Coherent Preferences and Reference Point Updating in Bargain, Competition, and Interactive Trading in Stock Market

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We welcome comments from the fields of behavior analysis, psychology, sociology, econophysics as well as from economics, finance, and management.
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Extended Abstract

(298 words)

Individuals have preferences and beliefs in their judgment and decision making. These are two broad topics in behavioral economics and finance. However, it is a challenge to find the right way to measure individual preferences in choice and beliefs in assessment because such behaviors are unobservable outside laboratory. We assume that a speculative trader might trade a stock as any another trader or collective traders have done in intraday trading although we observe that he trades it at a sure price. We measure individual decision weights in preferences by using collective cumulative trading volume distribution over a price range and determine a reference point concerning assessment value in beliefs by the maximum volume price. We test the hypothesis, from the microstructure of the distribution, that prospect theory traders are boundedly rational arbitrageurs and that they demonstrate coherent preferences in intraday trading in the stock market. That is, they search for a reference point intelligently in intraday trading, tend to have gains and losses arbitrage that brings stock price back to it, and adapt to any prospect or outcome by assigning decision weights in preferences in the allocation of final trading wealth. Testing the hypothesis against a set of explicit models of coherent preferences, we detected individual coherent preferences using high frequency data in the Chinese stock market. It holds true because 82.42 percent of total tests supports it. From time to time, moreover, speculative traders update the reference point, make it jump discontinuously, and generate a price volatility mean return in intraday trading. It has 11.92 percent chance to occur. To extent, we can infer that skewed cumulative trading volume distribution might reveal individual asymmetric preferences over gains and losses in the stock market. It suggests potential psychological and behavioral applications in economics, finance, management, and social sciences.

Key words: coherent preferences, boundedly-rational arbitrageur, prospect theory, volume distribution, market microstructure, decision weight, reference point

JEL Classifications: D03, C60, D30

1. INTRODUCTION

Finance literature has traditionally focused on asset price and return patterns and to a much lesser extent on trading volume. Trading volume plays no role in neoclassical finance models, such as CAPM (Sharpe, 1964), ICAPM (Merton, 1973), option pricing model (Black and Scholes, 1973), efficient market hypothesis—EMH (Fama, 1970), and arbitrage pricing theory (Ross, 1976), etc. Moreover, these models assume that market individuals behave in a rational and independent manner, and price volatility follows a Brownian motion (Samuelson, 1965). However, empirical investigations have demonstrated that rational arbitrage becomes ineffective in extreme circumstances and fail to bring stock price back to its fundamental value (Shleifer and Vishny, 1997), irrational traders can have significant impact on asset prices even when their wealth becomes negligible (Kogan et al., 2006; Chen, Huang, and Da, 2016), and the stock price index exhibits excessive volatility about 5~13 times larger than that supposed by the models (Shiller, 1981). Also, price changes do not follow random walk (Mandelbrot, 1963; Lo and MacKinlay, 1988), contrary to the predictions of rational models and Brownian motion of price in the neoclassical paradigm.

In the past 20 years, financial researchers have paid increasing attention to the behavioral implications of trading volume, where a list of explanations has been provided, such as attitude toward risk (Lo and Wang, 2006), a link between overreaction and underreaction (Lee and Swaminathan, 2000), overconfidence (Odean, 1999; Barber and Odean, 2000; Biais et al., 2005; and Barber et al., 2009), disagreement (Glaser and Weber, 2007; Hong and Stein, 2007; Chang et al., 2013), attention (Barber and Odean, 2008; Hou, Peng, and Xiong, 2009; and Preis, Susannah and Stanley, 2013), psychological biases (Barber, Odean, and Zhu, 2009), entertainment (Dorn and Sengmueller, 2009), sensation seeking (Grinblatt and Keloharju, 2001), sentiment (Han, 2007), and gender (Barber and Odean, 2001), etc. These studies suggest that there exists a link between assessment value in beliefs and trading volume in preferences over all prospects.

Insert Figures 1 about here

Trading individual stock gradually shows a limited number of cumulative trading volume distribution patterns over a price range on a trading day (Shi, 2006). There exists a microstructure of volume distribution in this typical complex system (see Figures 1), that is a consequence of coherence with a reference point at which there is a maximum volume price in collective interactive trading. The coherence is a constant interaction over a trading price range if the sum of momentum force (one variable) and reversal force (the other variable) is a constant. Analyzing the empirical data, Shi (2006) concludes that the price-volume joint behavior is coherent and stationary to a certain extent although the reference point jumps from time to time on a trading day[1]. For an illustration of this, please note an analogy between a partially stationary system

[1] Shi (2006) finds that a price-volume joint behavior exhibits coherence instead of random walk although a reference point is updated and jumps discontinuously from time to time on a trading day. It resembles a probability wave in which the intensity of price volatility is measured by cumulative trading volume distribution rather than the magnitude of price volatility relative to the reference point. Because economics, finance, management, and psychology specialists are not familiar with the probability wave that is borrowed from quantum physics (Born, 1926), we do not use this terminology in this paper.

in Figure 1 and a partially stable system in Figure 2.

Insert Figure 2 about here

However, the behavioral implications in cumulative trading volume distribution have not yet been understood by researchers working in economics, finance, and management because they violate the expected utility theory (Friedman and Savage, 1948), which goes back to Von Neumann and Morgenstern (1944), and the volume distribution does not exhibit a normal or log-normal form, contrary to the independent trading assumption in neoclassical finance (Samuelson, 1965). We address this important topic based on the empirical findings in stock markets, two explicit models for cumulative trading volume distribution (Shi, 2006), and theories in psychology and behavioral economics such as behavior analysis (Pavlov, 1904; Skinner, 1938; Pierce and Cheney, 2004; Staddon, 2010)[2], cognitive psychology, and prospect theory (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992).

It is a long-standing tradition in psychology that subjective judgment depends on a reference point in beliefs and the operant behavior adapts to an outcome relative to it by trading frequency or probability in preferences. A reference point has been introduced into behavioral economics by prospect theory, in which one of important behavioral characteristics is aversion to loss realization (Kahneman and Tversky, 1979). The feature is afterward tested in the financial market, depicted by individual disposition effect (Shefrin and Statman, 1985), and modeled by an S-shaped value

[2] Home of the science and practice of behavior analysis is at https://www.abainternational.org/welcome.aspx

function (Tversky and Kahneman, 1991). Disposition effect is the observation that investors tend to realize gains more than losses. In the past 20 years, the reference point has been extensively studied in finance (Arkes et al., 2010, and Barberis, 2013a). Several candidates have been suggested for a reference point in trading, for example, an initial price purchased (Shefrin and Statman, 1985), prior investment performance (Barberis, Huang, and Santos 2001), individual expectations about future outcomes in a full distribution (Kőszegi and Rabin, 2006), price as a combination of the first and the last price in a time series (Baucell, Weber, and Welfens, 2011), and even a trick valuation (Ariely, Loewenstein, and Prelec, 2003) or manipulated expectation (Song, 2016), etc. Arkes et al. (2008) note the faster adaptation of a reference point to gains than losses, explain this by mental accounting (Thaler, 1985), and relate it to asset pricing. Mental accounting is the set of cognitive operations used by individuals and households to organize, evaluate, and keep track of financial activities (Thaler, 1999). In experimental settings, Ariely, Loewenstein, and Prelec (2003) find that subjects respond to subsequent changes coherently although a reference point in anchoring a trick valuation is arbitrary. Anchoring, as a cognitive bias, occurs when individuals use an initial piece of information to make subsequent judgments. The coherence does not diminish when they provide valuations in a market context.

