

# How Does Stock Illiquidity Affect the Informational Role of Option Prices?

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## Abstract

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**Keywords:** Put-Call Parity, Stock Return Predictability, Hedging Costs, Short-Selling.

**JEL Classification:** G11, G12, C13.

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## Abstract

We uncover a mechanical relation in the predictability of stock returns from option prices. If the underlying stock is illiquid, the prices of calls and puts can deviate from the parity relation because of an asymmetry in their hedging costs, and the sign of the resultant implied-volatility spread depends on the expected return of the stock. We test this mechanism by separating the hedging demand from informed trading in options. Our identification strategy relies on the intersection of two sets of data: (i) stock-day pairs with zero option volume, and (ii) banned stocks in the short-sale ban period, when a limited exemption was granted to option dealers for the purpose of hedging. Our results significantly weaken prior evidence on informed trading in the options market.

# I Introduction

It has been argued in prior research that there is segmentation between the stock and options markets and that informed investors are more likely to trade in options before they trade in stocks. This could be because they can take advantage of the leverage implicit in options<sup>1</sup>, or because they can more easily hide their trades in the options market due to greater information asymmetry.<sup>2</sup> The abundant empirical evidence that informed trading occurs in the options market first is primarily based on the ability of option trading volume and implied-volatility spreads, skews, and changes therein, to predict future stock returns.<sup>3</sup>

In contrast, a more incipient strand of research has argued that the options market is not informationally superior to the stock market.<sup>4</sup> We contribute to this emergent research by proposing a mechanism that generates spurious predictability from option-based metrics to future stock returns. In particular, we argue that option dealers adjust call and put prices in order to be consistent with the costs that they expect to incur when hedging their option positions. Therefore, to the extent that it is more costly to hedge a position in a call (put) option when the price of the underlying stock is expected to rise (fall), this can lead the price of the call to be expensive compared to that of the put. This mechanically creates deviations from put-call parity that are consistent with the ability of option-based metrics, such as implied-volatility spreads and skews, to predict stock returns.

The innovation in our argument is that it holds even when information is reflected simultaneously in the stock and options markets, as well as when there is zero trading volume in

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<sup>1</sup> e.g. Back (1992), Biais and Hillion (1994), Ge, Lin, and Pearson (2016).

<sup>2</sup> e.g. Cao and Wei (2008).

<sup>3</sup> The research on the predictive ability of option volume includes Chan, Chung, and Fong (2002), Pan and Poteshman (2006), Roll, Schwartz, and Subrahmanyam (2010), Johnson and So (2012), and Hu (2014), among others. Cremers and Weinbaum (2010) focus on the implied-volatility spread. An, Ang, Bali, and Cakici (2014) examine first differences in implied volatilities, and Xing, Zhang, and Zhao (2010) explore the implied-volatility skew.

<sup>4</sup> e.g. Muravyev, Pearson, and Broussard (2013), Collin-Dufresne, Fos, and Muravyev (2015), Shang (2016), and Goncalves-Pinto, Grundy, Hameed, van der Heijden, and Zhu (2016).

options.<sup>5</sup> It also holds in the absence of price pressure in the stock market.<sup>6</sup>

We formalize our argument by extending the framework of Leland (1985) and Boyle and Vorst (1992). In particular, we consider an option dealer who writes European-type calls and puts on a stock. This option dealer trades on the underlying stock and on a risk-free bond to hedge her positions in options. In the presence of trading costs for stocks, the market is incomplete and the option dealer cannot use the standard replication method to determine the rational prices of options. Instead, we assume that the option dealer chooses optimal self-financing strategies that minimize the expected hedging error, and then endogenously determines the prices of options that are consistent with the costs incurred with the implementation of such strategies.

We then examine whether option prices convey information regarding the expected return of the underlying stock. In order to assess whether the options are mispriced, we measure the extent to which they create deviations from the put-call parity relation, similar to Cremers and Weinbaum (2010). If the put-call parity holds, then the options are fairly priced. If the parity does not hold, then the options are considered to be mispriced. Therefore, we investigate the circumstances under which deviations from put-call parity can emerge, and how such deviations relate to the expected returns of the underlying stock.<sup>7</sup> In the theoretical model, we do not require the information to be asymmetric across investors, or that markets are segmented, like in Easley, O'Hara, and Srinivas (1998). Therefore, the link that we uncover between put-call parity deviations and future stock returns is purely mechanical.

In our model, costly trading is a necessary condition for option prices to be informative

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<sup>5</sup> The fact that our argument holds even in the absence of option trading, makes it consistent with the evidence provided in Chan, Chung, and Fong (2002) that information in the options market is contained only in quote revisions, not in option trades. We also provide empirical evidence in Section V that option trading contributes very little to the predictability of the implied-volatility spread.

<sup>6</sup> Goncalves-Pinto, Grundy, Hameed, van der Heijden, and Zhu (2016) argue that price pressure in stocks is a primary driver of the stock return predictability associated with implied-volatility spreads, skews, and changes therein.

<sup>7</sup> In other words, we study the problem of an investor who seeks to infer the expected return of a stock from the imbalance in the prices of puts and calls quoted by the option dealer.

about the expected return of the underlying stock. In the absence of trading costs, the put-call parity always holds, in which case option prices convey no information about stock returns. However, in the presence of transaction costs, the hedging cost of the call option tends to be higher (lower) than that of the put option when the underlying stock exhibits a positive (negative) expected return. Moreover, the magnitude of the deviation from put-call parity increases with the transaction cost rate, which suggests that the strength of the predictability from option prices to stock returns is positively associated with the degree of illiquidity of the underlying stock. This positive relation has been documented in Cremers and Weinbaum (2010).

The mechanism behind the results in our model is as follows. The objective of the option dealer is to minimize the expected error of the hedging portfolio. In the absence of transaction costs, the option dealer can perfectly replicate a short position on a call by writing a put and taking a short position in a forward contract. In this case, the payoffs of these two portfolios are identical, because the forward contract can be exactly replicated by trading the stock and the bond at zero cost. As a result, the hedging costs of call and put options are tightly linked by the put-call parity relation. However, in the presence of transaction costs for stocks, the replication of the payoff of a forward contract is more costly. In this case, the put-call parity is unlikely to hold, and the relative hedging costs of calls versus puts become a function of the expected return of the underlying stock. Specifically, when the stock price is expected to appreciate in value, the dealer needs to gradually increase her long position in the stock to hedge the call option, and reduce her short position in the stock to hedge the put option. In a rational expectations framework, the dealer is expected to hold *ex-ante* a larger long position in stocks to hedge the position in a call option, and a smaller position in the stock to hedge the put. As a result, the optimal hedging portfolio for a position in a call option is more (less) costly compared to that for a put when the underlying stock price is expected to rise (fall).

We provide empirical evidence that is consistent with the novel predictions of our model. In particular, we test whether trading costs in the stock market affect the hedging costs of option dealers, which in turn affects option prices in a way that creates spurious predictability from option prices to stock returns. In order to test this prediction, we rely on an identification strategy that consists in intersecting two sets of data. First, we identify all the optionable stocks for which there is zero option volume on a given day. This helps us rule out the effect that option trading can have on option prices. Second, we take advantage of the short-selling ban imposed on U.S. financial stocks during the period from September 19 to October 8 of year 2008. This is a relatively clean experiment for the purpose of our empirical test, because during that period a limited exemption was granted by the SEC to option dealers so that they could carry on with their market making and hedging activities (e.g. Grundy, Lim, and Verwijmeren (2012) and Lin and Lu (2016)). Therefore, variation in the short-selling costs for this subset of banned stocks in the short-sale ban period is likely to affect option dealers through their hedging activity, which in turn is expected to affect option prices, as our model would predict. Following Saffi and Sigurdsson (2011), we measure short-sale costs using the equity lending fees.

We start by reporting that an increase in the hedging costs of option dealers, via an increase in the equity lending fees, is associated with a decrease in the implied-volatility spread (i.e. the difference between implied volatilities of call and put options with the same strike and maturity) for the banned stocks in the short-ban period. We use the implied-volatility spread as a proxy for deviations from put-call parity. This result suggests that put options are likely to become relatively more expensive than call options as the cost of hedging the puts increases. We then condition the analysis on option trading and find that the effect of short-selling costs on the implied-volatility spread during days in which option volume is null is about 70% of the effect found on days with positive volume. This suggests that option trading may not be that important of a driver of the relative mispricing in options.

We also study the effect of short-sale costs on the prices of puts and calls separately. We find that an increase in the equity lending fees is associated with an increase in the implied volatility of puts but not of calls. This means that the effect of shorting costs on the implied-volatility spread is driven by its effect on puts. This is expected because short-selling is more tightly linked to the hedging of put options.

Lastly, we show that the implied-volatility spread is a strong predictor of future stock returns for the banned stocks in the short-ban period, and remains strong for the subsample of stocks with zero option volume. Specifically, we find that the predictability of the implied-volatility spread on days with zero option volume is about 80% of that found for days with positive option volume. This suggests that the contribution of option trading to the predictability of the implied-volatility spread is relatively small. This is consistent with the idea that the predictive ability of the implied-volatility spread is due to the adjustment of option prices by option dealers as a result of changing conditions for hedging in the shorting market. This is consistent with Chan, Chung, and Fong (2002) who find that information in the options market is contained only in quote revisions, not in option volume. Our results also suggest that hedging costs are an important determinant of option prices, which is consistent with Muravyev (2016) who estimates that inventory risk and information asymmetry only explain 18% of option spreads, the remaining 82% being attributed to fixed costs such as the cost of hedging.

We conclude that one must account for the effect of hedging costs of options dealers on call and put prices when studying the information contained in the spread between their implied volatilities. We do not completely rule out the possibility of informed trading in options as being an important driver of the predictability from options to stocks. However, the extent to which the return predictability can be attributed to informed trading in options, and to asymmetric hedging costs incurred by option dealers, constitutes an important question for further investigation.

The remainder of the paper proceeds as follows. In the next section we present a review of the related literature. Section III describes our theoretical framework. Section IV discusses the main predictions of our model. We test the novel predictions of our model in Section V, and we conclude with Section VI. All technical issues, as well as some additional results, are relegated to the Appendix.

## II Related Literature

In a complete market, options are redundant securities that can be perfectly replicated in continuous time using stocks and bonds (Black and Scholes (1973)). In this paradigm, there is no role for options to convey any new information to market participants. However, markets are typically incomplete and options cannot be dynamically replicated. As a result, informed investors may prefer to trade options instead of the underlying stocks either because of the implicit leverage they offer (e.g. Back (1992), Biais and Hillion (1994), Ge, Lin, and Pearson (2016)), or because information asymmetry is greater for options than for the underlying stock, making the options market a more efficient venue for trading (Cao and Wei (2008)).

We argue that the stock return predictability derived from option prices is in large part mechanical. We show that the prices of calls and puts are likely to deviate from their parity relation due to an asymmetry in their hedging costs. In particular, in the presence of transaction costs for stocks, the hedging costs are higher (lower) for calls than puts when the expected return of the stock is positive (negative). This study relates to a recent strand of literature that argues that the options market is not informationally more efficient than the stock market (e.g. Muravyev, Pearson, and Broussard (2013), Collin-Dufresne, Fos, and Muravyev (2015), Shang (2016), and Goncalves-Pinto, Grundy, Hameed, van der Heijden, and Zhu (2016)).

We study the impact of stock illiquidity on option prices, which is related to the works



of Cetin, Jarrow, Protter, and Warachka (2006) and Chou, Chung, Hsiao, and Wang (2011). They show that a reduction in stock liquidity is associated with an increase in the level of the implied volatility of options. They argue that their results are attributable to the hedging costs incurred by option dealers, which is consistent with our findings. However, our paper differs from theirs in important respects. The main innovation in our analysis is that, we study the *asymmetric* impact of stock illiquidity on the hedging costs of calls and puts in different states of the underlying stock market. We then show that the asymmetric hedging costs for calls and puts creates differences in their relative prices, which leads to a mechanical relation between the implied-volatility spread between calls and puts and future stock returns. We argue that the predictive ability of implied-volatility spreads for future stock returns is unrelated to informed trading in options, contrary to what has been argued in Cremers and Weinbaum (2010) and An, Ang, Bali, and Cakici (2014), among others. We also provide empirical evidence that is consistent with our argument.

The model we propose in this paper builds on the framework of Leland (1985). However, we restrict our analysis to a set of self-financing hedging strategies so that we can account for the hedging costs incurred at the inception of the option positions. This is a feature that has been overlooked in Leland (1985). We work with an incomplete market in which hedging is inherently risky and imperfect, and in which the expected return of the underlying stock significantly affects the hedging and the pricing of options. This is in contrast with Boyle and Vorst (1992) which uses a binomial model to make the market complete even in the presence of transaction costs. In contrast to Davis, Panas, and Zariphopoulou (1993), Clewlow and Hodges (1997), Monoyios (2004), and Zakamouline (2006), we do not rely on the utility-based pricing of options, because such framework does not allow us to cleanly separate the incentives associated with option hedging from the incentives related to portfolio optimization.<sup>8</sup> We assume that our option dealer minimizes the expected error from hedging

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<sup>8</sup> A typical approach in the literature is to set the expected return of the underlying asset to the riskfree

her option positions, instead of minimizing the initial cost from a set of super-hedging policies like in Edirisinghe, Naik, and Uppal (1993).<sup>9</sup>

### III Theoretical Model

In this section, we present the theoretical framework of the paper. We model the choice of an options dealer who uses the stock market to hedge her option positions. We then examine how market conditions, including trading costs and the return of the stock, affect the costs of the dealer's hedging activities.

