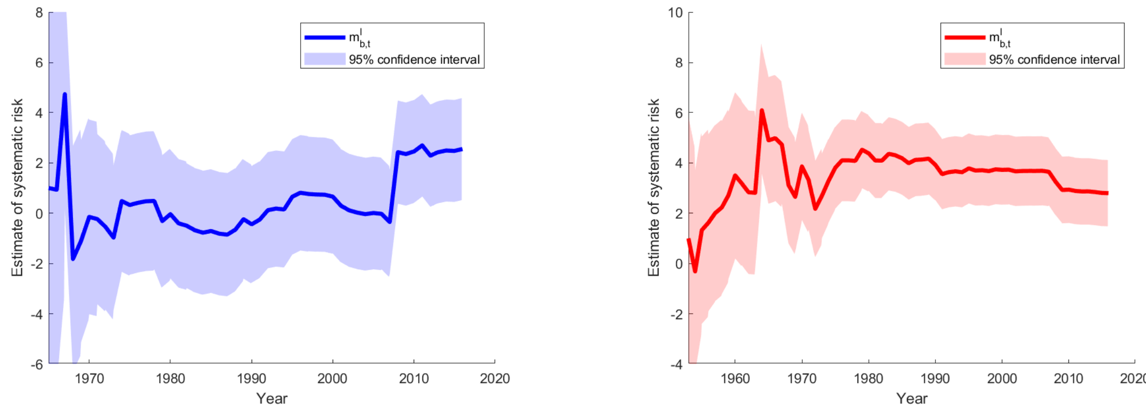
$$E_{i,t+1} = \alpha_0 + \beta_1 A_{i,t} + \beta_2 D_{i,t} + \beta_3 DD_{i,t} + \beta_4 E_{i,t} + \beta_5 NegE_{i,t} + \beta_6 AC_{i,t} + \epsilon_{i,t+1}$$

where $E_{i,t}$ denotes the earnings of firm i, $A_{i,t}$ is the total assets, $D_{i,t}$ is the dividend, $DD_{i,t}$ is a dummy variable that equals 1 for dividend pays and 0 otherwise, $NegE_{i,t}$ is a dummy variable that equals 1 for firms with negative earnings and 0 otherwise, and $AC_{i,t}$ is accruals. Following the prior study, I conduct 10-year rolling window regression each quarter and estimate the future earnings.

Figure 4: Examples of the Estimated Systematic Risk

Panel A. Distilled and Blended Liquors (SIC 2085)

Panel B. Retail - Variety Stores (SIC 5331)



This figure presents the time-series of the posterior beliefs about the systematic risk for selected industries (in this case, Distilled and Blended Liquors (SIC 2085) and Retail - Variety Stores (SIC 5331)). The shaded areas indicate the 95% confidence interval of the estimates from the standard deviation of the posterior distribution.

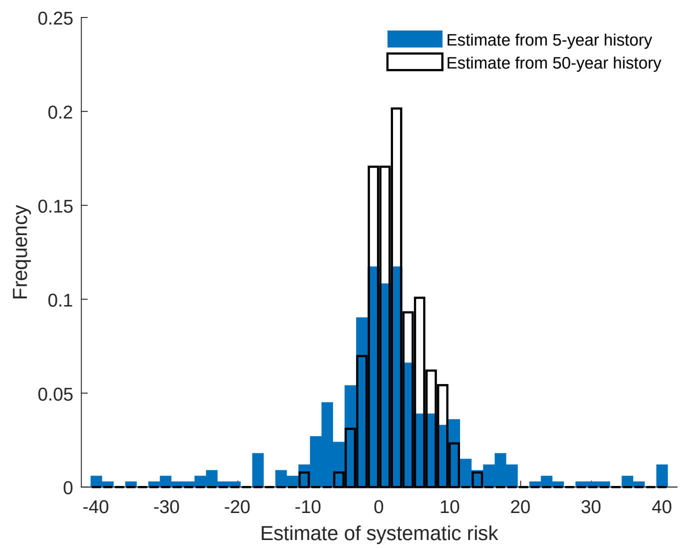
in these early years. This distinction, however, is not surprising because the parameter is updated through the realized productivity that is unique to each industry. As a result, the learning process is remarkably idiosyncratic, although the true exposures of the two industries may be similar as the recent estimates suggest.

Focusing on each industry, we find that the parameter beliefs change substantially over time. As an illustration, the risk estimate for Distilled and Blended Liquors ranges from -1.82 to 4.73. In the entire sample, each industry has, on average, a standard deviation of 4.33, larger than its mean of 1.69. Furthermore, as the confidence interval in the plot indicates, the precision of beliefs improves gradually due to the growing dataset that serves as reference for the learning.

Industries' uniqueness in updating the risk exposure naturally engenders a cross-sectional dispersion in the estimate, as depicted by Figure 5. To highlight the learning impact on the cross-sectional dispersion, we plot two histograms: One shows the distribution of the risk estimates from five-years' worth of data for each industry, and the other shows the distribution of the estimates from fifty-years' worth of data.

The figure reveals a particularly large dispersion in the early estimates obtained from the

Figure 5: Industry Age and the Distribution of Estimated Systematic Risk



This figure presents the cross-sectional distribution of the systematic-risk estimates across industries. Plotted are two histograms: one shows the distribution of the estimates from five-years' worth of data since the inception of each industry, and the other shows the distribution of the estimates from fifty-years' worth of data.

five-years' worth of data. This is plausible because these beliefs are formed based on just a few observations, thus particularly suffering from measurement errors. We therefore observe rather extreme values such as 40 or -40. On the other hand, once plenty of observations are obtained, the precision of beliefs improves. As a result, the distribution of the risk estimate becomes more concentrated, as documented by the histogram of the estimates from the fifty-years' worth of data. Nonetheless, the risk estimates from the long data are still distinct across industries. In the cross-section of these estimates, the standard deviation is 3.86, comparable to the mean of 2.35. This heterogeneity underscores the need to consider at the industry level in identifying the risk-exposure parameter.

In summary, we find that each industry has distinctively updated its respective risk exposure. Considering the uniqueness of the learning path, a finding of firm observables responding to this industry-specific history will offer strong evidence that the learning takes place in practice. We empirically test our model predictions in the following section.

4.3. Empirical Tests of Model Implications

4.3.1. Implications for Investment

We now test whether capital investment exhibits the learning-induced regularities. If decision makers, in practice, learn about a firm's exposure to systematic risk, we expect the firm's investment to respond to the posterior beliefs about the parameter as in Corollaries 1 and 2. To test this hypothesis, we conduct the following predictive regression:

$$\text{INVEST}_{i,t} = \alpha_i + \beta_1 \times m^{\mathbb{I}}_{b,t-1} + \beta_2 \times \left(1/\sigma^{\mathbb{I}}_{b,t-1}\right) + \gamma \times \text{Controls} + \epsilon_{i,t} \tag{13}$$

for which $\text{INVEST}_{i,t}$ denotes the firm i's investment-capital ratio. Other controls include variables that have been found in the literature to affect the investment: namely, firms' size, age, Tobin's q, cash flow, leverage ratio, and the indicator of financial constraints and industries' Herfindahl index.

In Table 3, we report our regression results. In specification (1), we use four-digit SIC codes for industry identification and obtain the time-series of beliefs about systematic risk for each industry. The first main finding is that capital investment responds negatively to shifts in the posterior mean, $m^{\mathbb{I}}_{b,t}$, with a strong significance documented by the t-statistic of -5.20.

Furthermore, this association is economically significant. The coefficient estimate suggests that a rise in the systematic-risk estimate by one standard deviation decreases investment by 9.5% (i.e., the annual investment-capital ratio changes from 0.217 to 0.196). Considering that the firm fixed effect is taken into account in this regression, this negative coefficient reveals the time-series response of firms to changes in $m^{\mathbb{I}}_{b,t}$. Hence, this finding implies that firms reduce (raise) capital investment when beliefs about the systematic risk is revised upward (downward).

Moreover, this negative association persists under alternative industry classifications. In specifications (2) and (3), we refer to NAICS or TNIC, instead of SIC, to identify industry peers and estimate industries' risk exposure accordingly. In these specifications, we find that $m^{\mathbb{I}}_{b,t}$ continues to be a negative predictor of the investment at a 1% level of significance; the t-statistic of NAICS (TNIC) measure is -4.63 (-2.91). All of these findings strongly support Corollary 1.

Apart from the posterior mean, the precision also characterizes the belief distribution. We find that the belief precision also significantly affects the investment, confirming Corollary 2. In all of the specification (1) through (3), the coefficient on $1/\sigma^{\mathbb{I}}_{b,t}$ is positive and statistically significant with t-statistics that range from 1.69 to 5.13. Concerning the economic magnitude, an improvement in the precision of beliefs by one standard deviation raises investment by 7.2% (e.g., the annual investment-capital ratio increases from 0.217 to 0.233). Consistent with the model prediction, firms indeed invest more as decision-makers are less ambiguous about the risk exposure.