Beliefs and preferences are two essential and broad topics in behavioral economics (Thaler, 2016). Prospect theory concerning value function over gains and losses in beliefs has advanced to cumulative prospect theory computing indirectly nonlinear decision weights in preferences (Tversky and Kahneman, 1992; Wakker, 2010 p. 342).

Decision weights are introduced in cumulative prospect theory (Barberis and Huang, 2008). In prospect theory, a risk attitude is a combination of both risk attitude towards outcomes relative to a reference point through a value function for beliefs in prospect theory and a risk attitude towards probabilities through a weighting function for preferences in cumulative prospect theory (Fennema and Wakker, 1997). It is descriptively accurate in psychological experiments.

However, there are some limitations in the application of prospect theory without a trading volume dimension in finance. Disposition is one of two behaviors in trading, such as purchases and sales in stock markets. Moreover, it is not normatively adequate for a power law based-value function in prospect theory. It has to be modified by decision weights in cumulative prospect theory. In addition, Shi et al. (2013) find the coexistence of apparently contradictory anomalies such as momentum and reversal trading in stock market by employing correlation analysis between return and change in trading volume in any two consecutive trading days. Individuals in the real world might show more behavioral characteristics than those in experimental settings (Hens and Vlcek, 2011). For example, investors are more likely to sell a security when the magnitude of gains or losses increases (Ben-David and Hirshleifer, 2012). There is the V-shaped disposition effect in response to extreme winners and losers, and An (2016) applies it to finance and finds that stocks with large unrealized gains or large unrealized losses outperform others in the following month.

A major challenge in the application of prospect theory to behavioral finance is how to find a measurable trading representation for risk of a particular subject outside the laboratory. In experimental settings, subjects are typically given a representation for any risk they are asked to consider, for example, an 85:15 bet in preferences to win 100 yuan or win nothing with the alternative of receiving 80 yuan for sure. Nevertheless, what are the probabilities of a subject preferring to trade individual stock at a price of 5.66 yuan and at a price of 5.70 yuan on a trading day in stock market, respectively?

We strive to fill this gap by using the microstructure of cumulative trading volume distribution because of several reasons. First, the cumulative trading volume distribution is observable and measurable. Second, it contains information about bargain (interaction) between buyers (bid) and sellers (ask) or between momentum traders and reversal traders, and competition among traders at each of two market sides in stock market. Third, it is normatively adequate in mathematics that there are two sets of explicit models describing the volume distribution (Shi, 2006). Fourth, it is reliable using a high frequency data test.

We shall explain why trading volume distribution is important in financial studies because it might represent individual trading preferences in judgment and decision making, and how we measure individual decision weights at each prospect or an outcome in trading because individual mental representation is always unobservable and decision weights are computed indirectly with the help of a weighting function in cumulative prospect theory (Tversky and Kahneman, 1992; Barberis, 2013a). We shall identify the determinant of a reference point in intraday trading if a stock fundament is not a reference point in beliefs because Kahneman and Tversky (1979) offer little guidance on how a reference point is determined (Arkes et al., 2008; Barberis, 2013a).

According to behavior analysis, observable variables such as trading frequency or trading volume distribution over a price range could represent, to a larger extent, intangible subjective behaviors in judgment and decision making. Barberis, Mukherjee, and Wang (2016) have suggested that, for many investors, their mental representation of a stock is given by the distribution of the stock past returns because those investors believe that the past return distribution is a well and easily accessible proxy for the distribution of the stock future returns, which is their only interest. This mental representation has been used in their model to predict the subsequent stock return. It could be traced back to Benartzi and Thaler’s (1995) influential work on the equity premium puzzle. However, a reference point in one’s beliefs is time varying and can change during analysis (Chen and Rao, 2002; Wakker, 2010 p. 234). It jumps from time to time on a trading day, generating a price volatility mean return in the market and causing, most probably, individual preference reversal. The probability weighting in the past is time inconsistent with that in the future (Barberis, 2013b). Thus, it is worthy to search for better mental representation *in a non-time series* to overcome this uncertainty in judgment and decision making.

Trading usually produces only costs but has utility if it involves some gains or losses. Cumulative trading volume distribution over a price range is a consequence of preferences in choices and beliefs in assessments in interactive trading. It can be simulated by two sets of explicit models (Shi, 2006). General speaking, it is stationary and time independent in a trading day. We consider that it represents individual trading probability or decision weight at each prospect or an outcome in intraday trading. That is, it might represent individual stationary trading preferences over a price range indirectly in stock market.

Prospect theory traders are those who follow the behaviors portrayed by prospect theory but show the trading preferences that respond to price volatility by cumulative trading volume distribution and the trading beliefs that are based on a reference point at which is the maximum volume price. We assume that they are boundedly rational arbitrageurs who tend to have gains-losses arbitrage between trading price and the reference point no matter whether the reference point corresponds to a fundamental value, and demonstrate coherent preferences in intraday trading. Coherent preferences are intelligent and cooperative choices in stock trading if the sum of momentum trading (a variable) and reversal (arbitrage) trading (the other variable) is a constant over all prices. It is time independent about coherent preferences and a reference point in beliefs in a certain time interval. A reference point is different from an equilibrium price in neoclassical economics and finance, where any trading price is an equilibrium price between buyers and sellers, and all significant factors such as price, supply, and demand remain more or less constant over a period. In addition, it jumps discontinuously from time to time in a trading day in stock market whereas a price mean value or an expected outcome moves smoothly over time. Thus, it is different from the reference point of expected outcomes in the KR model (Köszegi and Rabin, 2006). We are able to detect a reference point jump between two maximum volume prices in the volume distribution (see (d) in Figure 1).

The main contributions of this paper are the following: 1) We study individual trading probabilities and preferences that respond to price volatility in a given time interval in the stock market; 2) We measure the individual decision weights in uncertainty (preferences) through the collective cumulative trading volume distribution and determine a reference point for behavioral value in risk (beliefs) by the maximum volume price in the stock market; 3) We identify the individual boundedly rational arbitrage and detect coherent preferences in intraday trading; 4) We find that traders following descriptive prospect theory update their reference point from time to time in a trading day; 5) We explore a new framework to model nonlinear behavior and compare the model(s) we use in our tests with the functions in prospect theory. Finally, our test is robust and reliable using tick by tick high frequency data in the Chinese stock market "because financial markets have the features that should make it hardest to find evidence of misbehavior (Thaler, 2016)".

The most obvious limitation of the manuscript is that we currently focus only on data of the Chinese stock market. For that reason we plan to extend empirical tests for behavioral patterns across more countries such as developed markets in the United States and the EU. Second, we focus on boundedly rational arbitrage, coherent preferences, and the updating of a reference point, but we have not yet studied the *loss aversion* by skewed trading volume distribution and the *diminishing sensitivity* in prospect theory. In addition, although we test our hypotheses against a set of explicit models based on coherent preferences, we do not explain how they are derived from a behavioral theory and what relation there is between our models and prospect theory functions. All of these topics are left for the future research.

The organization of the paper is as follows: section 2 offers two hypotheses, one related to the measure of decision weights, and another related to the determinant of a reference point in competition for limited resources in interactive trading; section 3 introduces models and tests results; section 4 is devoted to discussions and potential applications; and the final section includes summaries and conclusions.