In particular, we assume a financial market in which two assets are traded: One is a risk free bond which earns constant interest rate  $r$ , and the other is a risky stock. We assume the bond market is perfectly liquid so trading the bond incurs no cost, while the stock market is less liquid hence trading the stock incurs proportional costs at rate  $\theta \geq 0$ .<sup>10</sup>

We consider an option market maker (the option dealer hereinafter) who writes European options on the stock and uses the stock and the bond to hedge her option positions. Due to the presence of trading costs, continuous reheding is prohibitively expensive. Therefore, following Leland (1985), we assume that the dealer rebalances her hedging portfolio only on a set of discrete time points  $0 = t_0 < t_1 < \dots < t_n = T$ , where  $T$  is the expiration date of the option. For simplicity, we assume these time points are equally spaced so that  $t_i = i\delta t$  for  $0 \leq i \leq n$ , where  $\delta t = T/n$ . During the life of the option, the stock return is assumed to

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rate in the option dealer's optimization problem. This approach is usually based on the argument that "the agent has already optimized her portfolio before pricing the option", which is a nontrivial argument to justify from the outset.

<sup>9</sup> In fact, it has been shown that nontrivial superhedging strategies do not exist with log-normal return and proportional transaction costs, e.g. Soner, Shreve, and Cvitanic (1995). Edirisinghe, Naik, and Uppal (1993) were able to find such strategies because they used a binomial model.

<sup>10</sup> We assume same transaction costs rate for purchase and sale to ensure that our results are not biased towards hedging call or put option.

have a log-normal distribution across the trading times

$$S_i = S_{i-1}e^{\phi_i}, \quad (1)$$

where

$$\phi_i = \left( \mu - \frac{\sigma^2}{2} \right) \delta t + \sigma \sqrt{\delta t} z_i, \quad (2)$$

is the log-return of the stock over the period  $(t_{i-1}, t_i)$ , and  $z_i$ s are i.i.d. standard normal random variables. Therefore,  $\mu$  and  $\sigma$  can be viewed as the annualized expected return, and the return volatility, of the stock.

Denote by  $x_i$  the number of stock shares in the dealer's hedging portfolio at time  $t_i$ , Leland (1985) assumes that the dealer chooses her hedging strategy exogenously according to  $x_i = \frac{\partial V}{\partial S}(t_i, S_i)$  and then prices the option accordingly, where  $V(t, S)$  is the option pricing function in Leland's model. However, Leland's approach suffers from the drawbacks that the resultant hedging strategy is not self-financing (cf. Boyle and Vorst (1992)), and that the costs of forming and unwinding hedging portfolio is not included. Therefore, the initial price of an option cannot fairly measure its hedging costs. To avoid these problems, we deviate from Leland (1985) and assume that the option dealer chooses a series of self-financing hedging strategies  $x_i : 0 \leq i \leq n - 1$  to minimize her time 0 expectation of the squared hedging error at time  $t = T$ , which is defined as follows

$$\min_{\{x_i: 0 \leq i \leq n-1\}} E_0 \left[ (1 - \theta)x_n^+ S_n - (1 + \theta)x_n^- S_n + y_n - f(S_n) \right]^2, \quad (3)$$

where  $x^+ = \max\{x, 0\}$ ,  $x^- = \max\{-x, 0\}$ ,  $y_i$  is the amount invested in the bond at time  $t_i$ , and the expectation is taken under the true probability measure  $P$ .<sup>11</sup>

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<sup>11</sup> Our objective function (3) is similar to that in Cont, Tankov, and Voltchkova (2005), which considers the hedging error minimization problem in the presence of jumps in stock return. However, Cont, Tankov, and Voltchkova (2005) does not include transaction costs in their model, and they assume the option hedger minimizes the expected hedging error under the risk-neutral probability measure  $Q$ , rather than the true

The first three terms in (3) represent the net value of the dealer's hedging portfolio, and the last term is her liability of delivering the option's payoff  $f(S_n)$ . For cash-settlement case, we have the following expressions<sup>12</sup>

$$f(S_n) = \begin{cases} (S_n - K)^+ & \text{for the call option,} \\ (K - S_n)^+ & \text{for the put option.} \end{cases}$$

The self-financing conditions that constrain the dealer's hedging strategy across time are specified as follows:

$$y_i = y_{i-} - (1 + \theta)(x_i - x_{i-})^+ S_i + (1 - \theta)(x_i - x_{i-})^- S_i, \quad (4)$$

$$x_{i+1-} = x_i, \quad (5)$$

$$y_{i+1-} = y_i e^{r\delta t}, \quad (6)$$

where  $x_i - x_{i-}$  in (4) is the number of stock shares that the option dealer trades at reheding time  $t_i$ . In particular, the option dealer purchases shares if  $x_i - x_{i-} > 0$ , and sells shares if  $x_i - x_{i-} < 0$ . (5) implies no reheding during period  $(t_i, t_{i+1})$ , and (6) is due to the accumulation of interest.

We solve the problem using dynamic programming method. For this purpose, we define the dealer's value function as follows

$$J(x_{i-}, y_{i-}, S_{i-}, t_{i-}) = \min_{x_j: i \leq j \leq n-1} E_{t_{i-}} [(1 - \theta)x_n^+ S_n - (1 + \theta)x_n^- S_n + y_n - f(S_n)]^2,$$

where the conditional expectation of the right hand side is taken under filtration  $\mathcal{F}_{t_{i-}}$ . Since

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probability measure  $P$ . We believe that minimizing the hedging error under the true probability measure is more relevant for the option dealer's risk management. Meanwhile, specifying the objective function this way helps us to separate the option dealer's hedging demand from other incentives.

<sup>12</sup> The expressions for asset-settlement case are more complicated. However, the mechanism that drives our main results in the cash-settlement case remains the same, and we have found similar results in the asset-settlement case.

at time  $t_i$ , the dealer determines her new hedging ratio  $x_i$ , the Bellman equation for this dynamic programming problem reads

$$J(x_{i-}, y_{i-}, S_{i-}, t_{i-}) = \min_{x_i} E_{t_i} [J(x_{i+1-}, y_{i+1-}, S_{i+1-}, t_{i+1-})], \quad (7)$$

where on the right hand side we have

$$\begin{aligned} x_{i+1-} &= x_i, \\ y_{i+1-} &= e^{r\delta t} (y_{i-} - (1 + \theta)(x_i - x_{i-})^+ S_i + (1 - \theta)(x_i - x_{i-})^- S_i), \\ S_{i+1-} &= S_i e^{\phi_{i+1}}, \end{aligned}$$

The terminal data is given by

$$J(x_n, y_n, S_n, t_n) = [(1 - \theta)x_n^+ S_n - (1 + \theta)x_n^- S_n + y_n - f(S_n)]^2.$$

Given the initial stock price  $S_0$ , initial stock position  $x_0$  and initial bond position  $y_0$ , the dealer chooses the hedging strategies  $\{(x_i, y_i) : 0 \leq i \leq n - 1\}$  according to objective (3). Moreover, the dealer chooses the optimal initial positions  $(x_0^*, y_0^*)$  such that

$$(x_0^*, y_0^*) = \arg \min_{(x_0, y_0)} J(x_0, y_0, S_0, t_0).$$

In order to obtain an expression for the associated hedging costs, we note that, in order to acquire  $x_0^*$  shares of the stock the dealer needs to pay  $(1 + \theta)x_0^* S_0$  dollars to cover the trading costs, and short selling  $x_0^*$  shares of the stock only generates  $(1 - \theta)x_0^* S_0$  dollars to the dealer. Therefore, the upfront costs of her hedging strategy is then given by

$$\xi^f = (1 + \theta)x_0^{*+} S_0 - (1 - \theta)x_0^{*-} S_0 + y_0^*,$$

where the superscript  $f$  indicates the dependence on the option's payoff function  $f$ . In the following discussions, we use  $\xi^C$  ( $\xi^P$ , resp.) to denote the hedging cost of a single share of call (put, resp.) option.

## IV Model Predictions

In this section we conduct theoretical and numerical analysis to uncover the main predictions of our model. We are primarily interested in when the hedging costs of call or put option are related via the well-known put-call parity relationship, and when such relationship can be violated. In case of violation, we examine how the relative hedging costs of put and call option are related to the expected return of the underlying stock.

### A Liquid Case: $\theta = 0$

First we examine the liquid case in which trading the stock incurs no cost. The following result shows that, in this case, the hedging costs of call option and of put option are related via the put-call parity relationship.<sup>13</sup>

**Proposition 1:** Absent trading costs (i.e.,  $\theta = 0$ ), the hedging costs of put and call option satisfy the parity relationship

$$\xi^C - \xi^P = S_0 - Ke^{-rT},$$

and the expected hedging errors are the same for call and put option.

The proof of Proposition 1 is presented in Appendix A. The intuition behind Proposition 1 is as follows: Absent transaction costs, the option dealer can replicate the writing of a call

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<sup>13</sup> However, one should keep in mind that satisfying put-call parity does not necessarily imply that the hedging costs equal Black-Scholes prices.

option by writing a put option and taking a short position in a forward contract. In this case, the payoff of these two portfolios are identical, and the forward contract can be exactly replicated by trading the stock and bond at zero cost. As a result, the hedging costs of call and put option are linked by put-call parity.

Proposition 1 implies that, in the absence of trading costs, the option prices are unlikely to deviate from the put-call parity, hence should exhibit no predictive power on the underlying's future return.<sup>14</sup> In the following section, we provide numerical analysis to show that this result does not hold anymore in the presence of stock trading costs.

## B Illiquid Case: $\theta > 0$

Now we examine the illiquid case which is of our major interest. In the dynamic model with trading costs, closed form solution is generally unavailable. In order to examine the effect of future stock returns on the costs of option dealer's hedging activities, we conduct, in this section, comprehensive numerical analysis for our model. The numerical results are generated via standard backward induction of Bellman equation (7) with suitably discretized state variables  $(x, y, S)$ .

In particular, we show that stock trading costs can lead to asymmetric hedging costs of the call option and of the put option, and this mechanically results in deviation from the put-call parity. In addition, the asymmetry in hedging costs largely depends on the expected return of the underlying stock, and thus provides spurious predictability on the stock's future return.

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<sup>14</sup> From the proof of Proposition 1, one can observe that this result does not depend on the distribution of the underlying stock's return  $\phi_i$ .

## B.1 Parameter Choice

We first describe the baseline parameter values we use to illustrate our results. For ease of exposition, we use the bond as the numeraire so that we normalize the interest rate to  $r = 0$ . We set initial stock price at  $S_0 = 100$  dollars. We consider at-the-money options only, therefore set  $K = S_0$ . As a result, the put-call parity reads simply  $\xi^C = \xi^P$ , and any case with  $\xi^C \neq \xi^P$  indicates deviation from the put-call parity. In particular, we say that the call option is more expensive to hedge than the put option if and only if  $\xi^C > \xi^P$ .

We assume the option expires in one month with two intermediate periods, i.e.,  $T = 1/12$ ,  $n = 2$  and consequently  $\delta t = 1/24$ . In other words, the option dealer's hedging portfolios are formed at  $t = 0$ , rebalanced at  $t = 1/24$ , and liquidated at  $t = 1/12$ .<sup>15</sup>

We consider two cases to characterize different future performance of the underlying stock. In the Bull case (resp. Bear case), we assume the underlying stock is more likely to rise (resp. drop) in value. We use the following parameter values to distinguish two cases. In the Bull Case (the Bear Case, resp.) we assume a monthly expected return of 0.015 (-0.015, resp.), which translates into annualized value of  $\mu = 0.18$  (-0.18, resp.). In both cases, we set the annualized return volatility at  $\sigma = 0.15$  and trading costs rate at  $\theta = 0.01$ .

We summarize these baseline parameter values in Table I.

[Insert Table I about here]

## B.2 Hedging Portfolios and Trading Costs

Due to the self-financing nature of our admissible hedging policies, the cost of a particular hedging strategy solely depends on the initial choice of stock position and bond position. Therefore, we start by examining how the option dealer chooses her initial positions and we

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<sup>15</sup> We focus our attention on the two-period setting because it is the simplest case which allows us to separately examine the costs of initial portfolio formation, of intermediate portfolio rebalancing, and of the eventual portfolio liquidation. Our analysis can be extended to more periods with the same underlying mechanism at work.



report the results in Table II. Panel A1 and B1 show the results for the Bull Case, and Panel A2 and B2 show the results for the Bear Case.