One might suspect that the response to this precision is due to effects distinct from the learning. A possible alternative force is the impact of within-industry competition. According to equation (8), the precision increases monotonically with the number of constituent firms in an industry, which likely reflects the amount of competition. In turn, this elevated competition potentially influences capital investment as found by Ghosal and Loungani (1996). To address this alternative explanation, we include the Herfindahl index in the regression. The coefficient on the Herfindahl index is found positive, suggesting that firms increase investment when facing heightened competition. More importantly, after controlling for this competition measure, the precision remains significant for predicting investment. This finding corroborates the model prediction, apart from the force of competition, that the parameter precision concerns firms' investment decisions.

Next, we examine another regularity that emerges from updating beliefs about the systematic risk. As described in Corollary 3, this learning mechanism causes a peculiar pattern in investment-cash flow association; firms reduce investment when the systematic component accounts for a larger part in cash-flow growth. By contrast, if the risk exposure were known with certainty, this composition of the growth would have no impact on investment, as shown in equation (12).

To test this model prediction, we slightly modify the regression equation (13). Among the explanatory variables, we decompose the one-year lagged estimate of cash flow into the two-year lagged estimate and its growth rate from $t-2$ to $t-1$. This is to examine the role of cash-flow growth in explaining investment.

Table 4 reports the regression results. First, specification (1) confirms that the investment is positively associated with the level of cash-flow growth, consistent with the literature. It turns out, however, that not every cash-flow growth encourages the investment. In specifications (2) through (4), we find that the fraction of systematic component in the growth negatively predicts the investment. This negative connection is highly significant with the 1% level in all specifications. This novel regularity, which is uniquely predicted by the learning mechanism, further strengthens our proposition that decision-makers' learning about the systematic risk forms corporate investment.

4.3.2. Implications for Valuation

We now shift our focus to firms' valuation as another dimension that manifests the parameter learning. Specifically, we test Corollaries 4 and 5 using the regression specification,

$$\mathrm{MB}_{i,t} = \alpha_i + \beta_1 \times m^{\mathbb{I}}_{b,t-1} + \beta_2 \times \left(1/\sigma^{\mathbb{I}}_{b,t-1}\right) + \gamma \times \text{Controls} + \epsilon_{i,t} \tag{14}$$

where $\mathrm{MB}_{i,t}$ is the firm i's market-to-book ratio. Controls variables are firms' size, age, return on equity, and leverage ratio and industries' Herfindahl index, which the literatures has noted to influence the valuation ratio.

Table 5 reports the regression results. We find that the market-to-book ratio also responds to the learning-related variables as hypothesized. For all industry classifications, the posterior mean of systematic risk negatively predicts $\mathrm{MB}_{i,t}$ at 1% level. At the same time, the precision of beliefs is a positive predictor of the valuation ratio at the 5% significance level in most specifications except for specification (1). These findings indicate that as the systematic-risk estimate is revised upward or downward through the learning, the firm value changes in accordance with the prediction. In addition, the market evaluates firms higher as the learning alleviates the parameter uncertainty. This evidence corroborates Corollaries 4 and 5.

In summary, we establish that empirical data for both investment and valuation display the time-series regularities induced by the collective learning about the systematic risk. Our evidence strongly suggests that decision-makers, both inside and outside firms, constantly revise their beliefs about the firms' risk profile.

4.3.3. Implications for Implied Cost of Capital

The concerted response of both investment and valuation to beliefs about systematic risk emerges from the parameter's impact on the discount rate. To see this connection more directly, we here examine the predictability of the parameter beliefs with respect to the cost of capital. In the spirit of the empirical asset pricing literature, we focus on whether the learning-related variables cause a cross-sectional dispersion of expected returns, testing Corollaries 6 and 7. The predictability in the cross-section would reveal that market participants compare the up-to-date perceived risk across firms for valuing assets in every time-snapshots.

In this test, we measure the cost of capital utilizing accounting data as suggested by Hou et al. (2012). The rationale for choosing the implied cost of capital over realized returns is two fold. First, realized returns are a noisy proxy for the discount rate, as pointed out by Blume and Friend (1973) and Elton (1999). Second, it is more sensible to link the time-t parameter beliefs to the implied cost of capital that we can also measure in a snapshot at time t. On the contrary, the realized returns from time t to $t+1$ are from the valuation of firms at two different points in time, which engage different beliefs. This fact complicates the empirical test.

Let $\mathrm{ICC}_{i,t}$ denote firm i's annualized cost of capital implicit in time-t accounting variables. We cross-sectionally regress this cost-of-capital estimate onto the parameter beliefs:

$$\mathrm{ICC}_{i,t} = \lambda_{0,t} + \lambda_{1,t} \times m^{\mathbb{I}}_{b,t} + \lambda_{2,t} \times \left(1/\sigma^{\mathbb{I}}_{b,t}\right) + \epsilon_{i,t}. \tag{15}$$

This cross-sectional regression is repeated every year and we calculate time-series average of coefficient estimates as in Fama and MacBeth (1973).

Table 6 presents the regression results. In specifications (1) through (3), the cost of capital is regressed only on the belief distribution – the mean and precision of beliefs about the risk exposure, obtained from the collective learning. Speaking of the precision impact first, we find that the belief precision is connected negatively to the cost of capital at 1% or 5% significance level. Economically, an increase in the precision by one standard deviation reduces the cost of capital by 0.7%. We interpret this finding as evidence that the market indeed penalizes ambiguity about firms' systematic risk and that the learning alleviates this penalty. This evidence is consistent with Corollary 7, supporting our proposition that market participants engage in learning about this parameter.

Furthermore, the posterior mean $m^{\mathbb{I}}_{b,t}$ is found to relate positively to the cost of capital. For all of the industry classifications, this positive connection is significant both statistically and economically; an increase in the risk estimate by one standard deviation raises the cost of capital by 0.9%. This positive association lends strong support for Corollary 6. Market participants indeed require a higher return on firms of which industry-level history implies a greater exposure to systematic risk.

In specifications (5) through (7), we additionally control for other risk factors noted by Fama and French (2015). We find that the predictive power of the learning-related variables is robust to the inclusion of these additional factors in all specifications. This robustness confirms that the belief distribution conveys return-relevant information that are not captured by the existing risk factors.

Although the above result may seem obvious from theoretical perspectives, prior studies have documented mixed results on the empirical connection between the consumption beta and the cross-section of expected returns. The association is found positive in Bansal et al. (2005) and Da (2009), whereas it is insignificant in Lettau and Ludvigson (2001) and Boguth and Kuehn (2013). We conjecture that this inconsistency arises due to difficulty in reliable identification of the consumption beta. Firms' cash flows, or productivity in our setting, contain idiosyncratic innovation that adds a substantial noise to a signal in this learning context, as discussed in section 4.3.4. As a result, we can reliably estimate the risk exposure only when we use a sufficient number of observations, such as analyzing at portfolio level (Bansal et al. (2005)) or using a very long firm-level data including future earnings (Da (2009)).

Our finding supports this interpretation. Note that the parameter beliefs that significantly predict discount rates in specifications (1) through (3) and (5) through (7) are product of the collective learning. In this particular form of learning, firms make use of their peers' observation, thus updating the beliefs through larger dataset than otherwise. To contrast, we try forcing each firm to learn from its own history only without reference to its peers. We then test whether this alternative beliefs from the individual learning predict the cost of capital. The result, reported in specifications (4) and (8), is that individual learning only leads to insignificant association between the perceived risk exposure and the cost of capital. This distinction in the predictability highlights that utilizing collective observations is critical to the determination of systematic risk. Further comparison between collective and individual learning is provided in the following section.

4.3.4. Is the Learning Collective or Individual?

We have documented, theoretically and empirically, that the parameter beliefs about firms' systematic risk have a widespread impact on firms' decisions and valuations. In obtaining these findings, we make an assumption as to which data is relevant for updating the risk-exposure estimate. Specifically, we assume that a target firm's peer observations are also informative about its risk profile, because constituents in one industry tend to have similar, if not identical, exposure to the systematic risk. Accordingly, this learning takes place at the collective level.

Meanwhile, one may conjecture another form of learning: that of individual learning, in which each firm's own history only is used without reference to peers. This alternative form is worth considering because the classification systems, which we employ to identify industries, might be only loosely defined. In other words, even firms in the same industry might have differential business profiles upon a closer look, so peers' observations might not accurately reflect each other's systematic risk. If this is indeed the case, focusing instead on individual history would result in a more precise estimate. Considering this possibility, we test whether the parameter beliefs $m^i_{b,t}$ and $1/\sigma^i_{b,t}$ from individual learning predict the investment and the valuation.

In Tables 3 and 5, we use specification (4) to report our regression results. Surprisingly, it turns out that $m^i_{b,t}$ is insignificant for predicting both the investment-capital ratio and the market-to-book ratio with t-statistics of 0.69 and -0.31, respectively. This is in stark contrast to the 1% level of significance of $m^{\mathbb{I}}_{b,t}$ that is associated with collective learning.

Why do firms not respond to the risk exposure that is estimated from their own history?