2. HYPOTHESES AND THE MEASURE OF DECISION WEIGHTS AND A REFERENCE POINT IN TRADING

Individuals are intelligent, adaptive, and competitive among themselves. They fail to have the arbitrage that brings stock price back to its fundamental value in a short term in the real world (Shleifer and Vishny, 1997). It requires a wide range of knowledge to study interdisciplinary fields such as behavior analysis, cognition, judgment and decision making, social psychology, etc.

2.1 Two Hypotheses

Shi (2006) has assumed that there exists a negative feedback, reversal, and restoring force towards a reference point in intraday trading, based on the microstructure of cumulative trading volume distribution over a price range.

To trade a stock on a trading day, an individual trader usually makes a decision in two phases. First, he evaluates its value in beliefs with expectation on return and decides whether it is worthy to be traded based on his own information. A trader has no impact on price at all without a trading action. Here, everyone might have their own reference point in judgment and decision making because of the decision frame (Tversky and Kahneman, 1981). Once he gets ready to buy or sell it with expectation on return in a time series, in the second phrase, he responds to price volatility accordingly and has chance to trade it at any trading price over all prospects in bargain and competition for a limited number of the shares consumed. That is, an individual trading preferences and beliefs in judgment and decision making should display over all prospects by trading volume distribution.

The prospect theory individuals usually follow what prospect theory suggests in decision making. They are sensitive to price volatility or gains-losses defined relative to a reference point in intraday trading. They prefer to buy an individual stock at the price as low as possible or to sell it as high as possible. Whereas a prospect theory individual is waiting to buy an individual stock at a lower price, he is also concerned that the price will go up, in which case he will have to buy it at a higher price later because of competition with other participants.

Individual traders adapt to an outcome relative to a reference point by operant frequency (Staddon, 2010). Trading frequency is approximately equal to trading volume probability or trading weights in stock market. We assume that a prospect theory individual might trade a stock as any other trader or collective traders have done in intraday trading. Whereas an investor buys a stock at a sure price, he has a chance to sell it at the same price at the moment if having different information. Thus, we might consider that the collective cumulative trading volume distribution of individual stock represent individual trading probability or decision weight in the allocation of his final trading wealth over all prospects.

In addition, a prospect theory individual is a reference point-dependent. The cumulative trading volume distribution has a maximum volume price in intraday trading (Shi, 2006). It is a consequence of the coherence that the sum of momentum trading and reversal trading is equal to a constant over all prospects. We assume that he assesses the value of individual stock with biases in beliefs and acts as a boundedly rational arbitrageur who tends to have the gains and losses arbitrage that brings stock price back to the reference point for a behavioral value in trading. Thus, we propose a coherent preferences hypothesis and a reference point updating hypothesis. We will examine both in Section 3.

A Coherent Preferences Hypothesis (Hypothesis One): Prospect theory individuals who are boundedly rational arbitrageurs demonstrate coherent preferential behavior in intraday trading in the stock market if, at each prospect or an outcome, they assess the behavioral value of trading individual stock with cognitive biases, search for a reference point in beliefs intelligently, and adapt to trade it by decision weights in preferences in uncertainty accordingly in terms of gains-losses arbitrage to the reference point at which is the maximum volume price in the volume distribution over a price range. The trading preferences or trading volume probabilities follow a set of explicit models of coherent preferences—a set of the absolute of zero-order Bessel eigenfunction (see Figure 3).

Insert Figure 3 about here

A Reference Point Updating Hypothesis (Hypothesis Two): Prospect theory traders update a reference point for assessment value of individual stock in beliefs and adapt to a new reference point intelligently from time to time on a trading day, generating the reference point jump and a price volatility mean return.

2.2 Individual Expectation and Collective Decision Weights in Actual Trading

There are two kinds of preferences in individual trading: individual expectation of return in a time series and individual choices over all prospects in uncertainty at the moment in a time interval.

There has been a tradition in the study of individual preferences in expectation of return in a time series. Individuals are boundedly rational in response to prior outcomes in beliefs—gain reinforcement and loss punishment (Rachlin, 1995). They tend to trade more frequently in general if they are reinforced by gain. Otherwise, they tend to trade less frequently if they are punished by loss[3]. For example, commonly there is a positive correlation between return and change in trading volume but sometimes there is a negative correlation (Shi et al. 2011). It has been supported that the prior outcomes affect subsequent risk-taking behavior in behavioral economics. Thaler and Johnson (1990) find that "when faced with sequential gambles, traders are more willing to take risk if they made money on prior gambles than if they lost (Barberis, Huang, and Santos, 2001)". In addition, individual traders are influenced not only by previous traders but also by subsequent traders. For example, a

[3] Some studies find contradictory results: there is negative correlation between return and trading volume or trading frequency (Shi et al., 2010; Ben-David and Hirshleifer, 2012; and Imas, 2016, etc)

momentum trader might be elicited to buy the stock he has sold at loss just before because the price bounces up. Thus, prospect theory individuals are interactive among themselves rather than being independent in the stock market.

As a response to information and news, prospect theory individuals assess the value of individual stock with cognitive biases in beliefs and trade stocks accordingly as trading frequency (trading volume) increases or decreases in preferences with expectation of return in a time series (see Figure 4)[4].

Insert Figure 4 about here

Second, individual trader expects a full distribution of outcomes and prefers trading a stock with a probability at each prospect over a price range in uncertainty. He prefers to buy individual stock at a price as low as possible or to sell it as high as possible. However, how do we measure individual trading probabilities or preferences at each prospect or any an outcome over a price range if individual mental representation is unobservable and hardly measurable in stock market?

To this end, let us consider how we measure collective decision weights and determine a reference point in interactive trading among all participants. According to behavior analysis, subjective behaviors should be exhibited to a larger extent by external and observable behaviors such as trading frequency in response to gain reinforcement or loss punishment. From a probabilistic point, if final trading volume

[4] Operant behavior is livingly illustrated in behavior analysis (Pierce and Cheney, 2004). Skinner (1938) is a pioneer in the field who designs an instrument to study how reinforcement affects the operant frequency of a mouse in learning. It can be traced back to the seminal classic conditioning of Pavlov (1904) who investigates a dogs' expectations on foods in learning by the measure of the animal saliva volume.

is much greater than any a tick-by-tick trading volume on a trading day, then the trading frequency at each price is close to cumulative trading volume probability in the volume distribution. It is true in our study on Chinese market because daily final trading volume is about 360,000,000 shares in average in our data. So, we could measure the collective trading frequency or trading probability by analyzing the trading volume distribution.

Here, we measure the number of collective traders estimating the final trading volume rather than the number of traders. A unit of trading volume, a share, represents a unit of collective traders. For example, an individual investor who buys 1,000 shares is a representative of 1,000 traders. Therefore, there are more collective traders among a few of institutional traders than those among a larger number of individual traders if the former trade more. A few institutional investors may have stronger impact in trading on stock price than a larger number of individual traders (Nofsinger and Sias, 1999; Gabaix et al., 2006).

Prospect theory traders are heterogeneous in risk attitude. They are intelligent and sensitive to price volatility. They adapt to any outcome or each prospect by trading frequency, decision weights, or trading volume probabilities over a price range in terms of gains and losses defined relative to a reference point. The larger the trading volume at a prospect over a trading price range in a time interval, the larger the trading volume probability the collective traders buy and sell at this price. That is, they prefer to trade more frequently. The cumulative trading volume distribution represents collective trading frequency and trading weight at each prospect over a price range in uncertainty. Thus, we can measure collective trading frequencies, decision weights, or preferences at each prospect over a price range by cumulative trading volume distribution we can observe every day (see Figures 1 and 3). The volume distribution follows a set of the absolute of a zero-order Bessel eigenfunction widely on a trading day (Shi, 2006).