[Insert Table II about here]

We first re-examine the cases without trading costs (i.e.,  $\theta = 0$ ) in Panel A1 and A2. Being consistent with Proposition 1, the difference in the optimal hedging ratios of call and put equals one, and their associated hedging costs satisfy the put-call parity (recall that with our particular parameter values, they share the same hedging costs). Moreover, the hedging costs do not depend on the stock's expected return. In either the Bull Case or the Bear Case, the hedging costs of call and put turn out to be both 1.70 dollars.

However, when trading costs are introduced to our model, asymmetric hedging costs of call and put can arise. For example, we show in Panel B1 and B2 the optimal choice of initial hedging portfolios and their associated costs when the trading costs rate is 1%. In the Bull Case, the underlying stock is more likely to appreciate in value. In response to such anticipation, the option dealer voluntarily chooses larger long stock position to hedge the call, and smaller short stock position to hedge the put. For example, Panel B1 shows that, when the transaction costs rate is 1%, the dealer chooses a hedging ratio of 0.659 for the call, and of -0.450 for the put. These asymmetric hedging ratios then result in differential upfront trading costs bill, and a spread of 0.41 dollars between the hedging costs for call and put. After further converting the hedging costs into Black-Scholes implied volatility, we find an implied volatility spread of 3.55%.<sup>16</sup>

The mechanism at work can be explained as follows. Since the stock price is more likely to rise in the Bull Case, the option dealer is more likely to purchase more stock shares to hedge her call option position (or to reduce her short position to hedge her put option position) in the future. The option dealer optimally chooses to increase the long stock position for

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<sup>16</sup>In the following analysis, we will always use the implied volatility spread to measure the difference in the hedging costs for call and for put, unless otherwise stated.

the call, and reduces the short stock position for the put, at an earlier time because it can provide two benefits: First, it reduces the average amount of portfolio rebalancing triggered by future stock price changes and the associated risk for option hedging; Second, it helps alleviate the impact of trading costs upon the final liquidation of the hedging portfolio by distorting the initial stock position towards the position at the time of liquidation.

Similarly, Panel B2 shows that in the Bear Case, the presence of trading costs makes the put more expensive to hedge than the call. The reason is analogous to that for the Bull Case: in response to the more likely scenario that the stock price will drop, the option dealer optimally reduces her long position to hedge the call and increases her short position to hedge the put, which makes it more costly to hedge the put option.

In addition to the relative hedging costs of call and put, we can also examine how the hedging strategy and costs of a particular option (the call option, for example) changes with the expected return of the stock. We find that the hedging ratios are positively related to the stock's expected return. For example, assuming a trading costs rate of 1%, for a call option, the initial hedging ratio is 0.659 in the Bull Case, and it reduces to 0.484 in the Bear Case. As a result, when the stock has good expected future return, the dealer buys more stock shares *ex ante* to hedge call option than she does when the stock has poor expected return. This indicates heavier liquidity costs of hedging the call option when the expected return is high than that when the expected return is low. This result is consistent with the finding that changes in the prices of particular options (in the form of changes in their implied volatilities) can predict future return of the underlying stock, as empirically documented in An, Ang, Bali, and Cakici (2014).

### **B.3 Expected Trading Cost Charge**

In this section, we provide an alternative way to understand our main results by calculating the average transaction costs bill incurred when hedging option positions. We note that,

during the entire hedging process in our model, trading costs are paid when the hedging portfolio is formed initially, is rebalanced intermediately, and is unwound eventually. In order to examine which part(s) of trading costs bills contributes most to the asymmetric hedging cost of call and put option, we calculate the discounted expected trading costs bill for each step during the hedging process. In particular, in the two-period case, these costs can be calculated as follows: The trading costs of portfolio formation is clearly  $\theta|x_0|S_0$ , the expected trading costs of portfolio rebalancing is

$$e^{-rT/2}E[\theta|x_1(S_1; x_0, y_0) - x_0|S_1],$$

and the expected trading costs of portfolio liquidation is

$$e^{-rT}E[\theta|x_1(S_1; x_0, y_0)|S_2],$$

where  $x_1(S; x_0, y_0)$ , as a function of  $S$ , is the optimal hedging strategy at time  $t = T/2$ , given initial hedging position  $(x_0, y_0)$ .

[Insert Table III about here]

We show in Table III the results. As can be seen, the major difference in the trading costs bills incurred is attributed to the difference in the costs of portfolio formation and unwinding, and the costs of intertemporal reheding do not differ much.<sup>17</sup> For example, in the Bull Case, acquiring the initial hedging portfolio of call option costs extra 0.21 dollars, and liquidating the final hedging portfolio costs extra 0.31 dollars. At the same time, rebalancing the hedging portfolio only costs 1 cent more. As a result, the total transaction costs bill for hedging the call option exceeds that for hedging the put option by 0.53 dollars. In the Bear Case,

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<sup>17</sup> This confirms our intuition that the dealer voluntarily adjusts her initial hedging position to reduce the amount of intermediate reheding.

we observe similar but reversed pattern, which indicates that the transaction costs bill for hedging the put option is heavier.

#### **B.4 Implications of Trading Costs, Expected Returns, and Return Volatility**

In this subsection ,we first examine the relation between the asymmetry of hedging costs (in the form of call-put implied volatility spread) and the transaction costs. Intuitively, the higher the trading costs, the greater the asymmetry between hedging costs for call and put options. This is because, as we have shown, the asymmetry of hedging costs is caused by the differential transaction costs bills incurred during the dealer’s hedging process. We plot in Figure 1 the implied volatility spread against trading costs rate  $\theta$ . As can be seen, the spread indeed increases with trading costs rate.<sup>18</sup> This pattern implies that, the probability of put-call parity violation and the robustness of option-based return predictability should both increase with stock trading costs, which is consistent with the results we obtain from our empirical tests presented in Section V.

[Insert Figure 1 about here]

Next we examine how the difference in option hedging costs changes with the expected return of the underlying stock. For this purpose, we plot in Figure 2 the difference in hedging costs against the stock’s monthly expected return. We exhibit the results for the Bear Case only to save the space, and the results for the Bull Case display similar pattern. As can be observed, the poorer the expected return of the underlying stock, the larger the spread between the implied volatility of the call option and of the put option. This is also consistent with the results we obtain from our empirical analysis.

[Insert Figure 2 about here]

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<sup>18</sup> The spread is slightly larger in the Bull Case, because in the Bull Case the stock price is more likely to rise, hence so does the transaction costs bill (recall that transaction costs bill is assumed to be proportional to the stock price).

Lastly, we examine how the implied volatility spread changes with respect to the underlying stock's return volatility. Figure 3 shows that, as the return volatility increases, the implied volatility spread tends to decrease. This is because, with greater volatility, the option dealer should less bias her hedging portfolio in response to the increased uncertainty in future stock price changes.

[Insert Figure 3 about here]

In summary, we have argued that there can be a mechanical relationship between the relative pricing of options and the future return of the stock which is solely attributed to asymmetric hedging costs, and the option-based return predictability does not necessarily imply informed trading activities in the options market. This is the main hypothesis we develop from our model, and will be tested empirically in Section V.

### **B.5 Option Moneyness and Implied Volatility Skew**

Besides implied volatility spread, there are empirical studies documenting the predictive power of implied volatility skew, which is the difference in the implied volatility of out-of-the-money put and that of at-the-money call (e.g., Xing, Zhang, and Zhao (2010)). Intuitively, in our theoretical framework, out-of-the-money options should be less costly to hedge. This is because hedging such options only involves trading small amount of the underlying stock, which in turn implies small amount of transaction costs.

[Insert Figure 4 about here]

We show in Figure 4 how the average trading costs change with the options' moneyness, using the parameter values in the Bear Case. When  $K/S_0$  decreases, the put option (call option, resp.) becomes deeper and deeper out-of-the-money (in-the-money). As can be seen from Figure 4, the trading costs incurred when hedging an out-of-the-money option are generally smaller indeed.

[Insert Figure 5 about here]

However, as we discussed before, the optimal choice of hedging portfolio is also significantly affected by the stock's expected return. Intuitively, when the stock's return is sufficiently negative, the put option can also have a large negative delta even when it is out-of-the-money. This means that the hedging costs for this out-of-the-money put can exceed that for an at-the-money call. To shed more light on our conjecture, we show in Figure 5 the implied volatility skew, which is defined as the difference in the implied volatility of an out-of-the-money put ( $K/S = 0.98$  in our calculation) and that of an at-the-money call. The results depicted in Figure 5 confirm our intuition that, when the stock's expected return is sufficiently negative, even an out-of-the-money put can be more costly to hedge than an at-the-money call. This implies that, the return predictability of implied volatility skew can also be a mechanical result of the asymmetric hedging costs incurred by option dealers.

## B.6 Robustness and Further Discussion

Before we proceed to present supportive empirical evidence in the next section, we close our theoretical analysis by briefly discussing some dimensions of the robustness of our results in this subsection.

In our baseline analysis, we concentrate on a two-period setting due to the heavy computational demand of solving the Bellman equation (7). We also implement the main model in settings with more intermediate periods, by utilizing the following trinomial random variable  $\hat{\phi}_i$  to approximate the normal random variable  $\phi_i$  in (1) as proposed in Kamrad and Ritchken (1991):

$$\hat{\phi}_i = \begin{cases} e^{\lambda\sigma\sqrt{\delta t}}, & \text{with probability of } \frac{1}{2\lambda^2} + \frac{\mu\sqrt{\delta t}}{2\lambda\sigma} \\ 1, & \text{with probability of } 1 - \frac{1}{\lambda^2} \\ e^{-\lambda\sigma\sqrt{\delta t}}, & \text{with probability of } \frac{1}{2\lambda^2} - \frac{\mu\sqrt{\delta t}}{2\lambda\sigma} \end{cases} \quad (8)$$

with the choice of parameter  $\lambda = \sqrt{3/2}$ .<sup>19</sup> This approximation effectively reduces the dimensionality of the optimization problem by allowing us to work on a two dimensional grid for  $(x, y)$  on each node of a recombining trinomial tree of  $S$ , rather than a three dimensional grid for  $(x, y, S)$ . Therefore, it significantly reduces the computational demand.<sup>20</sup> In the multi-period setting, we find qualitatively similar results. For example, in a four-period model with the same parameter values as listed in Table I (i.e., the hedging portfolios are rebalanced once per week), the difference in hedging cost of call and put option is 0.64 dollars in the Bull Case, and is -0.43 dollars in the Bear Case. These consistent results are not surprising, because our main argument can be well generalized to multi-period setting.

In addition, we note that the objective function defined in (3) penalizes large hedging error more severely than small hedging error, which makes the dealer trade more conservatively to avoid large hedging error. We have implemented a similar model in which the dealer's objective is to minimize the absolute value, rather than the squared value, of the hedging error. We find similar results in such a case, due to almost parallel intuition.

In our baseline model, the trading costs are assumed to be constant over time. If the option dealer is facing random variations of the trading costs, she may have stronger incentive to build more biased hedging portfolio ex ante, based on the expected market condition during the life of her options, to reduce the potential costs of future transactions. This argument suggests that our main results may still hold when the liquidity cost is varying stochastically.

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<sup>19</sup> This choice leads to equal probability of 1/3 for each realization, in case that  $\mu = 0$ .

<sup>20</sup> We do not use binomial tree structure as our approximation scheme because it produces trivial results. In particular, the options are perfectly replicable on a binomial tree, even in the presence of transaction costs (e.g., Boyle and Vorst (1992)). The possibility of perfect replication renders the expected return of the underlying stock irrelevant in option pricing. Instead, the trinomial tree structure keeps our incomplete market structure and provides a non-trivial case in which the expected return of the underlying stock does affect option pricing and hedging.

## V Empirical Analysis

In this section, we provide empirical evidence that is consistent with the novel predictions of our model. In particular, we show that the hedging costs incurred by option dealers affect the quality of the information signal provided by option prices. We also show that this mechanism is unlikely to be related to informed trading in the options market. We separate the hedging effects from informed trading by focusing on stocks with zero option volume, and on the short-sale ban of 2008, because during this event option dealers were the only ones allowed to short the banned stocks for the purpose of hedging.

### A Data and Descriptive Statistics

We extract equity lending data from the Markit database for the period between January 2007 and December 2010. We then merge this data with the OptionMetrics database, from which we obtain information on option prices, implied volatilities, open interest, trading volume, and other variables. We extract accounting information from Compustat, and stock characteristics from the CRSP database. We apply a number of filters to the options data, to exclude penny stocks with very illiquid options, and prevent data errors, following Grundy, Lim, and Verwijmeren (2012).

Table IV reports the descriptive statistics for our final sample. We use the equity lending fee as a proxy for the cost incurred by short sellers, similar to Saffi and Sigurdsson (2011). In our sample, the average loan fee (*Lending Fee*) is 53 bps, and the median is 9 bps. There is significant variation in the loan fee in our sample. The standard deviation for this variable is 238 bps. The loan fee is reported in the Markit database as an annualized measure.

[Insert Table IV about here]

We use the Black-Scholes implied-volatility spread to measure deviations from put-call parity. We extract implied-volatilities for call and put options from the OptionMetrics



database. There are many different options written on a single stock. Therefore, we compute a weighted-average of the implied-volatilities across all the options for each stock, across all the strikes and maturities. We use the open interest for each option as the weighting measure. We do this separately for calls and puts and report the summary statistics for the average implied volatility in Table IV. The average implied volatility for puts (*IV Put*) is 52%, while for calls (*IV Call*) it is 49%.