Certainly, the insignificance is not because of the theoretical design of the estimator; both individual and collective learning update the parameter through the Bayesian approach. Instead, the main difference between the two forms is the number of observations that enter the learning process. When we include peers' observations as the information source, this collective learning offers decision-makers much richer data from which to learn than does the individual learning. This seemingly mechanical difference plays a crucial role in this context of identifying the systematic risk. Here, the primary source of information, the realized growth in productivity, contains a substantial noise; in the calibrated model, the volatility of idiosyncratic shock (noise) to productivity is approximately 30 times as large as the volatility of the systematic component (signal). Due to the remarkably low signal-to-noise ratio, the reliable identification of the parameter requires a fairly large number of observations. Failing to do so, individual learning leads to an inaccurate risk-estimate that is incapable of explaining firms' decisions.

In summary, we confirm that updating beliefs about systematic risk is a collective process. It is the particular beliefs from collective learning, rather than the estimate that would otherwise emerge from individual history, that exert widespread effects on corporate decisions and market valuations.

5. Robustness Tests

5.1. Time-Variation in True Systematic Risk

One of the stylized assumptions in our model is that each industry's true exposure to systematic risk is constant over time. What causes the risk estimate to fluctuate is not changes in the true risk profile but the parameter learning from growing observations. One may conjecture that, if the true risk exposure itself fluctuates contrary to our assumption, then our learning-based risk estimate might misleadingly capture a variation in the true exposure. If so, our empirical evidence might not be interpreted as evidence of the learning. To address this concern, we explicitly model the dynamics of the true risk exposure and consider the learning in this context.

As an alternative form, we assume that the true risk exposure follows a first-order autoregressive process, specified as

$$b_t^{\mathbb{I}} = \varphi b_{t-1}^{\mathbb{I}} + (1-\varphi)\overline{b}^{\mathbb{I}} + \sigma_b \xi_t, \tag{16}$$

in which ξ_t is a standard normal innovation that is independent of η_t and $\epsilon_{i,t}$. This dynamic exposure $b_t^{\mathbb{I}}$ enters the law of motion of productivity in equation (5), replacing the constant exposure $b^{\mathbb{I}}$. Similar to the baseline model, we here consider the agent that observes neither $\overline{b}^{\mathbb{I}}$ nor $b_t^{\mathbb{I}}$ and instead infer those from the realized productivity.

Appendix E describes the updating of beliefs about the risk exposure in this setting. Briefly speaking, the distribution of the risk-exposure vector $[b_t^{\mathbb{I}} \;\; \overline{b}^{\mathbb{I}}]^T$ is revised over time, through the Kalman filter. To conduct this filtering, we estimate the parameters in equation (16) through the expectation-maximization algorithm. As a result, we obtain the posterior mean of $\overline{b}^{\mathbb{I}}$ conditional on all observations available in time t, denoted by $m_{\bar{b},t}^{\mathbb{I},\,\mathrm{KF}}$, and the standard deviation denoted by $\sigma_{\bar{b},t}^{\mathbb{I},\mathrm{KF}}$. Notice that we focus on the time-evolution of beliefs about the unconditional mean of systematic risk, of which true value is constant by nature. This focus is to isolate the learning effect while addressing variations in true conditional risk exposure. We then test whether the firm observables still respond to these beliefs about the long-run risk exposure.

The regression results are reported in Appendix Table E.7. Interestingly, the results here echo our baseline findings. The posterior estimate of unconditional mean, $m_{\bar{b},t}^{\mathbb{I},\,\mathrm{KF}}$, negatively predicts the investment and the market-to-book ratio, and it positively predicts the cost of capital. Simultaneously, an improvement in the precision, measured by $\left(1/\sigma_{\bar{b},t}^{\mathbb{I},\,\mathrm{KF}}\right)$, increases the investment and valuation, while reducing the cost of capital. Most of these associations are highly significant at the 1% level. These findings suggest that even after fluctuations in true risk exposure are taken into account, the parameter uncertainty about the unconditional systematic risk concerns decision-makers. We therefore conclude that the evolution of beliefs about this parameter are strongly relevant for corporate decisions and market valuations, irrespective of whether the true exposure is static or dynamic.

5.2. Does Industry Classification Matter?

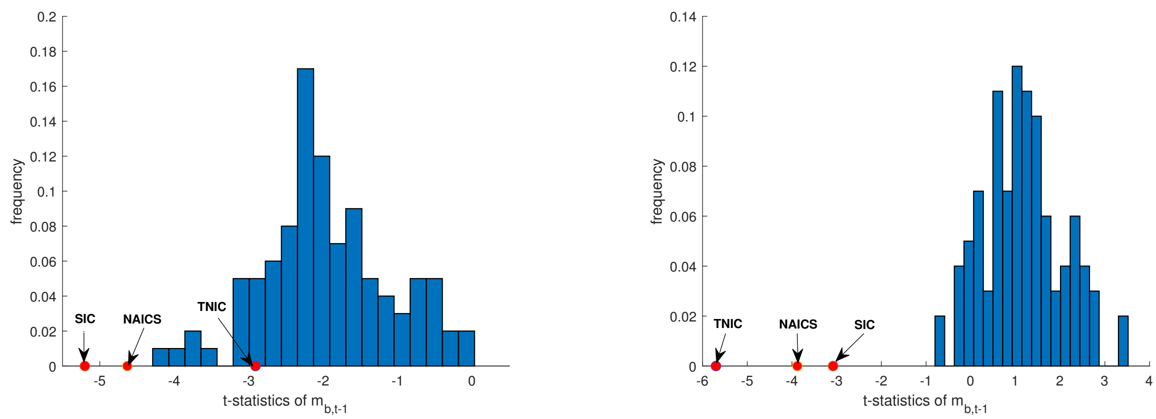
In our main analysis, the identification of each firm's peers depends on the particular systems of industry classifications. In turn, we question the extent to which these classification systems help identify the best sources of information with respect to systematic risk. To be critical on the role of these industry classifications is reasonable for a couple of reasons. First, the industry definition derived from either the SIC or NAICS system might not be as robust as possible because their classification criteria are sometimes chosen in a rather arbitrary way [see Bhojraj et al. (2003) and Weiner (2005)]. Second, if the risk exposure is similar for all firms in the economy irrespective of industry,[5] any group of firms would be informative about the parameter. If this is indeed the case, the industry codes would contribute nothing in identifying the best information source.

Motivated by these conjectures, we examine whether the reference to industry classification is critical for the learning. Essentially, we compare the actual systems of industry classification (i.e., SIC, NAICS, TNIC) to random grouping of firms. The test is designed as follows. First, we create 392 hypothetical industries, so we may match the total number of industries according to the four-digit SIC code. Second, each firm in Compustat is randomly classified into an industry, and each of these counter-factual industries has 38 constituents (i.e., the average number of firms for SIC industries). Once assigned, the industry code is fixed for each firm across time. Next, we let firms learn from past observations of their counter-factual peers and update the systematic risk accordingly. Importantly, firms here use the actual productivity data from Compustat, and we simulate only the grouping of firms. Lastly, we conduct regressions as in equations (13) and (14) to see whether the investment and valuation respond to this risk estimate from counter-factual peers. These steps constitute one simulated case, and we simulate 100 cases to obtain regression coefficients across simulations. Our conjecture is that the risk estimate from these simulated peers would similarly predict the firm observables as does our baseline estimate, if the actual systems of industry classifications are of no use.

[5] It may seem wild guess. However, we cannot rule this out because we do not observe the true values of systematic risk.

Figure 6: The Risk Estimates from Counter-Factual Peers and Firms' Investment and Valuation

Panel A. Investment-capital ratio | Panel B. Market-to-book ratio



This figure shows histograms of 100 estimates of the t-statistics of alternative measures of the systematic risk in investment and valuation regressions. The systematic risk is estimated from the observations of the counter-factual peers that are randomly grouped. The regression specifications are equations (13) and (14). The dots on the x-axis indicate t-statistics found in the regressions when we use actual industry classifications (i.e., SIC, NAICS, TNIC).

In Figure 5.2, we present the histograms of these 100 estimates of t-statistics for the risk estimate in prediction of investment and valuation. We find that the predictive power of the risk estimate from the counter-factual peers is noticeably lower than the baseline estimates. In the investment regression, in which the risk exposure is supposed to predict negatively, the t-statistics for the SIC-based and NAICS-based estimates are lower than all of the counter-factual estimates (which are larger in the absolute magnitude). Similarly, the t-statistic for the TNIC-based estimates are lower than 89 out of the 100 counter-factual estimates.

The significance of the actual industry classifications is even more pronounced in predicting the market-to-book ratio. The estimate from every actual classification outperforms all counter-factual estimates. In contrast, the parameter beliefs formed from hypothetical industries only lead to an insignificant relation between the risk exposure and the valuation ratio; the median of the t-statistics of the counter-factual estimates is 1.11, contradicting the model prediction.

These comparisons confirm that the actual systems of industry classification help determine which observations are informative about the systematic risk. As documented in Figure 5, industries are indeed heterogeneous with respect to the risk exposure. Therefore, without reference to the classification, firms are destined to use observations irrelevant to the parameter of their interest. As a result, firms cannot update the parameter estimate properly.

This finding lets us conclude that industry classification codes help firms identify their peers with similar risk exposure, despite some drawbacks of the classification that the literature might have noted. With guidance, decision-makers can learn about firms' systematic risk much more efficiently than they could otherwise.