2.3 A Reference Point and Its Updating in Collective Interactive Trading

Based on the microstructure of cumulative trading volume distribution over a price range, we study how collective participants trade individual stock. Suppose that there exists a reference point in beliefs when collective traders buy or sell a stock. They tend to realize gains defined relative to the reference point if the trading price is above it because of the disposition effect. The higher the trading price is above the reference point, the higher value it is in gains arbitrage, the stronger preferences collective traders tend to realize gains, the larger number of shares they tend to sell. It is behavioral preferences from sellers. In the viewpoint of buyers, however, they reluctant to buy because of the limited resources they have. When there are more supply from seller and less demand from buyers, trading price drops.

Moreover, the lower the trading price is below the reference point, the higher the gains arbitrage value over the losses is from a viewpoint of buyers. They have stronger preferences to buy the stock. Purchase quantity increases. In contrast, sellers are reluctantly to sell the stock because of risk seeking over losses and the limited resources the sellers have. When there are more demand from buyers and less supply from sellers, trading price goes up.

In a word, there is gains and losses arbitrage that brings trading price back to the reference point if it disperses.

The higher the absolute of the value is in the S-shaped disposition function in prospect theory, the stronger competition there is for gains and losses arbitrage that brings price back to the reference point, the less shares they are able to pair. Trading volume decreases. It reveals how collective traders are sensitive to price volatility and allocate final risk asset over a price range in a small supply-demand imbalance.

In brief, they tend to have gains and losses arbitrage that brings trading price back to the reference point in beliefs and prefer trading the most volume at the reference point where prospect value is zero. It is fair to both buyers and sellers because the reference point is the maximum utility price in optimal trading. Thus, we are able to determine a reference point in a trading day by the maximum volume price in the volume distribution.

In behavior analysis, collective traders are intelligent and adaptive to an outcome by operant frequency in learning. Learning is initiated by violation of expectation—surprise (Staddon, 2010). Whenever they generate a larger supply-demand quantity imbalance and update a reference point in trading, they adapt to the outcome of a new reference point intelligently in learning. We are able to find the updating of a reference point by two maximum volume prices in the volume distribution (see (d) and (e) in Figure 1).

Thus, we can measure collective decision weights, determine a reference point, and detect its updating by cumulative trading volume distribution.

2.4 Individual Representation via Collective Representation

Prospect theory traders are decision weight dependent in preferences and reference point dependent in beliefs. As discussed before, we can observe, measure, and determine collective decision weights and a reference point in trading by cumulative trading volume distribution.

A trader is a basic unit or element of collective traders. We consider that cumulative trading volume distribution might represent indirectly a trader's trading probabilities or decision weights over all prospects in preferences and determine his reference point concerning behavioral value in beliefs in intraday trading. To understand it, it is helpful to consider at what probability an individual investor might trade a stock at the beginning of a trading day. He expects a full distribution of outcomes in uncertainty and is inclined to trade it at any prospect over all prospects. If one gets ready to trade a stock, he might trade it at any price with a probability over a price range in uncertainty because there exists bargain between buyers and sellers, competition at each of two sides, and interaction among all participants in trading. Although we observe that an individual trader buys or sells a stock at a sure price in intraday trading, he might also trade it with a probability at another price the same as another trader or collective traders have done. Whereas we observe that an investor buys a stock at a sure price, he also has a chance to sell it at the same price. In an extreme scenario, a rational investor might behave as an "irrational" trader might do. In other words, an individual has a chance to trade a stock at this price or at that price in response to price volatility in a trading day because we can observe his trading probability by cumulative trading volume distribution. Thus, it is reasonable to assume that collective cumulative trading volume distribution might represent individual trading probabilities or decision weights over all prospects and the maximum volume price is a reference point in this trading. The advantage of the measure is that it is time independent and contains information about individual bargain, interaction, and competition for limited resources. It is easy to examine.

3. MODELS AND EMPIRICAL TESTS

As explained before, subjective behaviors in trading could be studied by using external and observable behaviors. We test a coherence preferences hypothesis and a reference point updating hypothesis using tick by tick high frequency data over more than two years in the Chinese stock market.

3.1 Data

We test our hypotheses using tick by tick high frequency data in Huaxia SSE 50ETF (510050) in China’s stock market from April 2, 2007 to April 10, 2009. The structure of the high frequency data includes the variables of trading volume and price or time independent cumulative trading volume distribution over all prices. There are two reasons for the selection of the data. First, it was tested in 2009. Second, it experienced a whole course from bubble growth to burst, shrink, and reversal again. SSE Composite Index started from 3252.59 points at the beginning, went up to 6124.04 points, dropped down to 1664.04 points, and further reversed up to 2444.25 points at the end of the period in the Shanghai Securities Exchange (SSE). Obviously, the stock values included in the ETF were overestimated during one period whereas underestimated during another period. Prospect theory traders were either overreaction or underreaction in stock market. Thus, we can infer that individual traders are boundedly rational. They might be irrational in trading (Kogan et al., 2006), contrary to the rational arbitrage assumption in neoclassical finance (Ross, 1976).

There are 740 days and 495 trading days. We obtained 495 cumulative volume distributions over a price range in tests. The data is from the HF2 database of Harvest Fund Management Co., Ltd. We processed the data in two steps. First, we reserved two decimal places in the price by the rounding-off method and added volume at a corresponding price[5]. Second, the cumulative trading volume at each price was divided by the final trading volume across all prices on a trading day. Thus, we had a trading volume probability at each price and obtained its distribution over a price range.

3.2 Models and Test Reports

We test our hypotheses by using two sets of regression models. They are obtained from a price-volume differential equation (Shi, 2006). One is a set of models of coherence preferences with a reference point in interactive trading, and the other is a set of multi-order models with a number of outcomes in independent trading.

3.2.1 Regression models

We examine a coherent preferences hypothesis and detect the updating of a

[5] Original data reserves three places of decimals.

reference point using a set of explicit models of coherent preferences (Shi, 2006). They are expressed by

$$|\psi_m(p)| = C_m |J_0[\omega_m(p - p_0)]|, \qquad (m = 0,1,2,\cdots) \qquad (1)$$

and

$$W(p) = A_m(p - p_o), \qquad (A_m > 0; m = 0,1,2\cdots) \qquad (2)$$

$$\omega_m^2 = v_{tt,m} - A_m = \frac{v}{V} v_{tt,m} = \pi \cdot v_{tt,m} = const.^{6}, \quad (\omega_m > 0; m = 0,1,2\cdots) \qquad (3)$$

where J_0 is a zero-order Bessel eigenfunction; p_0 is a reference point at which trading volume or trading weight is at its maximum; p is the trading price; p-p_0 is price volatility deviation or gain-loss defined relative to the reference point p_0; $W(p)$ is a nonlinear gains-losses utility because A_m, is a variable, $|W(p)|$ is a nonlinear V-shaped arbitrage utility that prospect theory traders tend to bring stock price back to the reference point p_0, and A_m is the magnitude of arbitrage force; π is actual nonlinear decision weight in trading, which is equal to cumulative trading volume v at a price p over final cumulative trading volume V across all prices, and is not an objective probability in expected utility theory in statistical mathematics (Shi, 2006); $\omega_m = \pi v_{tt,m}$ is an eigenvalue, intelligent constant, or coherent force, which is generated by interaction between momentum force $v_{tt,m}$ (a variable) and reversal (arbitrage) force $-A_m$ (the other variable), where the minus sign means that the arbitrage force is always toward the reference point p_0; C_m is a normalized constant; and $|\psi_m(p)|$ is the trading volume probability, trading weight, or trading frequency at each price p (see Figure 3).