Following Cremers and Weinbaum (2010), we compute the implied-volatility spread (*IVS*) as the difference between the implied-volatility of calls and puts with the same strike price and maturity, for a given stock. We aggregate the implied-volatility spreads by value-weighting across all pairs of options using the open interest of the pair (i.e. the sum of the open interests for the call and the put in the pair). The average (median) *IVS* in our sample is -1.35% (-0.77%). The negative sign of the *IVS* measure is consistent with the statistics we obtained for *IV Put* and *IV Call*, as *IVS* is essentially the difference between the implied volatilities of calls and puts.<sup>21</sup> These implied volatilities are provided by OptionMetrics. They compute these implied-volatilities for European options using the Black-Scholes model, while for American options they use binomial trees that account for early exercise rights.

Table IV also reports summary statistics for several variables that we use as controls in our regression models. This list of controls is similar to that used in Lin and Lu (2016). The list includes the contemporaneous return of the stock (*Stock Return (t=0, %)*); the turnover of the stock (*Turnover*), which we compute as the daily ratio of the trading volume to the amount of shares outstanding; the size of the stock, which is the product of the number of shares outstanding and the price. We use its log transformation ( $\ln(\text{Size})$ ) to alleviate the effects of skewness. *Stock Return Volatility* is the standard deviation of daily stock returns in the prior month. *VIX* is the level of the CBOE implied-volatility index for options on the

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<sup>21</sup> Note that *IVS* is not exactly equal to the difference between *IV Call* and *IV Put* in Table IV because *IVS* requires that we find a matched pair of calls and puts with the same strike price and maturity for a given stock.

S&P500 index. This proxies for market uncertainty. We also control for the return of the market using the *S&P 500 Index Return*. We also control for the stock return in the prior month (*Stock Return (Last Month, %)*), and the *Institutional Ownership* (i.e. the fraction of shares outstanding held by 13F institutions). The average (median) institutional ownership in our sample is 77% (80%). Lastly, the list of control variables includes the bid-ask spread for the stock as a proportion of the mid-quote (*Stock Bid-Ask Spread*), and the skewness in daily returns for the stock in the prior month (*Stock Return Skewness*).

In the following sections, we test the main predictions of our model. We are interested in studying the effect of lending fees on the hedging costs of option market makers, which in turn affect the pricing of options and the relation of these prices with future stock returns. We also try to rule out the potential alternative explanation that the effects we find are driven by informed trading in the options market, by focusing on stocks with zero option trading.

## **B Hedging Costs and Put-Call Parity Deviations**

In this section, we test the effect of the hedging costs incurred by option dealers on the relative pricing of call and put options. In particular, we examine how the equity lending fee (*Lending Fee*), which proxies for the short-sale cost, affects the implied-volatility spread (*IVS*), which proxies for the degree to which there are deviations from put-call parity (Cremers and Weinbaum (2010)). We also study the effects of *lending Fee* on the implied volatilities of puts and calls separately.

### **B.1 Equity Lending Fees and Implied-Volatility Spreads**

If the put-call parity holds, the implied-volatility spread (*IVS*) is null. Therefore, a non-zero *IVS* can capture the relative mispricing of calls versus puts, i.e. a positive (negative) *IVS* means that calls (puts) are relatively more expensive than puts (calls). In contrast to

the view that a non-zero *IVS* is the result of informed trading in the options market (e.g. Cremers and Weinbaum (2010)), we argue instead that it can be the result of asymmetric hedging costs incurred by the option dealers on their call and put positions, and that there is no additional information in *IVS* that is not already in the stock market.

We separate the hedging effects from informed trading in options, by examining put-call parity deviations for stocks with zero option volume on a given day. We create a dummy variable called *Zero Volume* which equals one for optionable stocks with zero option volume on a given day, and equals zero otherwise. This captures the idea that a non-zero *IVS* on a day with zero option volume is unlikely to be related to informed trading in the options market, but related to other factors instead.

We capture the variation in hedging costs for option dealers by exploiting changes in the fees charged in the equity lending market. The lending fees have been used as a proxy for short-selling costs (e.g. Saffi and Sigurdsson (2011)), which are costs that affect in particular the hedging of put options written by option dealers. In order to isolate the demand for short-selling initiated by option dealers for the purpose of hedging, we focus on an experiment associated with the the short-sale ban of 2008. During the period of the ban, which was in effect from September 19 to October 8 of year 2008, only option market makers could short the banned stocks and only for hedging purposes. The exemption granted by the SEC stated that option dealers were allowed to short “*when selling short as part of bona fide market making and hedging activities related directly to bona fide market in derivatives*” (SEC Emergency Order 34-58592). Therefore, this represents a relatively clean setting for us to examine how the variation in hedging costs for option market makers affects their pricing of options. Hence, we create a dummy variable called *Banned* which equals one for banned stocks in the ban period. This indicator allows us to disentangle the demand for short selling originated from option dealers, from shorting demand originated by speculators. We then interact this indicator with the *Lending Fee* variable to understand how shorting costs affect

the hedging activity of option dealers and their relative pricing of options. We also interact it with the indicator *Zero Volume* to isolate changes in option prices that are unrelated to informed trading in options.

We implement a model that captures the relationship between *IVS* and *Lending Fee*, as well as the multiple interactions between *Lending Fee* and the indicator variables for stocks with zero option volume on a given day (i.e. *Zero Volume*), and banned stocks in the short-ban period (i.e. *Banned*). The full specification of the pooled OLS regression is given as follows:

$$\begin{aligned}
IVS_{i,t} = & \beta_0 + \beta_1 Lending\ Fee_{i,t} + \beta_2 Zero\ Volume_{i,t} + \\
& + \beta_3 Lending\ Fee_{i,t} \times Zero\ Volume_{i,t} + \beta_4 Banned_{i,t} + \\
& + \beta_5 Lending\ Fee_{i,t} \times Banned_{i,t} + \beta_6 Banned_{i,t} \times Zero\ Volume_{i,t} + \\
& + \beta_7 Lending\ Fee_{i,t} \times Banned_{i,t} \times Zero\ Volume_{i,t} + \\
& + Controls + Effects + \epsilon
\end{aligned} \tag{9}$$

Table V reports the results of this regression, not only in its full form, but also five subsets of its full form. It shows that the relationship between *Lending Fee* and *IVS* is negative and significant across all the six specifications. This suggests that an increase in short-sale costs leads to an increase in the price of put options relative to that of call options, as captured by the difference between the implied volatilities of calls and puts (i.e. *IVS*).

[Insert Table V about here]

In column (1) of Table V, we study the effect of trading in the options market. The coefficient of  $-1.276$  for variable *Lending Fee* shows that an increase in *Lending Fee* by a one standard deviation (238 bps in Table IV) is associated with a change in *IVS* of  $-3.61\%$  (i.e. the product of  $-1.276$  by  $0.0238$ ). The magnitude of this effect is economically very large, as it compares with an average *IVS* of  $-1.35\%$  from Table IV. This is the estimated effect for days in which there is positive volume in options (i.e. *Zero Volume* = 0). For days

in which there is zero volume in options (i.e. *Zero Volume* = 1), the effect of *Lending Fee* on *IVS* is given by the sum of the coefficients  $\beta_1 + \beta_3$  in equation (9), which we report in column (1) of Panel B as *Lending Fee*  $\times$  ( $1 + \textit{Zero Volume}$ ). The sum of coefficients  $\beta_1 + \beta_3$  is also negative and significant at 1% level, which means that on days with zero option volume the shorting costs also significantly affect the relative prices of calls and puts. A one standard deviation increase in *Lending Fee* leads to a change in *IVS* of  $-2.46\%$  for zero option volume days, which is still very large compared to the average *IVS* of  $-1.35\%$  in Table IV. The effect of *Lending Fee* on *IVS* on days with zero option volume is 68% of its effect on days with positive volume. This suggests that option trading may not be the main driver of the relative mispricing of puts versus calls. In column (2) we control for the list of variables described in section V.A and the results remain qualitatively unchanged compared to column (1).

In columns (3) and (4) of Table V, we interact *Lending Fee* with the indicator *Banned*. This interaction captures the incremental effect of the short-sale cost on *IVS* attributable to the hedging activity of option dealers, because they are the only short sellers for the banned stocks during the ban period. The coefficient on the interaction *Lending Fee*  $\times$  *Banned* is negative at  $-1.877$  in column (4), and is significant at 1% level. This suggests that the hedging costs incurred by option dealers have a significantly stronger effect on the relative prices of puts versus calls for banned stocks in the short-ban period, compared to other stocks. The effect of *Lending Fee* on *IVS* for banned stocks in the short-ban period, is captured by the sum of the coefficients  $\beta_1 + \beta_5$ , which we report in Panel B as *Lending Fee*  $\times$  ( $1 + \textit{Banned}$ ). The sum of coefficients  $\beta_1 + \beta_5$  in column (4) is negative at  $-3.112$  and significant at 1% level. This suggests that a one standard deviation increase in *Lending Fee* is associated with a change in *IVS* of  $-7.41\%$ , which is more than five times large than the average *IVS* in Table IV.

However, even though the specifications in columns (3) and (4) may successfully isolate the demand for short-selling that is originated exclusively for the purpose of hedging, and the

effect of short-sale costs on hedging costs, it does not control for the fact that the short-ban in the stock market could have drifted investors' trading to the options market. As a result, the strong negative impact on *IVS* for banned stocks in the short-ban period could be due to a surge in demand for puts compared to calls due to the short-ban. Therefore, in order to address this potential concern, we also interact the indicators *Banned* and *Zero Volume*, to isolate the changes in *IVS* that are most likely related to the hedging activity of option dealers, instead of the potentially increased amount of trading in puts compared to calls in the options market as a result of the short-ban.<sup>22</sup> We report these additional tests in columns (5) and (6) of Table V, Panel A. We are particularly interested in the triple interaction *Lending Fee*  $\times$  *Banned*  $\times$  *Zero Volume*, which captures the incremental effect of short-sale costs on *IVS* for banned stocks in the short-ban period that have zero option volume, compared to the banned stocks in the short-ban period with positive volume in options. The coefficients on this triple interaction are negative but statistically not significant, which suggests that option trading has little incremental impact on *IVS*.

The effect of *Lending Fee* on *IVS* for banned stock with zero option volume in the short-ban period, is given by the sum of the coefficients  $\beta_1 + \beta_5 + \beta_7$ , which we report in Panel B as *Lending Fee*  $\times$   $(1+Banned \times (1+Zero Volume))$ . The sum of these three coefficients is negative at  $-3.977$  in column (6), and significant at 1% level. It suggests that a one standard deviation increase in *Lending Fee*, for banned stocks with zero option volume in the short-ban period, is associated with a change in *IVS* of  $-11.25\%$ , which is more than 8 times larger than the average *IVS* in Table IV. These are the stocks for which a change in *Lending Fee* is more likely to impact the relative prices of puts versus calls through the hedging channel, because only option dealers were allowed to short such stocks during the short-ban period, and exclusively for hedging purposes. Therefore, this is the first piece of

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<sup>22</sup> Lin and Lu (2016) and Grundy, Lim, and Verwijmeren (2012) show that the trading volume in put options actually decreased for banned stocks in the short-ban period. They argue that that is because the larger costs of hedging in the stock market refrained option dealers from making a market for put options.

evidence consistent with our working hypothesis that, asymmetric hedging costs for puts versus calls are strongly related to deviations from put-call parity. In section V.C we show that such parity deviations are strongly related to subsequent stock returns.

Next, we would like to study the effect of *Lending Fee* on the prices of puts and calls separately. We expect it to be much stronger for puts compared to calls, because short-selling is more tightly linked to the hedging of put options.

## B.2 Equity Lending Fees and Implied Volatilities of Puts and Calls

In order to separate the effects of short-sale costs on the prices of puts and calls, we perform a regression similar to equation (9), but we replace the implied-volatility spread (*IVS*) on the left-hand side with the implied volatilities of puts (*IV Put*) and calls (*IV Call*), respectively. We expect to find that short-sale costs have a much stronger effect on the prices of puts compared to calls.

Table VI splits the effect of *Lending Fee* on *IVS* into a part that is attributable to put options (columns (1) to (3)) and a part that is attributable to call options (columns (4) to (6)). The dependent variable in columns (1) to (3) is the implied volatility of puts (*IV Put*), and in columns (4) to (6) is the implied volatility of calls (*IV Call*). The coefficients on *Lending Fee* are positive and significant across the six specifications. This suggests that short-sale costs can increase the prices of both puts and calls. The effect of shorting costs on put prices is more intuitive than the effect on call prices. The positive effect of *Lending Fee* on *IV Call* in columns (4) to (6) can be the result of short sellers using the call option to hedge their short positions on the stock. Therefore, an increase in demand for lendable stocks to short, together with an increase in demand for calls to hedge short positions in stocks, could explain the positive association between *Lending Fee* and *IV Call* reported in Table VI.