6. Conclusion

Parameter uncertainty is present for virtually any decision in financial markets. Among parameters characterizing firms' operation, the systematic-risk exposure is particularly hard to identify quickly. The realized productivity, which is informative of the parameter in the production economy, is primarily driven by idiosyncratic innovation that acts as noise hampering the parameter identification. However, considering its essentiality in numerous decisions, decision-makers would strenuously engage in determining a firm's risk exposure. In this context, we propose a collective-learning framework in which beliefs about a target firm's risk profile is revised through its peers' observations.

Our model predicts that this learning process causes endogenous shifts in discount rates, in turn generating unique regularities in firm observables. Our empirical analysis strongly supports the model predictions. First, a revision in the mean of beliefs about systematic risk negatively predicts capital investment and market valuation, whereas the cost of capital responds positively. Second, ambiguity about this parameter creates additional risks that the market penalizes, and therefore an improvement in the precision of parameter beliefs lower the required return. Consistently, the reduced uncertainty raises investment and valuation. We further find that these empirical findings hold only when the parameter beliefs are updated through collective observations; otherwise, an attempt to shape the beliefs from each firm's individual history only leads to insignificant connections to the firm observables.

Nevertheless, our study does not provide a complete picture of how learning interacts with firms' decisions. A possible extension of our study is to incorporate firms' endogenous entry or exit. The decision on entry or exit, which will change the dataset for the learning, is likely to correlate with the aggregate state of the economy. This correlation, in turn, could amplify or mitigate the risk associated with the parameter uncertainty. We leave the exploration of this question to a future study.

Table 1: Calibration Results

This table presents the calibrated model parameters and the resulting moments on the simulated firm panel. The parameters in Panel A are used to simulate the quarterly time-series of the aggregate state of the economy and the firm-level variables. All moments in panel B are annualized.

Panel A. Parameters

Description	Parameter	Value
Rate of time preference	β	0.996
Relative risk aversion	γ	10
Elasticity of intertemporal substitution	ψ	2
Unconditional mean of consumption growth	g	0.005
Capital share of production	α	0.65
Depreciation rate	δ	0.03
Exposure to systematic risk	b	1.98
Volatility of idiosyncratic shock to productivity	ν	0.285
Coefficient for investment-adjustment costs	ϕ	6.5

Panel B. Moments

Moment	Data	Model
Average consumption growth	0.019	0.020
Volatility of consumption growth	0.022	0.025
Autocorrelation of quarterly consumption growth	0.316	0.299
Average risk-free rate	0.009	0.019
Volatility of risk-free rate	0.010	0.003
Average investment-capital ratio	0.211	0.204
Volatility of investment-capital ratio	0.261	0.354
Average stock return	0.072	0.054
Volatility of stock return	0.429	0.360

Table 2: Summary Statistics

This table presents the descriptive statistics of the annualized variables.

Variable	Mean	Std.Dev.	25%	50%	75%
$m^{\mathbb{I}}_{b,t}$ (SIC)	1.694	5.958	-0.911	1.551	4.355
$m^{\mathbb{I}}_{b,t}$ (NAICS)	1.773	4.987	-0.355	1.571	3.979
$m^{\mathbb{I}}_{b,t}$ (TNIC)	2.307	5.968	-0.915	1.807	5.266
$m^{i}_{b,t}$	2.186	16.45	-4.829	0.851	8.027
$1/\sigma^{\mathbb{I}}_{b,t}$ (SIC)	1.305	0.805	0.743	1.146	1.641
$1/\sigma^{\mathbb{I}}_{b,t}$ (NAICS)	1.702	0.960	0.953	1.611	2.290
$1/\sigma^{\mathbb{I}}_{b,t}$ (TNIC)	0.676	0.557	0.265	0.517	0.921
$1/\sigma^{i}_{b,t}$	0.235	0.132	0.116	0.220	0.328
Investment-capital ratio$_{i,t}$	0.217	0.242	0.130	0.211	0.368
Market-to-book ratio$_{i,t}$	1.569	1.375	0.951	1.207	1.736
ICC$_{i,t}$	0.121	0.385	0.033	0.065	0.126
Size$_{i,t}$	5.599	1.192	4.202	5.405	6.852
Cashflow$_{i,t}$	0.116	0.065	0.069	0.105	0.152
Leverage$_{i,t}$	0.247	0.198	0.091	0.231	0.364
Q$_{i,t}$	2.618	4.619	0.341	0.867	2.457
Age$_{i,t}$	2.404	1.042	1.792	2.485	3.178
ROE$_{i,t}$	0.033	0.860	-0.021	0.100	0.188
WW index$_{i,t}$	-0.084	1.464	-0.226	-0.142	-0.046
HHI$_{\mathbb{I},t}$ (SIC)	6.985	7.254	2.948	4.806	8.400
HHI$_{\mathbb{I},t}$ (NAICS)	17.387	33.079	3.971	7.071	12.494
$\beta^{i}_{\text{SMB},t}$	0.733	0.894	0.157	0.641	1.200
$\beta^{i}_{\text{HML},t}$	0.103	1.152	-0.388	0.153	0.651
$\beta^{i}_{\text{RMW},t}$	-0.144	1.416	-0.666	-9.49e^{-3}	0.503
$\beta^{i}_{\text{CMA},t}$	-0.031	0.625	0.557	0.914	1.224

Table 3: Capital Investment and Beliefs about Systematic Risk

This table presents panel regressions of capital investment on its determinant. The regression specification is

$$\text{INVEST}_{i,t} = \alpha_i + \beta_1 \times m^{\mathbb{I}}_{b,t-1} + \beta_2 \times \left(1/\sigma^{\mathbb{I}}_{b,t-1}\right) + \gamma \times \text{Controls} + \epsilon_{i,t}$$

for which $\text{INVEST}_{i,t}$ is firm i's investment-capital ratio. In specifications (1) through (3), beliefs about industry $\mathbb{I}$'s systematic risk – the posterior mean $m^{\mathbb{I}}_{b,t}$ and the precision $1/\sigma^{\mathbb{I}}_{b,t}$ – are calculated based on four-digit SIC codes, four-digit NAICS codes, or the text-based classification system (TNIC). In specification (4), the mean $m^{i}_{b,t}$ and the precision $1/\sigma^{i}_{b,t}$ are alternatively estimated through individual learning. The additional controls include firms' age, Tobin's q, size, leverage, cash flow, and the WW-index and industries' Herfindahl index. The standard errors are clustered by firms. The t-statistics are presented in parentheses below the parameter estimates. *, **, *** denote significance at the 10%, 5%, and 1% levels, respectively.

Specification:	(1)	(2)	(3)	(4)
$m^{\mathbb{I}}_{b,t-1}$(SIC)	-0.0035***			
	(-5.20)			
$1/\sigma^{\mathbb{I}}_{b,t-1}$(SIC)	0.0195***			
	(5.13)			
$m^{\mathbb{I}}_{b,t-1}$(NAICS)		-0.0035***		
		(-4.63)		
$1/\sigma^{\mathbb{I}}_{b,t-1}$(NAICS)		0.0054*		
		(1.69)		
$m^{\mathbb{I}}_{b,t-1}$(TNIC)			-0.0008***	
			(-2.91)	
$1/\sigma^{\mathbb{I}}_{b,t-1}$(TNIC)			0.0088**	
			(2.37)	
$m^{i}_{b,t-1}$				0.0000253
				(0.62)
$1/\sigma^{i}_{b,t-1}$				0.0740***
				(3.94)
$\text{Age}_{i,t-1}$	-0.0156***	-0.0072***	-0.0114***	-0.0104***
	(-9.38)	(-3.63)	(-4.58)	(-4.58)
$\text{Q}_{i,t-1}$	0.0157***	0.0149***	0.0167***	0.0149***
	(38.08)	(31.04)	(33.56)	(31.03)
$\text{Size}_{i,t-1}$	-0.0474***	-0.0433***	-0.0469***	-0.0443***
	(-27.17)	(-22.73)	(-20.84)	(-23.31)
$\text{Leverage}_{i,t-1}$	-0.143***	-0.116***	-0.121***	-0.115***
	(-17.07)	(-12.98)	(-11.33)	(-12.89)
$\text{Cashflow}_{i,t-1}$	0.647***	0.629***	0.543***	0.636***
	(37.47)	(32.84)	(24.79)	(33.22)
WW $\text{index}_{i,t-1}$	-0.0009*	-0.0001	-0.0002	-0.0001
	(-1.71)	(-0.11)	(-0.22)	(-0.05)
$\text{HHI}_{\mathbb{I},t-1}$	0.0016***	0.0018***	0.0016**	0.0018***
	(3.64)	(3.87)	(2.95)	(4.02)
N	121,055	96,995	66,787	96,995
adj. R^2	0.172	0.142	0.169	0.142

Table 4: Capital Investment and Systematic Components in Cash-Flow Growth

This table presents panel regressions of firms' investment. The regression specification is

$$\text{INVEST}_{i,t} = \alpha_i + \beta_1 \times g^{\text{cash flow}}_{i,t-1} + \beta_2 \times \left(\mathbb{1}_n^T \mathbb{F}^{\mathbb{I}}_{t-1}/n\right) + \gamma \times \text{Controls} + \epsilon_{i,t}$$

for which $\text{INVEST}_{i,t}$ is firm i's investment-capital ratio. $g^{\text{cash flow}}_{i,t-1}$ is the growth rate from $t-1$ to t of firm i's cash flow. $\mathbb{1}_n^T \mathbb{F}^{\mathbb{I}}_{t-1}/n$ is the industry $\mathbb{I}$'s average fraction of cash-flow growth which the systematic component accounts for, defined in equation (12). This fraction of the systematic component is calculated from industry classifications based on four-digit SIC codes, four-digit NAICS codes, or the text-based classification system (TNIC). The included controls are firms' age, Tobin's q, size, leverage, cash flow, and the WW-index and industries' systematic risk $m^{\mathbb{I}}_{b,t-2}$, the precision of the parameter beliefs $1/\sigma^{\mathbb{I}}_{b,t-2}$, and the Herfindahl index. The standard errors are clustered by firms. The t-statistics are presented in parentheses below the parameter estimates. *, **, *** denote significance at the 10%, 5%, and 1% levels, respectively.