We model actual trading volume distribution by equation (1). It represents a

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[6] Here, v_{tt} does not mean $\partial^2 v / \partial t^2$. It is the momentum force expressed by v/t^2 in a time interval [0,t] (Shi, 2006).

function of preferences or decision weights π in trading. Equation (3) is a mathematical expression for coherent preferences. We will discuss the models and the S-shaped value functions in prospect theory in Section 4.

There are three constant coefficients: a normalized constant C_m, a reference point p_0 in trading, and an intelligent constant ω_m in equation (1). They are determined by its nonlinear regression model,

$$\left|\psi_{m,i}(p_i)\right| = C_m \left|J_{0,i}\left[\omega_m(p_i - p_0)\right]\right| + \varepsilon_i\ , \qquad (i = 1,2,3\cdots,n) \tag{4}$$

where n is the number of trading prices over a price range in a trading day; ε_i is random error subject to $N(0,\sigma^2)$; $\left|\psi_{m,i}(p_i)\right|$ is an observable trading weight at a price, and $C_m\left|J_{0,i}\left[\omega_m(p_i - p_0)\right]\right|$ is a theoretical trading weight.

Insert Figure 5 about here

We tested volume distribution using the regression model, equation (4), using the Origin 6.0 Professional software. We ran the program by the Levenberg-Marquardt nonlinear least square method and got C_m, ω_m and p_0 from test reports (see (a) in Figure 5).

We tested significance by using the F statistic. The coefficient of determination R^2,

$$R^2 = \frac{ESS}{TSS} = \frac{TSS - RSS}{TSS} \tag{5}$$

Where $ESS = \sum_{i=1}^{n}\left(\hat{Y}_i - \overline{Y}\right)^2$, $RSS = \sum_{i=1}^{n}\left(Y_i - \hat{Y}_i\right)^2$, and $TSS = \sum_{i=1}^{n}\left(Y_i - \overline{Y_i}\right)^2$ are the explained sum of squares, the residual sum of squares, and total sum of squares, respectively. We have

$$F = \frac{ESS/k}{RSS/(n-k-1)}, \qquad (6)$$

where n and k are the sample size and the number of explanatory variables, respectively. If $F>F_{0.05}$

$$R^2 > R_{crit}^2 = \frac{k \cdot F_{0.05}}{k \cdot F_{0.05} + (n-k-1)}, \qquad (7)$$

then, the test holds true at a 95% significance level. Here, k=1.

Our test result is that 380 out of 495 distributions (about 76.77%) show significance (see (a) in Figure 5). The remainders (about 23.23%) lack significance.

There are two notable characteristics among the distributions without significance in the test. First, the number of trading prices or the sample size is not large enough for a statistical test. It is partly caused by previous data processing. We reserved two decimal places in the price by rounding-off three places. Because of this some information was lost in the data processing.

Insert Table Ⅰ about here

To solve the problem, we added 0.005 in three decimal places in the price and subdivided volume at corresponding prices. Then, we fitted and tested the remainders using equations (4) and (7) again. 28 more distributions showed significance. Thus, 408 distributions (about 82.42%) show significance (see Table Ⅰ). That is, prospect theory traders demonstrate coherence preferences in competition for limited resources.

Second, there are at least two maximum volume prices in the distribution. Prospect theory traders update the reference point, make it jump, and adapt to a new reference point quickly in a large supply-demand quantity imbalance. We can model the distributions by linear superposition of equation (1). It is expressed by

$$|\psi_m(p)| = \sum C_m \left|J_0\left[\omega_{m,n}(p - p_{0n})\right]\right|, \qquad (n = 1,2\cdots) \qquad (8)$$

where n is the number of reference points (here n=2). We test two maximum volume prices in distribution by following the regression model,

$$|\psi_{m,i}(p_i)| = \sum \sum C_m \left|J_{0,i}\left[\omega_{m,n}(p_i - p_{0,n})\right]\right| + \varepsilon_i \qquad (i = 1,2\cdots) \qquad (9)$$

where n=2.

In the tests, $k=2$ and $R_2^2 > R_{2crit}^2$. Our test result is: 59 (11.92% in total) show significance at a 95% level among 87 distributions (see (b) in Figure 5 and Table Ⅰ).

In addition, we have the other set of models from multi-order volume distribution functions (Shi, 2006). They are written as

$$|\psi_{n,m}(p)| = C_{n,m} e^{-\sqrt{A_{n,m}}|p - p_0|} \cdot \left|F\left(-n,1,2\sqrt{A_{n,m}}|p - p_0|\right)\right|, (n,m = 0,1,2,\cdots) \qquad (10)$$

and

$$U_{n,m} = pv_{tt} = const., \qquad (11)$$

$$\sqrt{A_{n,m}} = \frac{U_{n,m}}{1+2n} = const. > 0, \qquad (12)$$

where $F\left(-n,1,2\sqrt{A_{n,m}}|p - p_0|\right)$ is a set of multi-order eigenfunctions, n is the order of the multi-order function (For example, if $n=0$, then the function is exponential at (a) in figure 6. If $n=1$, the function is illustrated at (b) in Figure), $U_{n,m}$ is a constant trading liquidity utility expressed in terms of final trading wealth regardless of gains or losses, and $A_{n,m}$ is an eigenvalue or a constant arbitrage force or a reversal force; $|\psi_{n,m}(p)|$ is trading volume probability or trading weight and might be either exponential or uniform in distribution (see Figure 6).

Insert Figure 6 about here

Shi, Podobnik, and Njavro (2016b) explain that the volume distribution is exponential if prospect theory traders are homogenous in independent trading. Arbitrage force toward a reference point $A_{n,m}$ is a non-zero constant. Otherwise, it is uniform with a number of outcomes or reference points if they are heterogeneous in independent trading because of a decision frame; arbitrage force $A_{n,m}$ is approaching zero.

For convenience, we choose n=1. It is

$$\begin{aligned}|\psi_{1,m}(p)| &= C_{1,m} e^{-\sqrt{A_{1,m}}|p-p_0|} \cdot \left|F\left(-1,1,2\sqrt{A_{1,m}}|p-p_0|\right)\right| \\ &= C_{1,m} e^{-\sqrt{A_{1,m}}|p-p_0|} \cdot \left|1-2\sqrt{A_{1,m}}|p-p_0|\right|, \end{aligned} \tag{13}$$

and its regression model is

$$|\psi_{1,m,i}(p_i)| = C_{1,m} e^{-\sqrt{A_{1,m}}|p_i-p_0|} \cdot \left|F_i\left(-1,1,2\sqrt{A_{1,m}}|p_i-p_0|\right)\right| + \varepsilon_i \ .(i=1,2\cdots) \tag{14}$$

We test the remainders by using equation (14). 23 distributions show significance at 95% level (see (c) in Figure 5). Prospect theory traders tend to trade uniformly and have a number of reference points. The preferences are reference point independent. The remaining 5 distributions still lack significance (see (d) in Figure 5). There is not enough information or data for a significant test. It is about 5.66% in total (see Table Ⅰ).