[Insert Table VI about here]

In columns (1) and (4), we study the effect of option trading on the relation between *Lending Fee* and *IV Put* and *IV Call*, respectively. As expected, the effect of *Lending Fee* on *IV Put* than on *IV Call*. The coefficient on *Lending Fee* of 1.555 in column (1) means that a one standard deviation increase in *Lending Fee* of 283 bps (see Table IV) is associated with a change in *IV Put* of 4.40% (8.5% of the average *IV Put* in Table IV). The expected change in *IV Call* associated with a one standard deviation increase in *Lending Fee* is only 0.95% (1.9% of the average *IV Call* in Table IV). As expected, the effect of changes in shorting costs is stronger for puts than calls.

These are the estimated effects for stocks with positive option volume (i.e. *Zero Volume* = 0). For stocks with zero option volume (i.e. *Zero Volume* = 1), the relation between *Lending Fee* and *IV Put* is given by the sum of the coefficients  $\beta_1 + \beta_3$  reported in Panel B column (1) as *Lending Fee*  $\times$  ( $1 + \textit{Zero Volume}$ ). The coefficient of 1.565 in column (1) of Panel B is indistinguishable from the coefficient on *Lending Fee* in column (1) of Panel A. This suggests that, whether or not there is option trading, the relation between *Lending Fee* and *IV Put* remains unchanged. The coefficient on *Lending Fee*  $\times$  ( $1 + \textit{Zero Volume}$ ) in column (4) of Panel B is larger than the coefficient on *Lending Fee* in column (4) of Panel A. This suggests that, the effect of *Lending Fee* on the implied volatility of calls (*IV Call*) is actually stronger for stocks with zero option volume, compared to stocks with positive option volume.

In columns (2) and (5) of Table VI we study the effects of the short-sale ban of 2008. In these specifications, the coefficient on *Lending Fee* captures the effect of short-sale costs on the implied volatilities of puts and calls for non-banned stocks outside of the ban period. The non-banned stocks can always be shorted by speculators, but banned stocks in the short-ban period can only be shorted by option dealers for the purpose of hedging. We are particularly interested in the coefficients on the interaction between *Lending Fee* and *Banned* in columns (2) and (5) of Panel A. These coefficients represent the incremental effects of *Lending Fee* on the prices of puts and calls for banned stocks in the short-ban period, compared to non-



banned stocks outside of the ban period. A positive and significant coefficient in column (2), and the negative and insignificant coefficient in column (5), suggest that there is a positive incremental effect for puts but not for calls. This result is consistent with the predictions of our model, as we can attribute it to the hedging activity of option dealers.

We also test for the sum of coefficients  $\beta_1 + \beta_5$ , which we report in Panel B as *Lending Fee*  $\times$   $(1+Banned)$ . It shows a strong positive effect of *Lending Fee* on *IV Put* (column (2)) but an insignificant effect on *IV Call* (column (5)). If we compared the results in Table VI with the results in Table V, we conclude that the effect of *Lending Fee* on *IVS* is due to its effect on *IV Put* only.

Lastly, we intersect the group of stocks with zero option volume with the group of banned stocks in the short-ban period. The results of this intersection are reported in columns (3) and (6) of Table VI. We are particularly interested in the post-estimation tests of Panel B. The effect of *Lending Fee* on *IV Put* and *IV Call* for banned stocks with zero option volume in the short-ban period, is given by the sum of the coefficients  $\beta_1 + \beta_5 + \beta_7$ , which we report as *Lending Fee*  $\times$   $(1+Banned \times (1+Zero Volume))$ . The sum of these three coefficients is positive at 2.357 and significant at 5% level for *IV Put* (column (3)). It suggests that a one standard deviation increase in *Lending Fee*, for banned stocks with zero option volume in the short-ban period, is associated with a change in *IV Put* of 6.67%, which is 12.84% of the average *IV Put* in Table IV. These are the stocks for which a change in *Lending Fee* is more likely to impact the relative prices of puts versus calls through the hedging channel, because only option dealers were allowed to short such stocks during the short-ban period, and exclusively for hedging purposes.

The effect of the intersection of the group of stocks with zero option volume and the group of banned stocks in the short-sale period is insignificant for *IV Call*, which suggests that the results in Table V can be attributed to put options only.<sup>23</sup>

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<sup>23</sup> This does not represent a contradiction of our theoretical results, which suggest that the hedging cost

Next, we would like to test the relation between this asymmetric effect of short-sale costs on the pricing of puts versus calls and the ability of *IVS* to predict future stock returns.

## C Implied-Volatility Spreads and Future Stock Returns

Cremers and Weinbaum (2010) argue that the ability of  $\beta$  to predict stock returns is related to the idea that informed investors choose to trade in the options market first, and the information spills to the stock market with a delay. Instead, we show in section III that there is a mechanical relation between the costs incurred by option dealers when hedging their positions in puts and calls and the expected return of the underlying stock. In particular, negative (positive) expected returns for the underlying stock are associated negative (positive) implied-volatility spreads, because market makers choose relatively higher (lower) prices for puts compared to calls, as it is more expensive (cheaper) to hedge puts than calls under such stock market conditions. This suggests that, once we account for the hedging costs of options, there is not additional information in option prices that is not already available in the stock market.

In order to test this prediction, we use the following pooled OLS regression:

$$\begin{aligned}
 \text{Stock Return}_{i,t+1} &= \beta_0 + \beta_1 \text{IVS}_{i,t} + \beta_2 \text{Zero Volume}_{i,t} + & (10) \\
 &+ \beta_3 \text{IVS}_{i,t} \times \text{Zero Volume}_{i,t} + \beta_4 \text{Banned}_{i,t} + \\
 &+ \beta_5 \text{IVS}_{i,t} \times \text{Banned}_{i,t} + \beta_6 \text{Banned}_{i,t} \times \text{Zero Volume}_{i,t} + \\
 &+ \beta_7 \text{IVS}_{i,t} \times \text{Banned}_{i,t} \times \text{Zero Volume}_{i,t} + \\
 &+ \text{Controls} + \text{Effects} + \epsilon
 \end{aligned}$$

Table VII reports the results of this analysis. In columns (1) to (3), we run the specification in equation (10), where the main independent variable of interest is the continuous

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of puts are expected to be higher than those of calls when the underlying stock is expected to fall in value. This is the case even when shorting and longing the stock are subject to the same transaction cost rate.

variable  $IVS$ . However, in columns (4) to (6), we replace  $IVS$  in equation (10) with a dummy variable  $LowIVS$ , which equals one for stocks in the bottom decile of  $IVS$  in the cross-section of stocks on a given day, and equals zero otherwise. The indicator  $LowIVS$  captures the stocks with relatively more expensive puts compared to calls, which are expected to experience a drop in prices in the near future.

[Insert Table VII about here]

In columns (1) and (4), we study the contribution of option trading to the ability of  $IVS$  and  $LowIVS$  to predict the stock return on the next day. The positive and significant coefficient on  $IVS$  of 2.163 in column (1) suggests that, a one standard deviation increase in  $IVS$  of 0.0699 (see Table IV) is associated with an increase in the return of the stock on the next day of about 15 bps (i.e. the product of 0.0699 by 2.163). This corresponds to an annualized return of 37.5%, assuming a 250-day trading calendar. This is the estimated predictability for stocks with positive option volume (i.e.  $Zero\ Volume = 0$ ). The ability of  $IVS$  to predict the stock return on the next day, in the absence of option trading, is given by the sum of the coefficients  $\beta_1 + \beta_3$ , which we report in Panel B as  $IVS \times (1 + Zero\ Volume)$ . A coefficient of 1.728 suggests that a one standard deviation increase in  $IVS$  is associated with a 12 bps stock return on the next day (30.2% in annualized terms). This is about 80% of the effect found for stock with positive option trading, which suggests that the contribution of option trading to the predictability of  $IVS$  is relatively small.

In column (4), we focus on stocks in the lowest decile of  $IVS$ , which we identify with the indicator  $LowIVS$ . The coefficient of  $-1.182$  for  $LowIVS$  corresponds to an average negative stock return of  $-18$  bps on the next day ( $-45.5\%$  in annualized terms). This is the estimated effect for stocks with positive option volume (i.e.  $Zero\ Volume = 0$ ). In Panel B column (4) we report the sum of the coefficients  $\beta_1 + \beta_3$  in equation (10) for the subset of stocks in the lowest decile of  $IVS$  (i.e.  $LowIVS = 1$ ). A coefficient of  $-0.230$  corresponds to an average negative stock return of  $-23$  bps on the next day ( $-57.5\%$  in annualized terms).

This suggests that the predictability of *LowIVS* is even stronger for stocks with zero option volume, which is inconsistent with the informed trading hypothesis, as argued in Cremers and Weinbaum (2010) and others.

Next, we interact *IVS* and *LowIVS* with the indicator *Banned*. We understand that the short-ban period may be plagued with numerous confounding effects, such as increased stock spreads, and increased uncertainty, among others. However, the indicator *Banned* on its own is expected to absorb all the confounding effects associated with such a tumultuous period. We build our identification strategy around the interactions of *Banned* with our return predictors *IVS* and *LowIVS*.

In column (2), the coefficient on *IVS* is 1.838 (significant at 1%), and the coefficient on  $IVS \times (1 + Banned)$  is 8.002 (significant at 1%). These coefficients suggest that a one standard deviation increase in *IVS* is associated with a 13 bps return for non-banned stocks outside of the ban period, and a 56 bps return for banned stocks during the ban period. In column (5), the coefficient on *LowIVS* is  $-0.184$  (significant at 1%), and the coefficient on  $LowIVS \times (1 + Banned)$  is  $-2.081$  (significant at 1%). These coefficients suggest that stocks in the lowest decile of *IVS*, the stocks with the most expensive puts relative to calls (i.e.  $LowIVS = 1$ ), are expected to have a negative  $-18$  bps return on the next day for non-banned stocks outside of the ban period, and a negative  $-208$  bps return on the next day for banned stocks during the ban period.

In columns (3) and (6), we intersect the group of stock with zero option volume, and the group of banned stocks in the short-ban period. This intersection helps us separate the effects of hedging and informed trading on the predictive ability of *IVS* for future stock returns. We are particularly interested in the triple interaction term  $LowIVS \times Banned \times Zero Volume$ , which is insignificant. This indicates that the absence of option trading does not affect the predictability of *LowIVS* for banned stocks in the ban period (i.e.  $Banned = 1$ ). We also report the sum of coefficients  $\beta_1 + \beta_5 + \beta_7$  in Panel B as  $LowIVS \times (1 + Banned) \times (1 + Zero$

*Volume*)), with a magnitude of  $-2.082$  (significant at 5% level). This suggests that banned stocks in the ban period (i.e.  $Banned = 1$ ), that belong to the lowest decile of  $IVS$  (i.e.  $LowIVS = 1$ ), are expected to have a negative  $-208$  bps return on the next day. This is the same figure as for  $LowIVS \times (1 + Banned)$  in column (5) of Panel B, which that option trading has no contribution to the predictability of  $LowIVS$ .

Overall, these results are consistent with the idea that the ability of  $IVS$  and  $LowIVS$  to predict future stock returns is spurious and not related to informed trading taking place in the options market first. The results are consistent with our model, according to which, even when the information arrives simultaneously at both the stock and the options markets, the hedging activity of option dealers and the asymmetric costs they incur when hedging puts versus calls, can create a mechanical relation of  $IVS$  and  $LowIVS$  with future stock returns.

## VI Conclusion

We argue that differential hedging costs for call and put options under different stock market conditions, affect how option dealers optimally choose the relative prices of these options. This in turn leads to deviations from put-call parity and non-zero spreads between the implied-volatility of calls and puts. These spreads exhibit strong ability to predict future stock returns. However, we argue that the predictive ability of implied-volatility spreads can be spurious and unrelated to informed trading in the options market. In our theoretical model, there is no asymmetry of information across investors, no market segmentation, and the information arrives simultaneously at both the stock and the options markets. It is the higher costs incurred by option dealers when hedging a call (put) option in a rising (falling) market what results in an optimal choice of expensive prices for calls (puts) relative to puts (calls) in that market. Therefore, positive (negative) spreads between call and put implied volatilities indicate (mechanically) that the stock returns will likely be positive (negative) in

the near future.

We test this prediction of our model by intersecting two groups of stocks: (i) optionable stock with zero option volume, and (ii) banned stocks during the short-sale ban of 2008. The laboratory of the short-sale ban of 2008 is a relatively clean setting for our empirical analysis, because option dealers were granted an exemption by the SEC and were allowed to short the banned stocks exclusively for the purpose of hedging their option positions. Therefore, variation in the shorting costs for this subset of stocks only affects options dealers through their hedging activities. We show that an increase in hedging costs of option dealers, through an increase in equity lending fees, is associated with an increase in the implied volatility of puts, and a decrease in the implied-volatility spread (i.e. the difference between implied volatilities of call and put options with the same strikes and maturities) for the banned stocks in the ban period.