Specification:	(1)	(2)	(3)	(4)
$g^{\text{cash flow}}_{i,t-1}$	0.0634***	0.0634***	0.0621***	0.0551***
	(39.43)	(39.35)	(36.97)	(27.73)
$\frac{\mathbb{1}_n^T \mathbb{F}^{\mathbb{I}}_{t-1}}{n}$ (SIC)		-0.0010***		
		(-4.59)		
$\frac{\mathbb{1}_n^T \mathbb{F}^{\mathbb{I}}_{t-1}}{n}$ (NAICS)			-0.0013***	
			(-4.56)	
$\frac{\mathbb{1}_n^T \mathbb{F}^{\mathbb{I}}_{t-1}}{n}$ (TNIC)				-0.0018***
				(-4.86)
$\text{Cashflow}_{i,t-2}$	0.511***	0.510***	0.500***	0.452***
	(26.90)	(26.76)	(25.45)	(19.08)
Controls	Yes	Yes	Yes	Yes
N	108,383	108,163	98,114	61,710
adj. R^2	0.154	0.154	0.158	0.155

Table 5: Market-to-Book Ratio and Beliefs about Systematic Risk

This table presents panel regressions of the market-to-book ratio on its determinant. The regression specification is

$$\text{MB}_{i,t} = \alpha_i + \beta_1 \times m^{\mathbb{I}}_{b,t-1} + \beta_2 \times \left(1/\sigma^{\mathbb{I}}_{b,t-1}\right) + \gamma \times \text{Controls} + \epsilon_{i,t}$$

for which $\text{MB}_{i,t}$ is firm i's market-to-book ratio. In specifications (1) through (3), beliefs about industry $\mathbb{I}$'s systematic risk – the posterior mean $m^{\mathbb{I}}_{b,t}$ and the precision $1/\sigma^{\mathbb{I}}_{b,t}$ – are calculated based on four-digit SIC codes, four-digit NAICS codes, or the text-based classification system (TNIC). In specification (4), the mean $m^{i}_{b,t}$ and the precision $1/\sigma^{i}_{b,t}$ are alternatively estimated through the individual learning. The additional controls include firms' age, size, leverage ratio, and return on equity and industries' Herfindahl index. The standard errors are clustered by firms. The t-statistics are presented in parentheses below the parameter estimates. *, **, *** denote significance at the 10%, 5%, and 1% levels, respectively.

Specification:	(1)	(2)	(3)	(4)
$m^{\mathbb{I}}_{b,t-1}$(SIC)	-0.0137***			
	(-3.08)			
$1/\sigma^{\mathbb{I}}_{b,t-1}$(SIC)	0.0399			
	(1.44)			
$m^{\mathbb{I}}_{b,t-1}$(NAICS)		-0.0069***		
		(-3.88)		
$1/\sigma^{\mathbb{I}}_{b,t-1}$(NAICS)		0.0571**		
		(2.35)		
$m^{\mathbb{I}}_{b,t-1}$(TNIC)			-0.0095***	
			(-5.70)	
$1/\sigma^{\mathbb{I}}_{b,t-1}$(TNIC)			0.0586**	
			(2.63)	
$m^{i}_{b,t-1}$				-0.00006
				(-0.31)
$1/\sigma^{i}_{b,t-1}$				0.177
				(1.29)
$\text{Size}_{i,t-1}$	-0.190***	-0.172***	-0.203***	-0.169***
	(-16.38)	(-13.93)	(-14.18)	(-13.85)
$\text{Age}_{i,t-1}$	-0.0145	0.0372***	0.0142	0.0387***
	(-1.49)	(2.96)	(0.89)	(2.69)
$\text{ROE}_{i,t-1}$	0.105***	0.125***	0.0962***	0.126***
	(12.26)	(11.34)	(8.82)	(11.44)
$\text{Leverage}_{i,t-1}$	-0.353***	-0.377***	-0.358***	-0.378***
	(-6.73)	(-6.22)	(-5.05)	(-6.25)
$\text{HHI}_{\mathbb{I},t-1}$	-0.0039	-0.0045	0.0016	-0.0041
	(-1.25)	(-1.40)	(0.44)	(-1.30)
N	118,458	94,643	65,770	95,339
adj. R^2	0.040	0.030	0.037	0.029

Table 6: Cross-Section of the Implied Cost of Capital and Beliefs about Systematic Risk

This table presents the Fama-MacBeth regressions of firms' implied cost of capital. In the cross-section, we regress annualized cost of capital in year t ($\text{ICC}_{t,i}$) on the beliefs about industry $\mathbb{I}$'s systematic risk. In specifications (1) through (3), the posterior mean $m^{\mathbb{I}}_{b,t}$ and the precision $1/\sigma^{\mathbb{I}}_{b,t}$ are calculated based on four-digit SIC codes, four-digit NAICS codes, or the text-based classification system (TNIC). In specification (4), the mean $m^{i}_{b,t}$ and the precision $1/\sigma^{i}_{b,t}$ are alternatively estimated through the individual learning. In specifications (5) through (8), we additionally control for the exposures to the risk factors identified by Fama and French (2015). They are the exposure to the size factor $<\beta^{\mathbb{I}}_{\text{SMB},t}>$, the exposure to the value factor $<\beta_{\text{HML},t}>$, the exposure to the profitability factor $<\beta_{\text{RMW},t}>$ and the exposure to the investment factor $<\beta_{\text{CMA},t}>$. The t-statistics are presented in parentheses below the parameter estimates and they are based on Newey and West (1987) adjusted standard errors using two lags. *, **, *** denote significance at the 10%, 5% and 1% levels, respectively.

Specification:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$m^{\mathbb{I}}_{b,t-1}$ (SIC)	0.0015***				0.0013***			
	(2.91)				(2.84)			
$1/\sigma^{\mathbb{I}}_{t-1}$ (SIC)	-0.0044**				-0.0020*			
	(-1.96)				(-1.66)			
$m^{\mathbb{I}}_{b,t-1}$ (NAICS)		0.0014***				0.0008**		
		(3.45)				(2.00)		
$1/\sigma^{\mathbb{I}}_{t-1}$ (NAICS)		-0.0071***				-0.0057***		
		(-3.80)				(-4.55)		
$m^{\mathbb{I}}_{b,t-1}$ (TNIC)			0.0010*				0.0008*	
			(1.66)				(1.87)	
$1/\sigma^{\mathbb{I}}_{t-1}$ (TNIC)			-0.0067***				-0.0038**	
			(-2.95)				(-2.18)	
$m^{i}_{b,t-1}$				0.0004				0.0003
				(1.55)				(1.14)
$1/\sigma^{i}_{t-1}$				-0.0528**				-0.0316**
				(-2.39)				(-2.46)
$\beta_{\text{SMB},t-1}$					0.0071	0.0074	0.0059	0.0085*
					(1.32)	(1.47)	(1.28)	(1.87)
$\beta_{\text{HML},t-1}$					0.0169***	0.0183***	0.0193***	0.0250***
					(7.47)	(7.75)	(6.00)	(7.30)
$\beta_{\text{RMW},t-1}$					-0.0051**	-0.0063***	-0.0073**	-0.0071**
					(-1.99)	(-2.72)	(-2.30)	(-2.64)
$\beta_{\text{CMA},t-1}$					0.0084**	0.0096***	0.0097***	0.0107***
					(4.33)	(5.03)	(4.06)	(4.53)
N	71,544	77,481	55,664	67,396	71,544	77,481	55,664	67,396
adj. R^2	0.004	0.006	-0.002	-0.021	0.023	0.023	0.015	0.013

Appendix A. Wealth-Consumption Ratio

Let W_t denote the time-t wealth of the representative agent. The wealth satisfies the Euler equation,

$$W_t = \mathbb{E}_t\left[\beta^\theta \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \left(\frac{S_{t+1}+1}{S_t}\right)^{-(1-\theta)} (C_{t+1} + W_{t+1})\right]. \tag{A.1}$$

Dividing equation A.2 by time-t consumption and letting S_t denote W_t/C_t, we obtain the equation for the wealth-consumption ratio,

$$S_t^\theta = \mathbb{E}_t\left[\beta^\theta \left(\frac{C_{t+1}}{C_t}\right)^{1-\gamma} (S_{t+1}+1)^\theta\right]. \tag{A.2}$$