In our test, hypothesis one holds true if the test shows significance using the regression model, equation (4). Otherwise, we test a volume distribution using a linear superposition model, equation (9). If such a test shows significance, then a reference point is updated in a trading day. It has 11.92% chance to occur in the test. The reference point updating hypothesis is true. Moreover, we test the rest using the first-order eigenfunction in equation (10)—a uniform distribution in independent trading. Our test reports are listed in details in Table Ⅰ.

We did robust tests in a period of more than two years, during which the Shanghai Securities Exchange Composite Index went from 3252.59 points to 6124.04 points, then dropped to 1664.04 points, and further reverted to 2444.25 points at the end of the period. From test reports in Table Ⅰ, we conclude that both hypothesis one and hypothesis two hold true. Prospect theory traders demonstrate coherent preferences in intraday trading. They are boundedly rational arbitrageurs who tend to have the arbitrage that brings stock price back to a reference point concerning a behavioral value at which is the maximum trading volume price in the volume distribution. Moreover, a reference point is updated and jumps discontinuously from time to time on a trading day. We will have further discussions in Section 4.

4. DISCUSSIONS AND POTENTIAL APPLICATIONS

Based on decision weight dependence in preferences, reference point dependence in beliefs, and the microstructure of cumulative trading volume distribution in the stock market, we propose a coherent preferences hypothesis and a reference point updating hypothesis. We examine them using a set of explicit models of coherent preferences, equation (1). They hold true. Let us discuss some questions and explore potential research in the future.

4.1 Trading Volume, Decision Weights, Preferences, and Utility

Trading usually only produces costs but has utility if it involves some gains. When we study individual trading behaviors by the microstructure of cumulative trading volume distribution over a price range in the stock market, the trading is relevant to gains and losses defined relative to a reference point in beliefs as well as to decision weights at each prospect over all prospects in the allocation of final trading wealth in preferences.

The more the prospect theory traders buy and sell individual stock at a price, the stronger they prefer to trade at this price. They prefer to trade the most at a reference point in beliefs where cumulative trading volume is at its maximum. This is fair to both buyers and sellers because it is the maximum utility price in full competition. Thus, a set of explicit trading volume distribution models, equation (1), is a utility function for prospect theory traders who respond to price volatility in intraday trading.

One might ask whether we can determine a reference point by the maximum trading amount price. If the price of a reference point is much greater than the price deviation from the reference point, then cumulative trading amount distribution is close to cumulative trading volume distribution. The maximum trading amount price is close to the maximum trading volume price. It is true in the stock market.

We select cumulative trading volume distribution instead of trading amount distribution as an individual representation for decision weights because of two reasons. First, we are interested in studying the relation between price, trading volume, and trading asset liquidity. The trading liquidity or trading amount is a controllable variable for the change of price and trading volume in a time interval (Shi, 2006; Shi, Podobnik, and Njavro, 2016b). Second, cumulative trading volume distribution does not have a price and thus represents actual nonlinear trading preferences in the allocation of final trading wealth over a price range. However, expected trading liquidity utility expressed in terms of trading wealth is the total sum of the product of trading amount probability and trading liquidity utility at each prospect over all prospects, according to mathematical statistics. It describes independent behavior in probability and mathematical statistics. The trading amount probability and the trading liquidity utility is linear each other. It does not add any new information concerning trading behavior.

4.2 The Nonlinear Models of Value in Beliefs and Decision Weight in Preferences and Asymmetric Preferences over Gains and Losses

Prospect theory is descriptively accurate in experimental settings. Contrary to expected utility theory about rational traders in beliefs in neoclassical economics and price random walk hypothesis about preferences in neoclassical finance, individual risk attitude is nonlinear in beliefs in terms of gains and losses defined relative to a reference point in disposition (Shefrin and Statman, 1985). Tversky and Kahneman (1991) model it by a power law based value function in prospect theory, in which parameters α and β are not a whole number and λ is a constant (Kahneman and Tversky, 1979). That is, a value function is fractal (Mandelbrot, 1967). It is expressed by

$$V(p) = \begin{cases} \lambda_1(p-p_0)^{\alpha} & if \quad p-p_0 \geq 0 \\ -\lambda_2(p-p_0)^{\beta} & if \quad p-p_0 < 0 \end{cases}, \tag{14}$$

where *V(p)* is a S-shaped value function in beliefs, *p* is a trading price, any a prospect, or an outcome, p_0 is a reference point in beliefs, and p-p_0 is the price deviation or gains-losses defined relative to the reference point.

The theory is extended to study individual behavior from risk to uncertainty by engaging cumulative prospect theory (Wakker, 2010, p342), in which decision weights are computed indirectly with the help of probability weighting (Tversky and Kahneman, 1992). They are a function of probabilities π (probability), where π (probability)≠ the objective probability of the state of economy in expected utility theory (Thaler, 2016). These aspects of the theory were inferred from studying the choices subjects made when asked to choose between gambles in experimental settings.

There is another approach in how we model a nonlinear subject behavior such as beliefs in risk and preferences in uncertainty. We follow the direction that was explored by Shi (2006). We assume that α and β are equal to one but λ is a variable in equation (14), i.e. arbitrage force -*A* is a variable in equation (2) if equation (3) is satisfied. Equation (3) is a condition for coherence preferences in stock market. It holds true in our high frequency data tests. Equation (2) is a gains and losses utility in beliefs. It is linear if the arbitrage force -*A* is a constant. Otherwise, it is nonlinear if the arbitrage or reversal force is a variable. We measure nonlinear individual decision weights in preferences directly by cumulative trading volume distribution in the real world and simulate them using equation (1) where we specify an eigenvalue ω_m which is an intelligent constant between momentum trading (one variable) and reversal trading (the other variable).

One might ask a question how you detect the disposition effect in prospect theory from the microstructure of cumulative trading volume distribution using a set of explicit models, equation (1). Cumulative trading volume has the microstructure of skewed distribution relative to a reference point at which is the maximum volume price in the distribution. It might be either a left or a right skewed distribution in the stock market (see (b) and (c) in Figure 1). That is, there is asymmetric selling pressure over gains and losses. We can explain it by disposition effect or its opposite effect in details in another paper.

There is similar evidence of the disposition effect (Shefrin and Statman, 1985; Arkes et al., 2010) and its contrary effect (Hens and Vlcek, 2011; Ben-David and Hirshleifer, 2012; An, 2016) in the financial market. The apparently contradictory behaviors coexist in intraday trading in the stock market because trading volume is a consequence of trading behaviors from buyers and sellers who have contradictory attitudes toward risk in trading (Shi et al., 2013). Kahneman (McGraw et al., 2010), together with others, explains the mix of risk attitudes in comparative judgments of feelings: an individual might consider losses (bad) against other losses (worse) if they tend towards loss realization.

4.3 The Measure of Intangible Individual Preferences in Psychology

We measure individual trading probabilities or trading preferences by collective cumulative trading volume distribution. Whereas we observe that an individual buys or sells a stock at a certain price for sure, he might also have chance to trade it at another price as other individuals have done in intraday trading. That is, an individual trader might trade a stock with a probability at this price or with another probability at that price in uncertainty. In a word, a prospect theory individual might trade a stock the same way as any other trader or collective traders have done although we observe that he trades it at a sure price in intraday trading. We annotate on the cumulative trading volume distribution, inspired by the concept of a probability wave in quantum physics which is proposed by Born (1926)—a Nobel laureate in physics.