We also show that the predictability associated with the implied-volatility spread is strong for the banned stocks in the short-ban period. Moreover, the predictability remains strong and significant when we restrict our analysis to a subsample of stocks with zero trading in options. This is the strongest evidence we find that is consistent with the idea that, the predictability of the implied-volatility spread can be attributed in large part to the adjustment of option quotes by option dealers due to changing conditions for hedging in the shorting market.

Overall, we conclude that one must account for the effect of hedging costs of options dealers on call and put prices when studying the information contained in the spread between their implied volatilities. Although we cannot completely rule out the possibility that informed trading in options also contributes to the stock return predictability using option-based predictors, the degree to which the asymmetry in hedging costs incurred by option dealers accounts for such predictability is a question that deserves further investigation.

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# Appendix

The content of the Appendix is as follows. In Appendix A, we present the proof of Proposition 1. In Appendix B, we present the analysis of a single period trinomial model to show that our results hold in a static setting as well.

## A. Proof of Proposition 1

*Proof.* For any self-financing hedging strategy for the call option  $\{(x_{C,i}, y_{C,i}) : i = 0, \dots, n-1\}$ , we can construct a corresponding hedging strategy for the put option as follows:

$$x_{P,i} = x_{C,i} - 1, \quad y_{P,i} = y_{C,i} + Ke^{-r(T-t_i)}, \quad 0 \leq i \leq n-1.$$

It is easy to verify that this hedging strategy is also self-financing, and the expected hedging error for the put option under this strategy equals that for the call option under strategy  $\{(x_{C,i}, y_{C,i}) : i = 0, \dots, n-1\}$ . Symmetrically, for any hedging strategy for the put option, we can also construct a corresponding hedging strategy for the call option in a similar manner. As a result, the optimal hedging strategies  $\{(x_{C,i}^*, y_{C,i}^*)\}$  and  $\{(x_{P,i}^*, y_{P,i}^*)\}$  must be linked through equations  $x_{P,i}^* = x_{C,i}^* - 1$ ,  $y_{P,i}^* = y_{C,i}^* + Ke^{-r(T-t_i)}$ , for all  $0 \leq i \leq n-1$ , and the associated hedging costs are related via the put-call parity.  $\square$

## B. A Simple One-Period Model

In this Appendix we develop a simple one-period model to show that our results hold in static settings as well.

We first sketch the model. There are two dates, 0 and 1. On day 0, the stock price is  $S_0$ . On day 1, there are three possible outcomes of the world:  $S_1 = S_0(1 + \omega)$  with probability  $p$ ,

$S_1 = S_0(1 - \omega)$  with probability  $q$ , and  $S_1 = S_0$  with probability  $1 - p - q$ , where  $\omega \in [0, 1]$  is constant. As before, we assume trading the stock incurs proportional transaction costs at rate  $\theta \geq 0$ . In addition to the stock, there is also a riskfree bond which earns interest rate  $r = 0$  and is costless to trade.<sup>24</sup>

On day 0, the option dealer sells at-the-money European options on the stock, and acquires (or shorts)  $x$  shares of the stock and  $y$  shares of the bond as hedging portfolio. For notational simplicity, we assume cash-settlement of the option on day 1. Then the dealer's net position value on day 1 is given by

$$Z = x^+ S_1(1 - \theta) - x^- S_1(1 + \theta) + y - f(S_1) \quad (11)$$

where the first three terms represents the net value of her hedging portfolio, and  $-f(S_1)$  is her liability of delivering the option's payoff. The dealer chooses a hedging portfolio  $(x, y)$  on day 0 to minimize her expected mean-square hedging error defined as

$$V(x, y) = E[Z^2] \quad (12)$$

For the case of call option, the expected hedging error expands as

$$\begin{aligned} V_C(x, y) &= [xS_0(1 + \omega)(1 - \theta) + y - S_0\omega]^2 p \\ &+ [xS_0(1 - \theta) + y]^2 (1 - p - q) \\ &+ [xS_0(1 - \omega)(1 - \theta) + y]^2 q \end{aligned}$$

where we have used  $K = S_0$ , and have assumed ex ante that the optimal hedging ratio  $x$  is positive for the call option, which can be verified ex post from the solution. Similarly, for

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<sup>24</sup> We view this bond as the numeraire so that we can normalize the interest rate to  $r = 0$  without loss of generality.

the case of put option, the expected hedging error reads

$$\begin{aligned}
V_P(x, y) &= [xS_0(1 + \omega)(1 + \theta) + y]^2 p \\
&+ [xS_0(1 + \theta) + y]^2 (1 - p - q) \\
&+ [xS_0(1 - \omega)(1 + \theta) + y - S_0\omega]^2 q
\end{aligned}$$

where we have assumed ex ante that the optimal hedging ratio  $x$  is negative for the put option, which can also be verified ex post from the solution.

The following proposition summarizes the main results we derive from this model:

**Proposition 2:** Let  $R = S_1/S_0$  be the gross return on the stock,  $E[R] = 1 + (p - q)\omega$ ,  $E[R^2] = 1 + 2(p - q)\omega + (p + q)\omega^2$ , and  $Var[R] = E[R^2] - (E[R])^2$  be the expectation, the second moment, and the variance of  $R$ , respectively, then we have the following statements:

1. The optimal hedging portfolio for a call option and a put option is given by

$$x_C^* = \frac{p\omega(1 + \omega - E[R])}{(1 - \theta)Var[R]}, \quad y_C^* = \frac{S_0p\omega(E[R^2] - (1 + \omega)E[R])}{Var[R]}, \quad (13)$$

$$x_P^* = \frac{q\omega(1 - \omega - E[R])}{(1 + \theta)Var[R]}, \quad y_P^* = \frac{S_0q\omega(E[R^2] - (1 - \omega)E[R])}{Var[R]}, \quad (14)$$

2. The initial costs of hedging the call option and the put option are

$$\xi_C = \frac{S_0p\omega}{Var[R]} \left[ \frac{1 + \theta}{1 - \theta} (1 + \omega - E[R]) + E[R^2] - (1 + \omega)E[R] \right], \quad (15)$$

$$\xi_P = \frac{S_0q\omega}{Var[R]} \left[ \frac{1 - \theta}{1 + \theta} (1 - \omega - E[R]) + E[R^2] - (1 - \omega)E[R] \right], \quad (16)$$

3. We can decompose the total hedging cost into an intrinsic component  $\xi^{in}$  and a trading

cost component  $\xi^{tr}$  as follows:

$$\xi_C = \xi_C^{in} + \xi_C^{tr}, \quad \xi_P = \xi_P^{in} + \xi_P^{tr}$$

where

$$\begin{aligned} \xi_C^{in} &= \frac{S_0 p \omega}{Var[R]} [1 + \omega - E[R] + E[R^2] - (1 + \omega)E[R]], \\ \xi_C^{tr} &= \frac{S_0 p \omega}{Var[R]} \left[ \frac{2\theta}{1 - \theta} (1 + \omega - E[R]) \right], \end{aligned}$$

and

$$\begin{aligned} \xi_P^{in} &= \frac{S_0 q \omega}{Var[R]} [1 - \omega - E[R] + E[R^2] - (1 - \omega)E[R]], \\ \xi_P^{tr} &= \frac{S_0 q \omega}{Var[R]} \left[ \frac{-2\theta}{1 + \theta} (1 - \omega - E[R]) \right] \end{aligned}$$

In addition, we have  $\xi_C^{in} = \xi_P^{in}$ .

4. The difference in the total hedging costs of call and put option is given by

$$\xi_C - \xi_P = \frac{2S_0\omega^2}{Var[R]}(p - q)(1 - p - q)\theta + O(\theta^2) \quad (17)$$

where the second term on the right hand side reflects higher order effect of transaction costs.

*Proof.* 1. Using the first order conditions of the quadratic minimization problem, we yield the following linear system of equations

$$\begin{aligned} S_0(1 - \theta)E[R]^2x + E[R]y &= S_0p(1 + \omega)\omega, \\ S_0(1 - \theta)E[R]x + y &= S_0p\omega, \end{aligned}$$

for the case of call option, and

$$S_0(1 + \theta)E[R]^2x + E[R]y = S_0q(1 - \omega)\omega,$$

$$S_0(1 + \theta)E[R]x + y = S_0q\omega,$$

for the case of put option. Solving these equations gives the desirable results.

2. Acquiring  $x_C^*$  shares of stock at  $t = 0$  will cost the dealer  $(1 + \theta)x_C^*S_0$  dollars due to the transaction costs payment, therefore the total cost is given by  $(1 + \theta)x_C^*S_0 + y_C^*$ . Similarly, short selling  $x_P^*$  shares of stock only gives the dealer proceeds of  $(1 - \theta)x_P^*$ , and the total cost is  $(1 - \theta)x_P^*S_0 + y_P^*$ . The results then follow by part 1.

3. We only prove the last assertion. By straightforward calculation, we have

$$\begin{aligned} \xi_C^{in} &= \frac{S_0\omega p}{Var[R]} [1 + \omega - E[R] + E[R^2] - (1 + \omega)E[R]] \\ &= \frac{S_0\omega p}{Var[R]} [1 + \omega - (p(1 + \omega) + 1 - p - q + q(1 - \omega)) \\ &\quad + p(1 + \omega)^2 + 1 - p - q + q(1 - \omega)^2 - (1 + \omega)(p(1 + \omega) + 1 - p - q + q(1 - \omega))] \\ &= \frac{2S_0}{Var[R]} pq\omega^3 \end{aligned}$$

Similarly, we also have

$$\xi_P^{in} = \frac{2S_0}{Var[R]} pq\omega^3$$

and the result follows.

4. Due to 2 and 3, we have

$$\begin{aligned}
\xi_C - \xi_P &= \xi_C^{tr} - \xi_P^{tr} \\
&= \frac{S_0 \omega p}{Var[R]} \left[ \frac{2\theta}{1-\theta} (1 + \omega - E[R]) \right] - \frac{S_0 \omega q}{Var[R]} \left[ \frac{-2\theta}{1+\theta} (1 - \omega - E[R]) \right] \\
&= \frac{2\theta S_0}{Var[R]} \left[ \frac{\omega p (1 + \omega - E[R])}{1-\theta} - \frac{\omega q (E[R] - 1 + \omega)}{1+\theta} \right] \\
&= \frac{2\theta S_0}{Var[R]} [p\omega(1 + \omega - E[R]) - q\omega(E[R] - 1 + \omega) + O(\theta)] \\
&= \frac{2\theta S_0}{Var[R]} [p\omega(1 + \omega - (p(1 + \omega) + 1 - p - q + q(1 - \omega))) \\
&\quad - q\omega(p(1 + \omega) + 1 - p - q + q(1 + \omega) - 1 + \omega) + O(\theta)] \\
&= \frac{2\theta S_0}{Var[R]} [(\omega^2 p(1 - p) - \omega^2 q(1 - q) + O(\theta))] \\
&= \frac{2\theta S_0 \omega^2}{Var[R]} (p - q)(1 - p - q) + O(\theta^2),
\end{aligned}$$

where we have applied the following Taylor's expansions

$$\begin{aligned}
\frac{1}{1-\theta} &= 1 + \theta + \frac{1}{2}\theta^2 + \dots = 1 + O(\theta), \\
\frac{1}{1+\theta} &= 1 - \theta + \frac{1}{2}\theta^2 + \dots = 1 + O(\theta).
\end{aligned}$$

□

Part 4 of Proposition 2 has at least the following implications: (1) in the absence of transaction costs ( $\theta = 0$ ), the hedging costs of put and call option are equal; (2) in the presence of transaction costs ( $\theta > 0$ ), then at leading order,  $\xi_C > \xi_P$  if and only if  $p > q$ , i.e., it is more likely that the stock price will increase on day 1. These results are consistent with those we derive from the dynamic model with log-normal returns.

### Table I: Model Parameters

This table summarizes the baseline parameter values we use in the numerical analysis to demonstrate the main results. Note that the return parameter  $\mu$  is annualized, hence the true expected return of the stock during the option's life is  $\mu T$ .

Parameter	Symbol	Baseline Value
Time to Expiration	$T$	1/12
Strike Price	$K$	100
Initial Stock Price	$S_0$	100
Risk-free Rate	$r$	0.00
Annualized Expected Return of the Stock	$\mu$	0.18 (Bull Case), -0.18 (Bear Case)
Annualized Volatility of the Stock Return	$\sigma$	0.15
Transaction Cost Rate	$\theta$	0.01



**Table II: Initial Hedge Portfolio and Hedging Costs**

This table shows the initial hedging position and the associated hedging cost of the option dealer, in different market conditions. The “Initial Stock Position” is the number of stock shares held, and the “Initial Bond Position” is the number of bond shares held (the bond is assumed to have a face value of  $K$  dollars per share). The parameter values used to generate the results are as follows:  $r = 0$ ,  $S_0 = 100$ ,  $K = 100$ ,  $T = 1/12$ ,  $n = 2$ ,  $\sigma = 0.15$ ,  $\mu = 0.18$  for the Bull Case, and  $\mu = -0.18$  for the Bear Case.