Appendix B. Derivation of the Posterior Distribution of Risk Exposure

Suppose that beliefs about the risk exposure $b^{\mathbb{I}}$ conditional on observations until time $t-1$ are normally distributed with mean $m^{\mathbb{I}}_{b,t-1}$ and the standard deviation $\sigma^{\mathbb{I}}_{b,t-1}$. Our goal is to update the beliefs using new observations in time t. We start by linearizing equation (5) with respect to $b^{\mathbb{I}}$ through the Taylor expansion around $m^{\mathbb{I}}_{b,t-1}$. The approximation leads to

$$\underbrace{\ln\frac{A_{i,t}}{A_{i,t-1}} - g - \mu_c - \frac{(m^{\mathbb{I}}_{b,t-1})^2\sigma_c(\omega_{t-1})^2}{2}}_{\equiv \overline{A}_{i,t}} \approx \underbrace{\left(-m^{\mathbb{I}}_{b,t-1}\sigma_c(\omega_{t-1})^2 + \sigma_c(\omega_{t-1})\eta_t\right)}_{\equiv \overline{C}_t} b^{\mathbb{I}} + \nu\epsilon_{i,t}. \tag{B.1}$$

Next, we let $\overline{\mathbb{A}}_t$ denote a $(n \times 1)$ column vector with ith element equal to $\overline{A}_{i,t}$ and $\overline{\mathbb{C}}_t$ denote a $(n \times 1)$ column vector with all elements equal to $\overline{C}_t$. Using these vectors, we calculate the conditional distribution in time t as follows:

$$\begin{aligned}
p\left(b^{\mathbb{I}}|\mathbb{A}_1,\ldots,\mathbb{A}_t,\mathbb{C}_1,\ldots,\mathbb{C}_t\right) &\propto p\left(\mathbb{A}_t|b^{\mathbb{I}},\mathbb{C}_t\right)\times p\left(b^{\mathbb{I}}|\mathbb{A}_1,\ldots,\mathbb{A}_{t-1},\mathbb{C}_1,\ldots,\mathbb{C}_{t-1}\right)\\
&\propto \exp\left(-\frac{(\overline{\mathbb{A}}_t-b^{\mathbb{I}}\overline{\mathbb{C}}_t)^T(\overline{\mathbb{A}}_t-b^{\mathbb{I}}\overline{\mathbb{C}}_t)}{2\nu^2}\right)\exp\left(-\frac{(b^{\mathbb{I}}-m^{\mathbb{I}}_{b,t-1})^2}{2(\sigma^{\mathbb{I}}_{b,t-1})^2}\right)\\
&\propto \exp\left(-\frac{1}{2}\left[\frac{1}{(\sigma^{\mathbb{I}}_{b,t-1})^2}+\frac{\overline{\mathbb{C}}_t^T\overline{\mathbb{C}}_t}{\nu^2}\right]\times\right.\\
&\left.\left[b-\frac{\frac{1}{(\sigma^{\mathbb{I}}_{b,t-1})^2}\times m^{\mathbb{I}}_{b,t-1}+\frac{\overline{\mathbb{C}}_t^T\overline{\mathbb{C}}_t}{\nu^2}\times\overbrace{\left(\overline{\mathbb{C}}_t^T\overline{\mathbb{C}}_t\right)^{-1}\overline{\mathbb{C}}_t^T\overline{\mathbb{A}}_t}^{\equiv\widehat{b_t}}}{\frac{1}{(\sigma^{\mathbb{I}}_{b,t-1})^2}+\frac{\overline{\mathbb{C}}_t^T\overline{\mathbb{C}}_t}{\nu^2}}\right]^2\right).
\end{aligned}\tag{B.2}$$

Thus, beliefs about $b^{\mathbb{I}}$ conditional on time t are normally distributed with the mean and standard deviation,

$$\begin{aligned}
m^{\mathbb{I}}_{b,t} &= \frac{\frac{1}{(\sigma^{\mathbb{I}}_{b,t-1})^2}\times m^{\mathbb{I}}_{b,t-1}+\frac{\overline{\mathbb{C}}_t^T\overline{\mathbb{C}}_t}{\nu^2}\times\widehat{b_t}}{\frac{1}{(\sigma^{\mathbb{I}}_{b,t-1})^2}+\frac{\overline{\mathbb{C}}_t^T\overline{\mathbb{C}}_t}{\nu^2}}\\
\frac{1}{(\sigma^{\mathbb{I}}_{b,t})^2} &= \frac{1}{(\sigma^{\mathbb{I}}_{b,t-1})^2}+\frac{\overline{\mathbb{C}}_t^T\overline{\mathbb{C}}_t}{\nu^2}.
\end{aligned}\tag{B.3}$$

Appendix C. The Ex-Dividend Price of Unit Capital

The first-order condition of the firm's problem given by equation (9) is

$$-1-2\phi\left(\frac{I_t}{K_t}\right)+\underbrace{\mathbb{E}_t\left[M_{t,t+1}\frac{\partial V_{t+1}}{\partial K_{t+1}}\right]}_{\equiv P_{i,t}}=0 \tag{C.1}$$

In addition, from the Envelope theorem,

$$\frac{\partial V_t}{\partial K_t} = \alpha A_t K_t^{\alpha-1} + \phi \left(\frac{I_t}{K_t}\right)^2 + (1-\delta) \underbrace{\mathbb{E}_t \left[M_{t,t+1} \frac{\partial V_{t+1}}{\partial K_{t+1}}\right]}_{\equiv P_{i,t}}. \tag{C.2}$$

Using equation (C.2), we can obtain the ex-dividend value of unit capital

$$\begin{aligned} P_{i,t} &= \mathbb{E}_t \left[M_{t,t+1} \frac{\partial V_{t+1}}{\partial K_{t+1}}\right] \\ &= \mathbb{E}_t \left[M_{t,t+1} \left(\alpha A_{t+1} K_{t+1}^{\alpha-1} + \phi \left(\frac{I_{t+1}}{K_{t+1}}\right)^2 + (1-\delta) P_{i,t+1}\right)\right] \\ &= \mathbb{E}_t \left[M_{t,t+1} \left(\alpha A_{t+1} K_{t+1}^{\alpha-1} + \frac{P_{i,t+1}^2}{4\phi} + \left(1-\delta-\frac{1}{2\phi}\right) P_{i,t} + \frac{1}{4\phi}\right)\right], \end{aligned} \tag{C.3}$$

where the last equality is obtained from substituting the investment-capital ratio from equation (C.1) for the marginal value of capital.

Appendix D. Covariance Between a Revision in the Risk Estimate and Consumption Shock

From equation (8), the revision in the posterior mean of beliefs is

$$m_{b,t+1}^{\mathbb{I}} - m_{b,t}^{\mathbb{I}} = \underbrace{\frac{\frac{\overline{\mathbb{C}}_{t+1}^T \overline{\mathbb{C}}_{t+1}}{\nu^2}}{\frac{1}{(\sigma_{b,t}^{\mathbb{I}})^2} + \frac{\overline{\mathbb{C}}_{t+1}^T \overline{\mathbb{C}}_{t+1}}{\nu^2}}}_{\equiv \mathbb{K}_{t+1}} \times \left(\widehat{b_{t+1}} - m_{b,t}^{\mathbb{I}}\right). \tag{D.1}$$

Using the definition of the sample estimate in equation (B.2) and the approximation in equation (B.1), we express the right-hand side as follows:

$$
\begin{aligned}
&\widehat{b_{t+1}} - m^{\mathbb{I}}_{b,t} \qquad\qquad \text{(D.2)}\\
=&\frac{-\frac{(m^{\mathbb{I}}_{b,t}\sigma_c(\omega_t))^2}{2} + m^{\mathbb{I}}_{b,t}\sigma_c(\omega_{t+1})\eta_{t+1} + (-m^{\mathbb{I}}_{b,t}\sigma_c(\omega_t)^2 + \sigma_c(\omega_t)\eta_{t+1})(b^{\mathbb{I}} - m^{\mathbb{I}}_{b,t}) - \frac{(m^{\mathbb{I}}_{b,t}\sigma_c(\omega_t))^2}{2} + \frac{1}{n}\sum_{i=1}^{n}\nu\epsilon_{i,t+1}}{-m^{\mathbb{I}}_{b,t}\sigma_c(\omega_t)^2 + \sigma_c(\omega_t)\eta_{t+1}}\\
&- m^{\mathbb{I}}_{b,t}\\
\approx&\frac{-b^{\mathbb{I}} m^{\mathbb{I}}_{b,t}\sigma_c(\omega_t)^2 + (b^{\mathbb{I}} - m^{\mathbb{I}}_{b,t})\sigma_c(\omega_t)\eta_{t+1} + \frac{1}{n}\sum_{i=1}^{n}\nu\epsilon_{i,t+1}}{\sigma_c(\omega_t)\eta_{t+1}},
\end{aligned}
$$

where the last approximation is based on the fact that $\sigma_c(\omega_t) << 1$, so its quadratic terms can be ignored.