There are some advantages to using this measure. First, we are able to measure intangible individual trading preferences in uncertainty and determine a reference point in beliefs in risk choice by observable cumulative trading volume distribution in the stock market. Second, the volume distribution contains information about competition from each side in trading and interaction between buyers and sellers which is determined by an intelligent constant ω_m. Third, it overcomes imperfections in the application of prospect theory such as time inconsistence in probability weighting and the updating of a reference point which might generate individual preference reversal from time to time in a trading day in stock market. Finally, our test is robust and reliable using high frequency data in the Chinese stock market.

4.4 Coherent Preferences in Bargain and Competitive Trading, a Reference Point, and Its Updating

A reference point is the maximum volume price in the volume distribution. There exists one reference point in intraday trading no matter whether stock prices in the

ETF are overestimated or underestimated in an empirical test. Obviously, it does not correspond to its fundamental value in a trading day because of anchoring. That is, prospect theory traders are sensitive to price volatility in intraday trading and tend to arbitrage gains and losses between trading price and a reference point rather than between trading outcome and stock fundamental value. They prefer to trade the most at the reference point and trade less and less in general when the price deviates from it. They are boundedly rational arbitrageurs.

Prospect theory traders demonstrate coherent preferences in intraday trading if there is a reference point or a maximum volume price in cumulative trading volume distribution. In our test, 82.42 percent of total tests shows that prospect theory traders demonstrate coherent preferences significantly in the stock market. It is consistent with the findings in the tests of individual stock by Shi (2006).

There are some precedents in relevant studies about boundedly rational arbitrageurs in finance. Shleifer and Vishny (1997) evidence that arbitrage becomes ineffective in extreme circumstance when prices diverge far from fundamental value. Ross (Kogan et al., 2006), who proposes a rational arbitrage pricing theory (Ross, 1976), together with other scholars in finance, shows himself that speculators' portfolio policies can deviate from their limits long after the price process approaches its long-run limit. Even when they do not survive, they can still have a persistent impact on asset prices. We might further study whether coherence preferences in uncertainty generate a bubble in the stock market, somewhat different from the viewpoint that noise traders limit arbitrage to stock fundamental values (Hu, Pan, and Wang, 2013).

If there are two maximum volume prices in the volume distribution, then it indicates that there are two reference points in trading. Prospect theory traders update a reference point on the trading day. It jumps discontinuously and has 11.92% chances to happen. A reference point jumps from time to time rather than moving smoothly with time on a trading day. The jump of a reference point or the quantum cognition in beliefs might be studied in a perspective of neuroscience.

If there are more than two maximum volume prices in the volume distribution, for example, a uniform distribution, then prospect theory traders are heterogeneous and show reference point independent preferences. They often violate consistency and coherence. They buy and sell individual stock independently with any a number of outcomes because of the decision frame (Tversky and Kahneman, 1981 and 1986). We simulate a linear arbitrage behavior by equation (2) subject to a condition of equation (12) in which the constant arbitrage force -*A* is approaching zero. We model independent preferences in trading by using a set of multi-order models, equation (9). Prospect theory individuals are reference point dependent in beliefs if they are homogeneous. Otherwise, they are reference point independent in beliefs if they are heterogeneous. The reference point independent in beliefs are illustrated at (f) in Figure 1 and at (c) in Figure 6.

4.5 Other Behavioral Features in Trading

Because we restrict ourselves to studying individual coherent preferences and the updating of a reference point in the stock market, we have not yet taken into consideration other individual trading behaviors behind the microstructure of cumulative trading volume distribution such as loss aversion in skewed volume distribution and diminishing sensitivity in prospect theory. In addition, it is still unclear what relation there is between a set of explicit models of coherent preferences and prospect theory functions and how we associate the microstructure of cumulative trading volume distribution with the existing microstructure theory that focuses on how specific trading mechanisms affect the price formation process (O'Hara, 1995). For example, can we predict individual bid-ask behavior and V-shaped asymmetric selling propensity in response to profits (Ben-David and Hirshleifer, 2012; An, 2016) by using cumulative trading volume distribution in financial market?

4.6 Possible Application

There are many possible applications and practices in our study. First, psychology such as behavior analysis, cognitive science, and social psychology, is a basic course for students in economics, finance, and management, similar to how mathematics and physics are basic courses for students in electronic engineering. Second, a return model should have a trading volume dimension in behavioral finance because we could measure individual decision weights in trading by using cumulative trading volume distribution and determine a reference point in interactive trading by using the maximum trading volume price. Third, it helps us to understand market anomalies, for examples, excessive price volatility (Shiller, 1981) because prospect theory traders pair shares in terms of the reference point that is determined by the maximum trading volume price rather than the fundamental value that is hardly estimated in daily trading. Specifically, prospect theory traders are boundedly rational arbitrageurs. They might adapt to outcomes for survival by trading volume and show animal spirits sometimes (Akerlof and Shiller, 2009). They might either over- or under-assess the behavioral value of individual stock in trading. An individual's behavior in social activity such as trading in stock market might be quite different from his or her independent rational behavior (Le Bon, 1982).

5. SUMMARIES AND CONCLUSIONS

Prospect theory is descriptively accurate about individual disposition behavior in economic activity, consistent with a vast number of experimental tests in psychology. We apply it to study individual trading behavior, based on the microstructure of cumulative trading volume distribution over a price range in the stock market.

We measure individual decision weights in trading at each prospect over all prospects by using collective cumulative trading volume distribution and determine a reference point in intraday trading by the maximum volume price in the stock market. It is based on the assumption that a prospect theory individual might trade a stock the same as any other trader or collective traders have done in intraday trading although we observe that he trades it at a sure price. We model individual nonlinear decision weights or trading preferences by using a set of explicit models of coherence preferences, equation (1), if equations (2) and (3) are satisfied.

By empirical testing, we evidence that prospect theory traders are boundedly rational arbitrageurs and demonstrate coherent preferences in intraday trading. They tend to have gains and losses arbitrage that brings stock price back to a reference point concerning behavioral value at which is the maximum volume price. The reference point does not have to correspond to the fundamental value that is hardly estimated in a trading day. In addition, they adapt to any outcome or prospect relative to the reference point in beliefs by decision weights in the allocation of final trading wealth over all prospects in preferences when trading individual stock. They prefer to trade the most at the reference point. It is fair to both buyers and sellers because it is the maximum utility price in the volume distribution model(s). In addition, prospect theory traders update the reference point, which jumps discontinuously from time to time in intraday trading, and adapt to a new reference point or the outcome intelligently from one state of coherent preferences to another.

We predict that coherent preferences, a consequence of individual trading behavior in beliefs and preferences, might explain bubbles in the stock market and the puzzle of high peaked, fat tailed, diminishing, and clustered characteristics in return distribution since Mandelbrot (1963). The behavioral anomaly might also explain asymmetric preferences over gains and losses by skewed trading volume distribution over a price range relative to a reference point in the stock market. We need a unified theory that explains detailed behavior of speculative traders in each contingency and further examines the prediction in the future. It might be interesting to psychologists and social scientists to conduct experiments as well as to theorists to model market dynamics and actual trading behaviors in a unified framework (Shi, Podobnik, and Njavro, 2016b). We expect that someone else would propose better theories in the applications of behavior analysis, psychology, and prospect theory to economics, finance, management, and social sciences.