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	Panel A1: Bull Case		Panel A2: Bear Case	
	Call Option	Put Option	Call Option	Put Option
	Initial Stock Position	0.586	-0.414	0.440
Initial Bond Position	-0.569	0.431	-0.423	0.577
Hedging Cost	1.70	1.70	1.70	1.70
Difference in Hedging Costs	0.00	-	0.00	-
IV spread (% , call-put)	0.00	-	0.00	-

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	Panel B1: Bull Case		Panel B2: Bear Case	
	Call Option	Put Option	Call Option	Put Option
	Initial Stock Position	0.659	-0.450	0.484
Initial Bond Position	-0.634	0.473	-0.461	0.641
Hedging Cost	3.16	2.75	2.78	3.12
Difference in Hedging Costs	0.41	-	-0.34	-
IV spread (% , call-put)	3.55	-	-2.88	-

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**Table III: Expected Transaction Cost Payment**

This table shows the expected discounted trading costs bill incurred during the entire hedging process, in a two-period setting. We decompose the total trading costs payment into three components: the trading costs of forming, of rebalancing, and of unwinding the hedging portfolio. The parameter values used to generate the results are as follows:  $S_0 = 100$ ,  $K = 100$ ,  $r = 0$ ,  $\theta = 0.01$ ,  $T = 1/12$ ,  $n = 2$ ,  $\sigma = 0.15$ , for the Bull Case  $\mu = 0.18$ , and for the Bear Case  $\mu = -0.18$ .

	Panel A: Bull Case		Panel B: Bear Case	
	Call Option	Put Option	Call Option	Put Option
Formation	0.659	0.450	0.484	0.616
Rebalancing	0.235	0.225	0.229	0.235
Unwinding	0.667	0.358	0.372	0.610
Total	1.561	1.032	1.084	1.460
Difference	0.529	-	-0.376	-

**Table IV: Summary Statistics**

This table provides summary statistics for our final sample, which merges information on the equity lending market from Markit, and information on the options market from OptionMetrics. The sample covers the period between January 2007 and December 2010. A detailed description of the variables in this table can be found in Section IV.A.

	N (1000s)	Mean	Std	P5	P25	Median	P75	P95
Lending Fee	1,783	0.0053	0.0238	-0.0003	0.0006	0.0009	0.0015	0.0206
Implied Volatility of Puts (IV Put)	1,783	0.5195	0.2388	0.2392	0.3591	0.4687	0.6234	0.9689
Implied Volatility of Calls (IV Call)	1,783	0.4874	0.2236	0.2185	0.3359	0.4416	0.5889	0.9094
Implied-Volatility Spread (IVS)	1,783	-0.0135	0.0699	-0.1025	-0.0274	-0.0077	0.0086	0.0589
Stock Return (t=0, %)	1,783	0.0612	3.6277	-4.9971	-1.4603	0.0000	1.4808	5.2442
Turnover	1,783	0.0136	0.0185	0.0026	0.0057	0.0094	0.0159	0.0366
Ln(Size)	1,783	21.18	1.53	18.96	20.08	21.01	22.08	23.99
Stock Return Volatility	1,783	3.0143	2.0738	1.0252	1.7276	2.4951	3.6727	6.7839
VIX	1,783	0.2578	0.1157	0.1283	0.1888	0.2321	0.2799	0.5093
S&P 500 Index Return	1,783	0.0064	1.6927	-2.7090	-0.6561	0.0837	0.6918	2.4151
Stock Return (Last Month, %)	1,783	0.8870	16.0269	-21.5879	-6.8175	0.5620	7.7273	23.3478
Institutional Ownership	1,783	0.7701	0.2269	0.3405	0.6453	0.8000	0.9205	1.0889
Stock Bid-Ask Spread	1,783	0.0017	0.0036	0.0002	0.0005	0.0010	0.0019	0.0049
Stock Return Skewness	1,783	0.1491	0.9308	-1.2671	-0.3104	0.1330	0.5960	1.6397

**Table V: The Effect of Equity Lending Fees on Deviations from Put-Call Parity**

This table reports the results of panel regressions with year and industry dummies. The sample period covers the years 2007-2010. Standard errors (in parenthesis) are adjusted by clustering that accounts for heteroskedasticity and dependence of observations across the same stock and same year. We denote by \*\*\*, \*\*, \* the significance at the 1%, 5%, and 10% levels, respectively.

Panel A: Panel Regressions						
Dependent Variable:	Implied-Volatility Spread (IVS)					
	(1)	(2)	(3)	(4)	(5)	(6)
Lending Fee	-1.276*** (0.050)	-1.293*** (0.051)	-1.212*** (0.055)	-1.235*** (0.057)	-1.269*** (0.050)	-1.287*** (0.051)
Zero Volume	0.003*** (0.000)	0.001 (0.000)			0.003*** (0.000)	0.001 (0.000)
Lending Fee × Zero Volume	0.406*** (0.086)	0.394*** (0.087)			0.413*** (0.084)	0.400*** (0.085)
Banned			-0.018** (0.008)	-0.025*** (0.008)	-0.007 (0.008)	-0.014* (0.008)
Lending Fee × Banned			-1.930*** (0.444)	-1.877*** (0.428)	-1.788*** (0.415)	-1.739*** (0.394)
Banned × Zero Volume					-0.055** (0.023)	-0.054** (0.022)
Lending Fee × Banned × Zero Volume					-0.968 (1.202)	-0.952 (1.186)
Stock Return (t=0)		-0.004*** (0.000)		-0.004*** (0.000)		-0.004*** (0.000)
Turnover		0.004 (0.014)		-0.006 (0.014)		0.005 (0.014)
Ln(Size)		-0.001*** (0.000)		-0.001*** (0.000)		-0.001*** (0.000)
Stock Return Volatility		-0.001*** (0.000)		-0.001*** (0.000)		-0.001*** (0.000)
VIX		-0.035*** (0.003)		-0.034*** (0.003)		-0.034*** (0.003)
S&P 500 Index Return		-0.004*** (0.000)		-0.004*** (0.000)		-0.005*** (0.000)
Stock Return (Last Month)		-0.000*** (0.000)		-0.000*** (0.000)		-0.000*** (0.000)
Institutional Ownership		-0.010*** (0.002)		-0.011*** (0.002)		-0.010*** (0.002)
Stock Bid-Ask Spread		-0.076 (0.091)		0.027 (0.088)		-0.042 (0.090)
Stock Return Skewness		0.000 (0.000)		0.000 (0.000)		0.000 (0.000)
Intercept	-0.005*** (0.001)	0.030*** (0.004)	-0.004*** (0.001)	0.033*** (0.004)	-0.005*** (0.001)	0.029*** (0.004)
Year Dummies	Yes	Yes	Yes	Yes	Yes	Yes
Industry Dummies	Yes	Yes	Yes	Yes	Yes	Yes
Adjusted $R^2$	0.181	0.262	0.180	0.262	0.183	0.265
Observations (1000s)	1,783	1,783	1,783	1,783	1,783	1,783

Panel B: Post-Estimation Tests						
	(1)	(2)	(3)	(4)	(5)	(6)
Lending Fee × (1 + Zero Volume)	-0.870*** (0.094)	-0.899*** (0.097)				
Lending Fee × (1 + Banned)			-3.142*** (0.439)	-3.112*** (0.423)		
Lending Fee × (1+ Banned × (1+ Zero Volume))					-4.025*** (1.181)	-3.978*** (1.173)

**Table VI: The Effect of Lending Fees on Implied Volatilities of Puts and Calls**

This table reports the results of panel regressions with year and industry dummies. The sample period covers the years 2007-2010. Standard errors (in parenthesis) are adjusted by clustering that accounts for heteroskedasticity and dependence of observations across the same stock and same year. We denote by \*\*\*, \*\*, \* the significance at the 1%, 5%, and 10% levels, respectively.

Panel A: Panel Regressions						
Dependent Variable:	IV Put			IV Call		
	(1)	(2)	(3)	(4)	(5)	(6)
Lending Fee	1.555*** (0.098)	1.608*** (0.098)	1.546*** (0.097)	0.337*** (0.078)	0.424*** (0.076)	0.335*** (0.078)
Zero Volume	-0.042*** (0.002)		-0.042*** (0.002)	-0.033*** (0.002)		-0.033*** (0.002)
Lending Fee × Zero Volume	0.010 (0.178)		0.018 (0.179)	0.305** (0.133)		0.317** (0.135)
Banned		0.092*** (0.019)	0.117*** (0.020)		0.043*** (0.016)	0.072*** (0.017)
Lending Fee × Banned		1.697*** (0.581)	1.902*** (0.609)		-0.007 (0.491)	0.297 (0.415)
Banned × Zero Volume			-0.135*** (0.031)			-0.152*** (0.025)
Lending Fee × Banned × Zero Volume			-1.090 (1.081)			-1.839** (0.855)
Stock Return (t=0)	0.001*** (0.000)	0.001*** (0.000)	0.001*** (0.000)	-0.002*** (0.000)	-0.002*** (0.000)	-0.002*** (0.000)
Turnover	1.374*** (0.073)	1.503*** (0.078)	1.374*** (0.072)	1.287*** (0.069)	1.382*** (0.074)	1.287*** (0.069)
Ln(Size)	-0.040*** (0.001)	-0.035*** (0.001)	-0.040*** (0.001)	-0.045*** (0.001)	-0.042*** (0.001)	-0.045*** (0.001)
Stock Return Volatility	0.050*** (0.003)	0.050*** (0.003)	0.050*** (0.003)	0.047*** (0.003)	0.048*** (0.003)	0.047*** (0.003)
VIX	0.400*** (0.013)	0.395*** (0.013)	0.399*** (0.013)	0.340*** (0.011)	0.337*** (0.011)	0.339*** (0.011)
S&P 500 Index Return	0.004*** (0.000)	0.004*** (0.000)	0.004*** (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
Stock Return (Last Month)	-0.001** (0.000)	-0.001** (0.000)	-0.001** (0.000)	-0.001*** (0.000)	-0.001*** (0.000)	-0.001*** (0.000)
Institutional Ownership	-0.074*** (0.006)	-0.070*** (0.006)	-0.075*** (0.006)	-0.094*** (0.005)	-0.092*** (0.005)	-0.094*** (0.005)
Stock Bid-Ask Spread	3.799*** (0.360)	3.556*** (0.338)	3.740*** (0.355)	3.772*** (0.358)	3.662*** (0.346)	3.756*** (0.356)
Stock Return Skewness	-0.001 (0.002)	-0.001 (0.002)	-0.001 (0.002)	-0.001 (0.002)	-0.001 (0.002)	-0.001 (0.002)
Intercept	1.140*** (0.033)	1.036*** (0.031)	1.142*** (0.033)	1.271*** (0.031)	1.191*** (0.028)	1.272*** (0.031)
Year Dummies	Yes	Yes	Yes	Yes	Yes	Yes
Industry Dummies	Yes	Yes	Yes	Yes	Yes	Yes
Adjusted $R^2$	0.675	0.672	0.676	0.674	0.672	0.675
Observations (1000s)	1,783	1,783	1,783	1,783	1,783	1,783

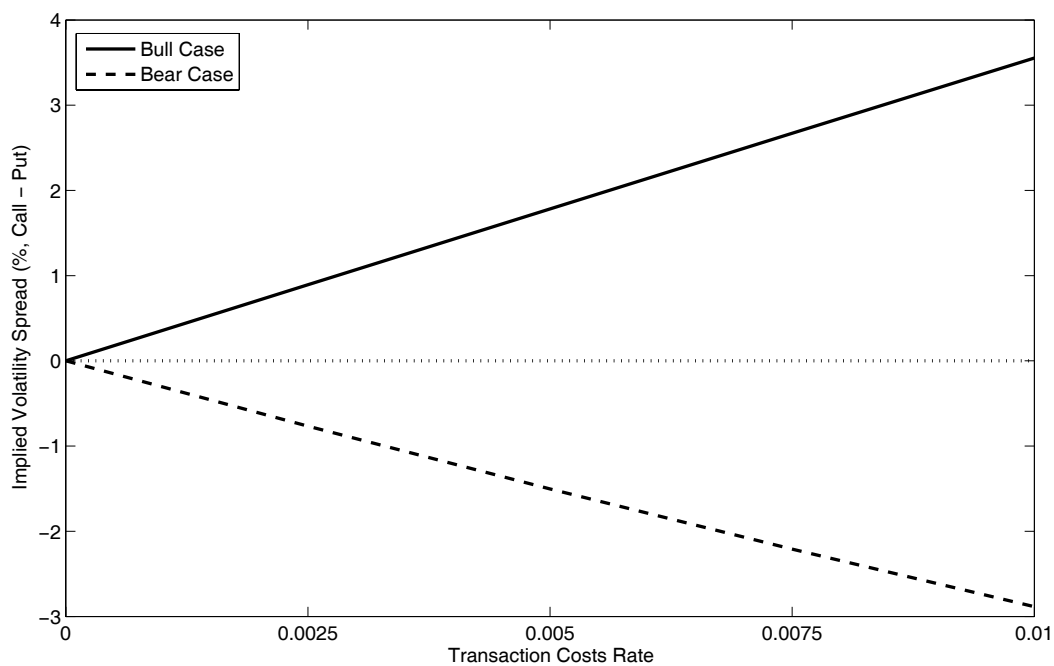
  

Panel B: Post-Estimation Tests						
	(1)	(2)	(3)	(4)	(5)	(6)
Lending Fee × (1 + Zero Volume)	1.565*** (0.187)			0.642*** (0.127)		
Lending Fee × (1 + Banned)		3.305*** (0.583)			0.417 (0.494)	
Lending Fee × (1+ Banned × (1+ Zero Volume))			2.358** (1.014)			-1.207 (0.879)