Next, the covariance of our interest is

$$
\begin{aligned}
&\mathrm{cov}_t\left(m^{\mathbb{I}}_{b_t,t+1} - m^{\mathbb{I}}_{b_t,t}, \sigma_c(\omega_{t+1})\eta_{t+1}\right) \qquad\qquad \text{(D.3)}\\
\approx&\mathbb{E}_t\left[\mathbb{K}_{t+1}\left(\widehat{b_{t+1}} - m^{\mathbb{I}}_{b,t}\right)\sigma_c(\omega_t)\eta_{t+1}\right] - \mathbb{E}_t\left[\mathbb{K}_{t+1}\left(\widehat{b_{t+1}} - m^{\mathbb{I}}_{b_t,t}\right)\right]\underbrace{\mathbb{E}_t\left[\sigma_c(\omega_t)\eta_{t+1}\right]}_{=0}\\
=&-\mathbb{E}_t\left[\mathbb{K}_{t+1}b^{\mathbb{I}} m^{\mathbb{I}}_{b,t}\sigma_c(\omega_t)^2\right] + \mathbb{E}_t\left[\mathbb{K}_{t+1}(b^{\mathbb{I}} - m^{\mathbb{I}}_{b,t})\sigma_c(\omega_t)\eta_{t+1}\right] + \mathbb{E}_t\left[\mathbb{K}_{t+1}\frac{1}{n}\sum_{i=1}^{n}\nu\epsilon_{i,t+1}\right]\\
=&-(m^{\mathbb{I}}_{b,t})^2\sigma_c(\omega_t)^2\mathbb{E}_t\left[\mathbb{K}_{t+1}\right] + \mathbb{E}_t\left[\mathbb{K}_{t+1}\sigma_c(\omega_t)\eta_{t+1}\right]\underbrace{\mathbb{E}_t\left[b^{\mathbb{I}} - m^{\mathbb{I}}_{b,t}\right]}_{=0} + \mathbb{E}_t\left[\mathbb{K}_{t+1}\right]\underbrace{\mathbb{E}_t\left[\frac{1}{n}\sum_{i=1}^{n}\nu\epsilon_{i,t+1}\right]}_{=0}\\
=&-(m^{\mathbb{I}}_{b,t})^2\sigma_c(\omega_t)^2\mathbb{E}_t\left[\mathbb{K}_{t+1}\right],
\end{aligned}
$$

where the second last line obtains from the fact that η_{t+1} and $\mathbb{K}_{t+1}$ are independent of $\epsilon_{i,t+1}$ and $m^{\mathbb{I}}_{b,t}$. Furthermore, from the definition of $\mathbb{K}_{t+1}$ in equation (D.1), it can be easily seen that $\mathbb{K}_{t+1}$ increases with $\sigma^{\mathbb{I}}_{b,t}$, for every $\overline{\mathbb{C}}_t$. Hence, $\mathbb{E}_t\left[\mathbb{K}_{t+1}\right]$ also increases with $\sigma^{\mathbb{I}}_{b,t}$.

Appendix E. Learning about the Risk Exposure when the True Parameter is Dynamic

In this section, we formulate the updating of beliefs about risk exposure when the true parameter changes over time. To facilitate the formulation, we express the law of motion of systematic risk described in equation (16) using a state-space representation:

$$\underbrace{\begin{bmatrix} b^{\mathbb{I}}_{t+1} \\ \bar{b}^{\mathbb{I}} \end{bmatrix}}_{\equiv B_{t+1}} = \underbrace{\begin{bmatrix} \varphi & 1-\varphi \\ 0 & 1 \end{bmatrix}}_{\equiv \Phi} \underbrace{\begin{bmatrix} b^{\mathbb{I}}_{t} \\ \bar{b}^{\mathbb{I}} \end{bmatrix}}_{\equiv B_{t}} + \sigma_b \begin{bmatrix} \xi_{t+1} \\ 0 \end{bmatrix}. \tag{E.1}$$

Suppose that B_t conditional on all observations until time t is normally distributed with mean $m^{\mathbb{I}}_{B,t}$ and covariance $\Sigma^{\mathbb{I}}_{B,t}$. Our goal is to update beliefs about B_{t+1} using new observations in time $t+1$ along with the above transition equation.

From the new observation, we calculate the sample estimate $\widehat{b_{t+1}}$ defined in equation (B.1). This estimate can be written as

$$\begin{aligned} \widehat{b_{t+1}} &= \left(\overline{\mathbb{C}}^T_{t+1}\overline{\mathbb{C}}_{t+1}\right)^{-1}\overline{\mathbb{C}}^T_{t+1}\overline{\mathbb{A}}_{t+1} \\ &= \left(\overline{\mathbb{C}}^T_{t+1}\overline{\mathbb{C}}_{t+1}\right)^{-1}\overline{\mathbb{C}}^T_{t+1}\overline{\mathbb{C}}_{t+1}b^{\mathbb{I}}_{t+1} + \left(\overline{\mathbb{C}}^T_{t+1}\overline{\mathbb{C}}_{t+1}\right)^{-1}\left(\nu\sum_{i=1}^{n}\overline{C}_{t+1}\epsilon_{i,t+1}\right) \end{aligned} \tag{E.2}$$

Consequently, $\mathbb{E}_t\left[\widehat{b_{t+1}}\right] = b^{\mathbb{I}}_{t+1}$, and $var_t\left(\widehat{b_{t+1}}\right) = \nu^2/\left(n^2\overline{C}^2_{t+1}\right)$. Based on these properties of $\widehat{b_{t+1}}$, we can obtain an observation equation

$$\widehat{b_{t+1}} = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{\equiv N} \underbrace{\begin{bmatrix} b^{\mathbb{I}}_{t+1} \\ \bar{b}^{\mathbb{I}} \end{bmatrix}}_{=B_{t+1}} + \underbrace{\frac{\nu}{n\sqrt{\overline{C}^2_{t+1}}}}_{\equiv s_{t+1}}\epsilon_{\mathbb{I},t+1} \tag{E.3}$$

where $\epsilon_{\mathbb{I},t+1}$ is a standard normal innovation that is independent of the innovation ξ_{t+1} in B_{t+1}.

We now have the state-space model consisting of transition equation (E.1) and observation equation (E.3). Applying the Kalman filter, we can derive the distribution of B_{t+1} conditional on all observations until time $t+1$. To apply the filter, we calculate the relevant statistics:

$$cov_t\left(B_{t+1}, \widehat{b_t}^T\right) = \left(\Phi\Sigma_{B,t}\Phi^T + \begin{bmatrix} \sigma_b^2 & 0 \\ 0 & 0 \end{bmatrix}\right) N^T \equiv \Sigma_{B,t+1|t}N^T,$$

$$var_t\left(\widehat{b_t}\right) = N\Sigma_{B,t+1|t}N^T + s_{t+1}^2. \tag{E.4}$$

Finally, based on the property of a joint-normal distribution, B_{t+1} conditional on new observation $\widehat{b_{t+1}}$ in addition to the previous information is also normally distributed. Its mean and covariance are updated as follows:

$$m_{B,t+1} = \Phi m_{B,t} + \left[cov_t\left(B_{t+1}, \widehat{b_t}^T\right)\right]\left[var_t\left(\widehat{b_t}\right)\right]^{-1}\left(\widehat{b_t} - N\Phi m_{B,t}\right) \tag{E.5}$$

$$= \Phi m_{B,t} + \underbrace{\left[\Sigma_{B,t+1|t}N^T\right]\left[N\Sigma_{B,t+1|t}N^T + s_{t+1}^2\right]^{-1}}_{\equiv \mathbb{K}_{t+1}}\left(\widehat{b_t} - N\Phi m_{B,t}\right),$$

$$\Sigma_{B,t+1} = \left(I - \mathbb{K}_{t+1}N\right)\Sigma_{B,t+1|t}. \tag{E.6}$$

Of course, this filtering depends on the model parameters. We estimate the parameters $\left(\varphi^{\mathbb{I}}, \sigma_b^{\mathbb{I}}, \nu^{\mathbb{I}}\right)$ for each industry using the expectation-maximization algorithm. As a result, the time-t posterior beliefs about industry $\mathbb{I}$'s systematic risk, $B_t^{\mathbb{I}}$, are normally distributed with the following conditional mean and covariance:

$$m_{B,t}^{\mathbb{I},\text{ KF}} \equiv \begin{bmatrix} m_{b_t,t}^{\mathbb{I},\text{ KF}} \\ m_{\bar{b},t}^{\mathbb{I},\text{ KF}} \end{bmatrix}, \quad \Sigma_{B,t}^{\mathbb{I},\text{ KF}} \equiv \begin{bmatrix} \left(\sigma_{b_t,t}^{\mathbb{I},\text{ KF}}\right)^2 & \rho\sigma_{b_t,t}^{\mathbb{I},\text{ KF}}\sigma_{\bar{b},t}^{\mathbb{I},\text{ KF}} \\ \rho\sigma_{b_t,t}^{\mathbb{I},\text{ KF}}\sigma_{\bar{b},t}^{\mathbb{I},\text{ KF}} & \left(\sigma_{\bar{b},t}^{\mathbb{I},\text{ KF}}\right)^2 \end{bmatrix}. \tag{E.7}$$