**Table VII: Continued**

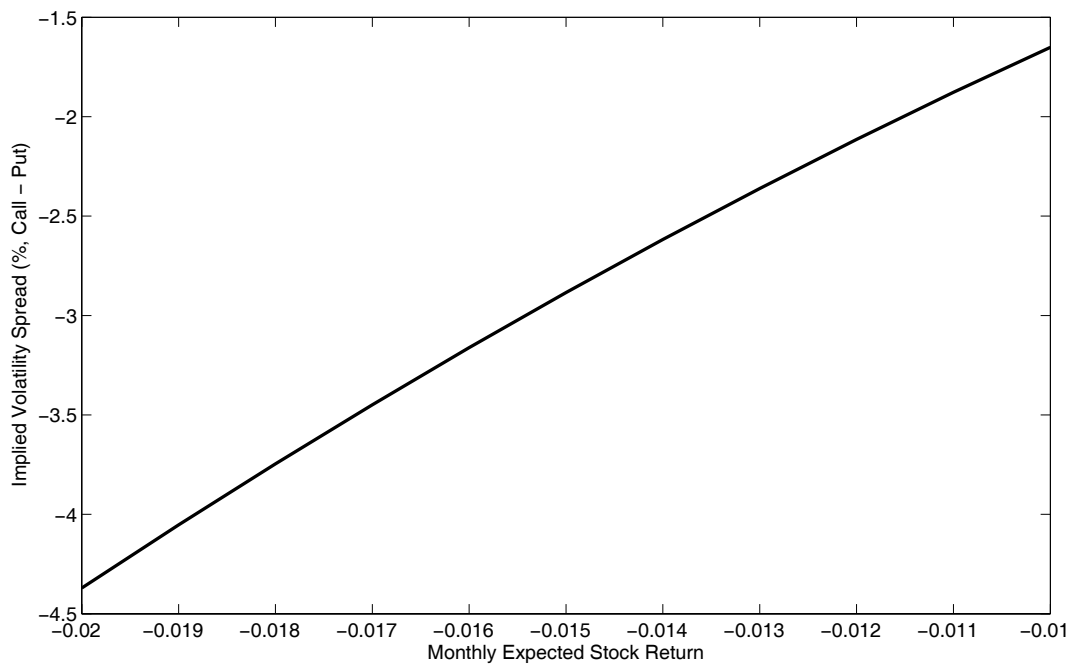
Panel B: Post-Estimation Tests	(1)	(2)	(3)	(4)	(5)	(6)
IVS $\times$ (1 + Zero Volume)	1.728*** (0.126)					
IVS $\times$ (1 + Banned)		8.002*** (1.055)				
IVS $\times$ (1+ Banned $\times$ (1+ Zero Volume))			3.654** (1.528)			
LowIVS $\times$ (1 + Zero Volume)				-0.230*** (0.021)		
LowIVS $\times$ (1 + Banned)					-2.081*** (0.457)	
LowIVS $\times$ (1+ Banned $\times$ (1+ Zero Volume))						-2.082** (0.827)



**Figure 1: Implied Volatility Spread and Transaction Costs**

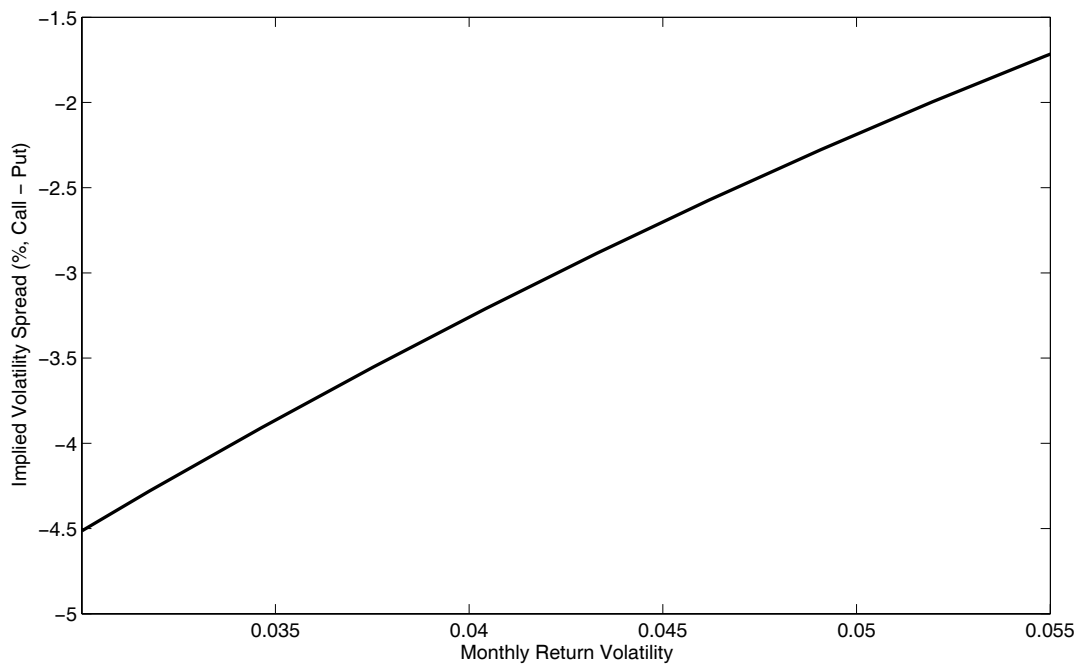
This figure shows how the difference in the hedging cost of call and put option (expressed in terms of the spread between their implied volatilities) varies with trading costs rate  $\theta$ . We show the results for both the Bull Case and the Bear Case. The parameter values used to generate the results are as follows:  $r = 0$ ,  $S_0 = 100$ ,  $K = 100$ ,  $T = 1/12$ ,  $n = 2$ ,  $\sigma = 0.15$ , for the Bull Case,  $\mu = 0.18$ , and for the Bear Case,  $\mu = -0.18$ .





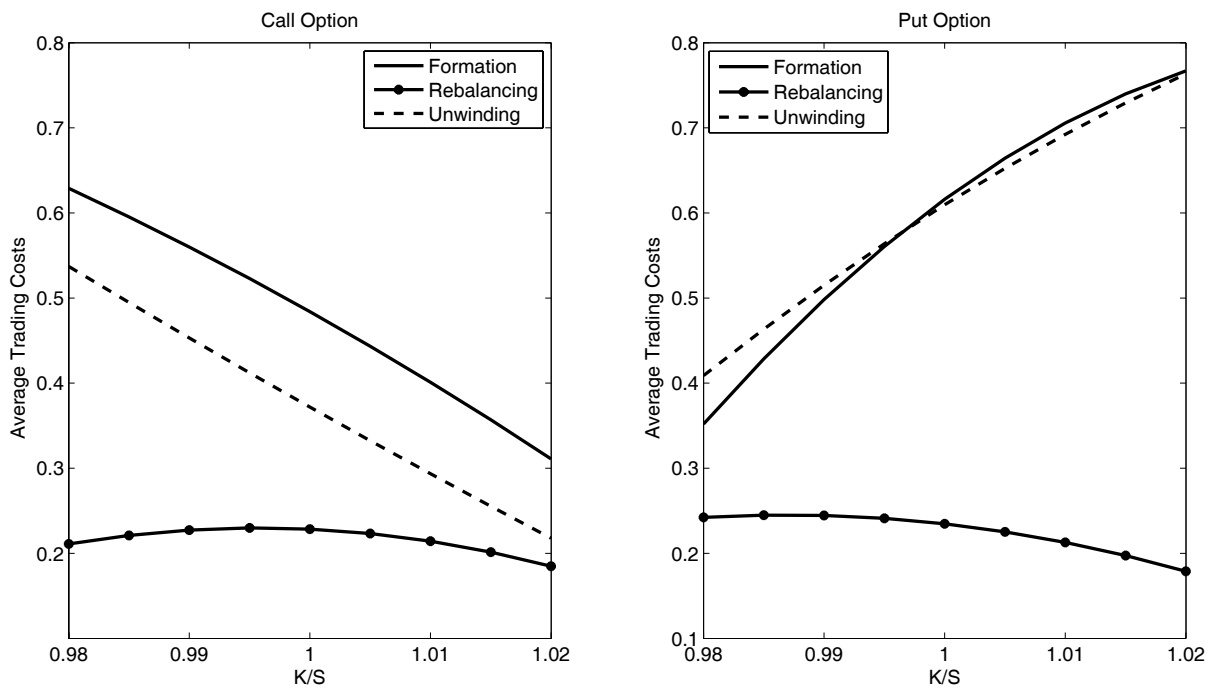
**Figure 2: Implied Volatility Spread and Expected Return**

This figure shows how the difference in the hedging cost of call and put option (expressed in terms of the spread between their implied volatilities) varies with the monthly expected return of the underlying stock. The parameter values used to generate the results are as follows:  $r = 0$ ,  $S_0 = 100$ ,  $K = 100$ ,  $\theta = 0.01$ ,  $T = 1/12$ ,  $n = 2$ , and  $\sigma = 0.15$ .



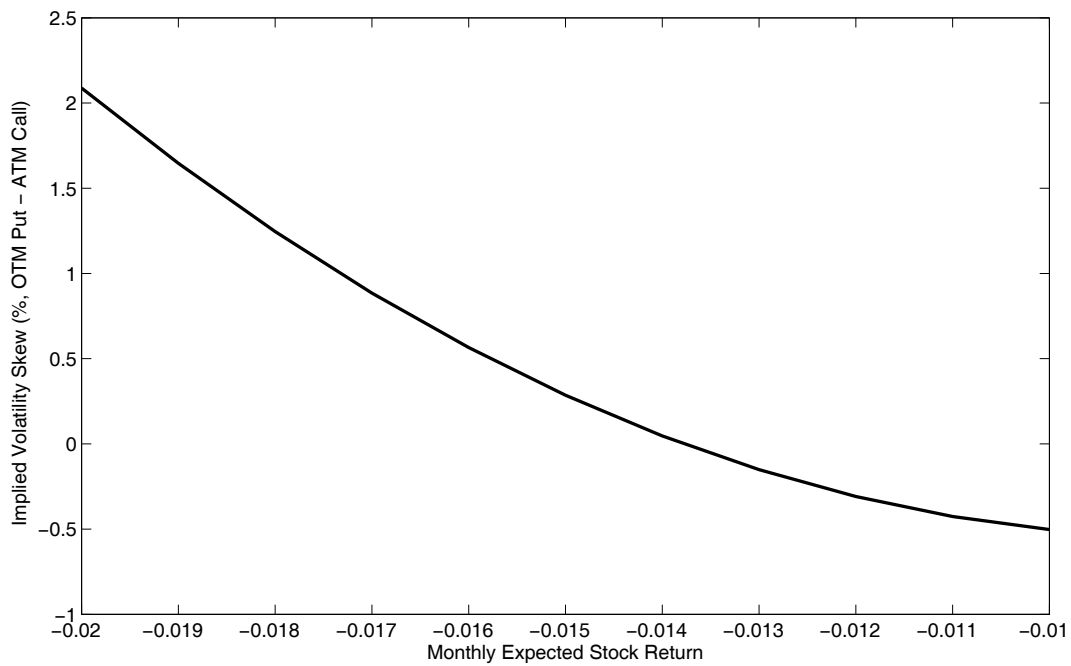
**Figure 3: Implied Volatility Spread and Return Volatility**

This figure shows how the difference in the hedging cost of call and put option (expressed in terms of the spread between their implied volatilities) varies with the monthly return volatility of the underlying stock. The parameter values used to generate the results are as follows:  $r = 0$ ,  $S_0 = 100$ ,  $K = 100$ ,  $\theta = 0.01$ ,  $T = 1/12$ ,  $n = 2$ , and  $\mu = -0.18$ .



**Figure 4: Average Trading Costs and Option Moneyness**

This figure shows how the average trading costs incurred when hedging the call or put option change with the options' moneyness. The parameter values used to generate the results are as follows:  $r = 0$ ,  $S_0 = 100$ ,  $\theta = 0.01$ ,  $T = 1/12$ ,  $n = 2$ ,  $\mu = -0.18$ , and  $\sigma = 0.15$ .



**Figure 5: Implied Volatility Skew and Expected Return**

This figure shows how the implied volatility skew (the difference in the implied volatility of an OTM put and that of an ATM call) varies with the monthly expected return of the underlying stock. The moneyness used for the OTM put is  $K/S_0 = 0.98$ . The other parameter values used to generate the results are as follows:  $r = 0$ ,  $S_0 = 100$ ,  $K = 100$ ,  $\theta = 0.01$ ,  $T = 1/12$ ,  $n = 2$ , and  $\sigma = 0.15$ .