Table E.7: Response of the Firm Observables to Beliefs about Systematic Risk Estimated from Kalman Filter

In the table, specifications (1) and (2) present panel regressions of firms' investment and valuation. The regression specification is:

$$\text{Dependent Variable}_{i,t} = \alpha_i + \beta_1 \times m^{\mathbb{I},\text{KF}}_{\bar{b},t-1} + \beta_2 \times \left(1/\sigma^{\mathbb{I},\text{KF}}_{\bar{b},t-1}\right) + \gamma \times \text{Controls} + \epsilon_{i,t}$$

for which the dependent variable is either firm i's investment-capital ratio ($\text{INVEST}_{i,t}$) or market-to-book ratio ($\text{MB}_{i,t}$). The beliefs about the unconditional mean of systematic risk – the posterior mean $m^{\mathbb{I},\text{ KF}}_{\bar{b},t}$ and the precision $1/\sigma^{\mathbb{I},\text{ KF}}_{\bar{b},t}$ – are obtained from the Kalman filter, in which the true risk exposure follows an AR(1) as in equation (16). For each of investment and valuation regression, controls in equation (13) and (14) are included; they are firms' age, size, Tobin's q, leverage ratio, cash flow, the WW-index, return on equity, and industries' Herfindahl index. The standard errors are clustered by firms. Specifications (3) present Fama-MacBeth regression of firms' implied cost of capital. For each year t, a cross-sectional regressions is conducted with the specification:

$$\text{ICC}_{i,t} = \lambda_{0,t} + \lambda_{1,t} \times m^{\mathbb{I},\text{KF}}_{\bar{b},t} + \lambda_{2,t} \times \left(1/\sigma^{\mathbb{I},\text{KF}}_{\bar{b},t}\right) + \gamma \times \text{Controls} + \epsilon_{i,t}.$$

We then report the time-series average of these coefficients. The controls included are the exposure to the size factor $<\beta^{\mathbb{I}}_{\text{SMB},t}>$, the exposure to the value factor $<\beta_{\text{HML},t}>$, the exposure to the profitability factor $<\beta_{\text{RMW},t}>$ and the exposure to the investment factor $<\beta_{\text{CMA},t}>$. The t-statistics are presented in parentheses below the parameter estimates and they are based on Newey and West (1987) adjusted standard errors using two lags. *, **, *** denote significance at the 10%, 5%, and 1% levels, respectively.

Specification:	(1)	(2)	(3)
Dependent variable:	$\text{INVEST}_{i,t}$	$\text{MB}_{i,t}$	$\text{ICC}_{i,t}$
$m^{\mathbb{I},\text{ KF}}_{\bar{b},t-1}$(SIC)	-0.0128***	-0.0343***	0.0027***
	(-7.32)	(-2.67)	(2.84)
$1/\sigma^{\mathbb{I},\text{ KF}}_{\bar{b},t-1}$(SIC)	0.0454***	0.1653*	-0.0042***
	(3.97)	(1.94)	(-3.89)
Controls	Yes	Yes	Yes
N	116,191	112,165	68,631
adj. R^2	0.17	0.04	0.03

References

Ai, H., Croce, M., Diercks, A. M., Li, K., 2018. New shocks and the production-baed term structure of equity returns. Review of Financial Studies 31, 2423–2467.

Alti, A., 2003. How sensitive is investment to cash flow when financing is frictionless? Journal of Finance 58, 707–722.

Bansal, R., Dittmar, R. F., Lundblad, C. T., 2005. Consumption, dividends, and the cross section of equity returns. Journal of Finance 60, 1639–1672.

Bansal, R., Yaron, A., 2004. Risk for the long run: a potential resolution of asseet pricing puzzles. Journal of Finance 59, 1481–1509.

Berk, J. B., Green, R. C., Naik, V., 1999. Optimal investment, growth options, and security returns. Journal of Finance 54, 1553–1607.

Bhojraj, S., Lee, C. M., Oler, D. K., 2003. What's my line? a comparison of industry classfication schemes for capital market research. Journal of Accounting Research 41, 745–774.

Blume, M. E., Friend, I., 1973. A new look at the capital asset pricing model. Journal of Finance 28, 19–33.

Boguth, O., Kuehn, L.-A., 2013. Consumption volatility risk. Journal of Finance 68, 2589–2615.

Brealey, R. A., Myers, S. C., Allen, F., 2016. Principles of Corporate Finance, 12th Edition. McGraw-Hill/Irwin, New York, N. Y.

Cappe, O., Moulines, E., Ryden, T., 2011. Inference in Hidden Markov Models. Springer.

Carlson, M., Fisher, A., Giammarino, R., 2004. Corporate investment and asset price dynamics. Journal of Finance 59, 2577–2603.

Chava, S., Purnanandam, A., 2010. Is default risk negatively related to stock returns? Review of Financial Studies 23, 2523–2559.

Cochrane, J. H., 1991. Production-based asset pricing and the link between stock returns and economic fluctuations. Journal of Finance 46, 209–237.

Cooley, T. F., Prescott, E. C., 1995. Frontiers of Business Cycle Research. Princeton University Press, Ch. Economic Growth and Business Cycle.

Cooper, R., Ejarque, J., 2003. Financial frictions and investment: A requiem in q. Review of Economic Dynamics 6, 710–728.

Da, Z., 2009. Cash flow, consumption risk, and the cross-section of stock returns. Journal of Finance 64, 923–956.

Elton, E. J., 1999. Expected return, realized return, and asset pricing tests. Journal of Finance 54, 1198–1220.

Epstein, L. G., Zin, S. E., 1989. Substitution, risk aversion, and the temporal behavior of consumption and asset returns: a theoretical framework. Econometrica 57, 937–969.

Erickson, T., Jiang, C. H., Whited, T. M., 2014. Minimum distance estimation of the errors-in-variables model using linear cumulant equations. Journal of Econometrics 183, 211–221.

Fama, E. F., French, K. R., 1997. Industry costs of equity. Journal of Financial Economics 43, 153–193.

Fama, E. F., French, K. R., 2015. A five-factor asset pricing model. Journal of Financial Economics 116, 1–22.

Fama, E. F., MacBeth, J. D., 1973. Risk, return, and equilibrium - empirical test. Journal of Political Economy 81, 607–636.

Fazzari, S. R., Hubbard, G., Petersen, B., 1988. Financing constraints and corporate investment. Brookings Papers on Economic Activity, 141–195.

Foucault, T., Fresard, L., 2014. Learning from peers' stock prices and corporate investment. Journal of Financial Economics 111, 554–577.

Frank, M. Z., Shen, T., 2016. Investment and the weighted average cost of capital. Journal of Financial Economics 119, 300–315.

Ghosal, V., Loungani, P., 1996. Product market competition and the impact of price uncertainty on investment: Some evidence from us manufacturing industries. Journal of Industrial Economics 44, 217–228.

Gomes, J., Kogan, L., Zhang, L., 2003. Equilibirum cross section of returns. Journal of Political Economy 111, 693–732.

Hoberg, G., Phillips, G., 2010. Product market synergies and competition in mergers and acquisitions. Review of Financial Studies 23, 3773–3811.

Hoberg, G., Phillips, G., 2016. Text-based network industries and endogenous product differentiation. Journal of Political Economy 124, 1423–1465.

Hoshi, T., Kashyap, A., Scharfstein, D., 1991. Corporate structure, liquidity, and investment: Evidence from japanese industrial groups. Quarterly Journal of Economics 106, 33–60.

Hou, K., van Dijk, M. A., Zhang, Y., 2012. The implied cost of capital: A new approach. Journal of Accounting and Economics 53, 504–526.

Johnson, T. C., 2007. Optimal learning and new technology bubbles. Journal of Monetary Economics 54, 2486–2511.

Kuehn, L.-A., Schmid, L., 2014. Investment-based corporate bond pricing. Journal of Finance 69, 2741?776.

Lettau, M., Ludvigson, S., 2001. Resurrecting the (c)capm: A cross-sectional test when risk premia are time-varying. Journal of Political Economy 109, 1238–1287.

Levi, Y., Welch, I., 2016. Assessing cost-of-capital inputs. Working Paper.

Liu, L. X., Whited, T. M., Zhang, L., 2009. Investment-based expected stock returns. Journal of Political Economy 117, 1105–1139.

Newey, W. K., West, K. D., 1987. A simple, positive semidefinite, heteroskedasticity and autocrrelation consistent covariance-matrix. Eonometrica 55, 703–708.

Pastor, L., Veronesi, P., 2003. Stock valuation and learning about profitability. Journal of Finance 58, 1749–1789.

Pastor, L., Veronesi, P., 2009. Learning in financial markets. Annual Review of Financial Economics 1, 361–381.

Weiner, C., 2005. The impact of industry classification schemes on financial research. Working paper.

Weitzman, M., 2007. Subjective expectations and asset-return puzzles. American Economic Review 97, 1102–1130.

Whited, T. M., Wu, G., 2006. Financial constraints risk. Review of Financial Studies 19, 531–559.

Zhang, L., 2005. The value premium. Journal of Finance 60, 67–103.