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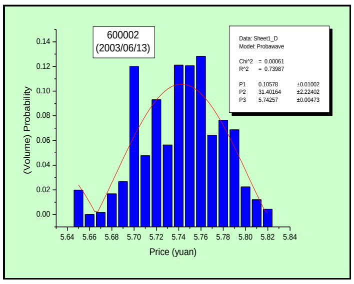
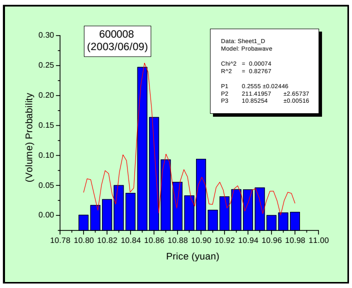
Table Ⅰ: Test Reports on a Reference Price on a Trading Day

	No. of Distributions	Percentage (%)
Total Number of Distributions	495	100
A Reference Price in Trading	408	82.42
Updating of A Reference Price	59	11.92
Multiple Reference Prices in Independent Trading	23	4.65
Inadequate Information from Trading	5	1.01

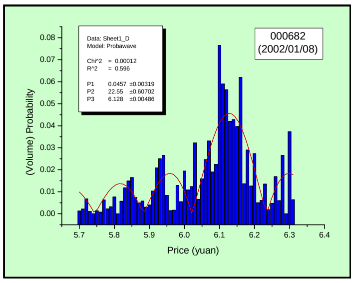
Note: There are 4 trading hours per day in China stock market.



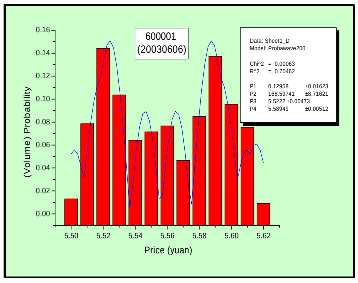
(a) close to but not a normal distribution



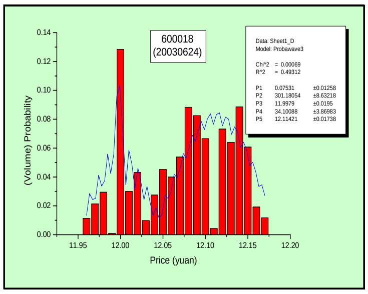
(b) close to but not a log-normal distribution



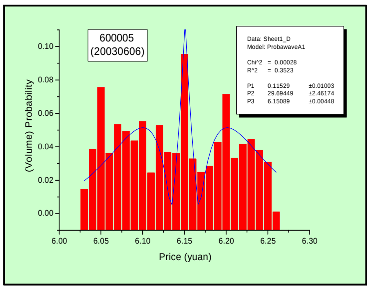
(c) a distribution with
high peaked, heavy tailed, and clustered features



(d) a distribution with
two maximum volume prices



(e) two maximum volume prices



(f) a uniform distribution

Figure 1: The cumulative trading volume distribution over a price range of an individual stock on a trading day[7]

[7] Price is the horizontal coordinate and cumulative trading volume probability is the vertical coordinate, respectively. In test reports, P3 is a reference point in trading.

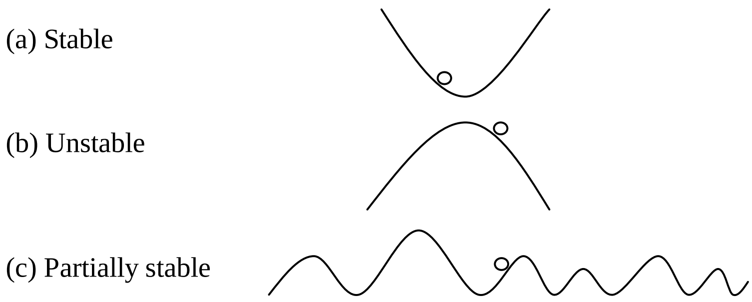


Figure 2: A partially stable system

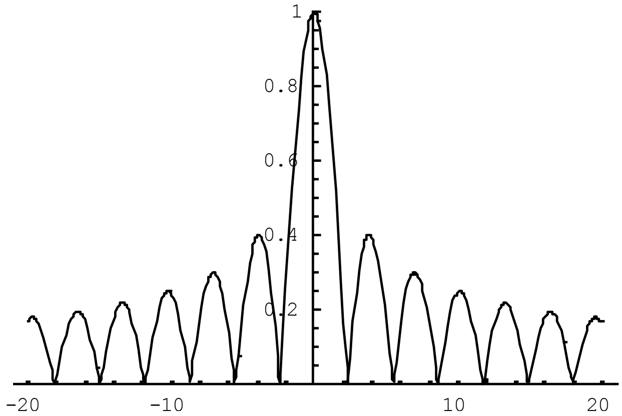


Figure 3: The absolute of zero-order Bessel eigenfunctions[8]

[8] In Figure 3, price is the horizontal coordinate and cumulative trading volume probability in a time interval is the vertical coordinate, respectively. The origin is a reference point.

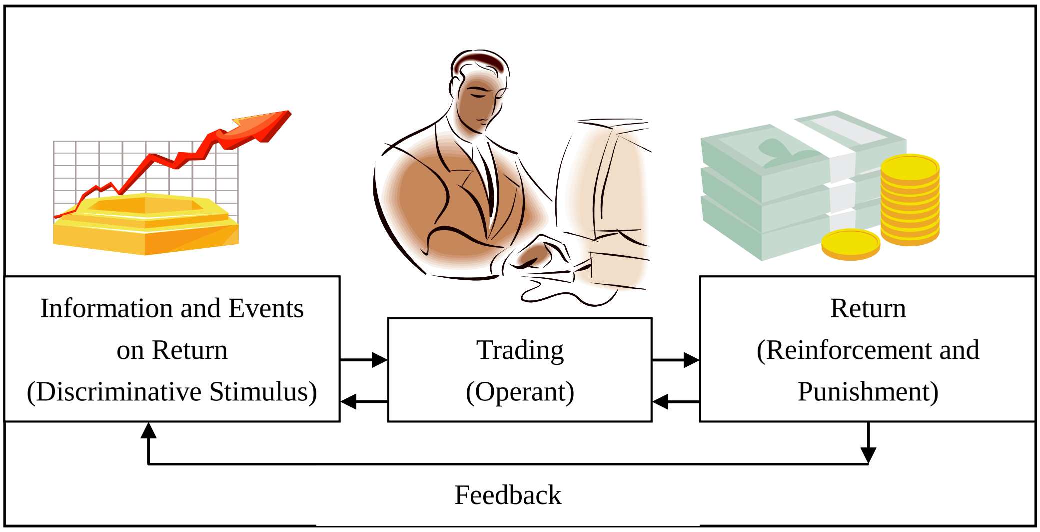
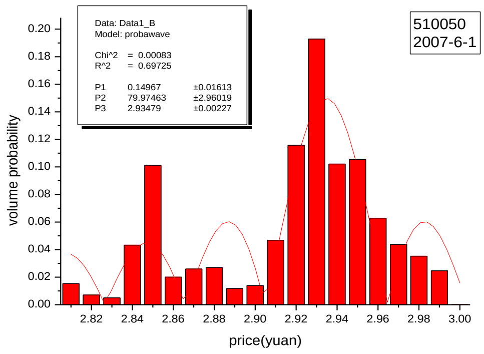
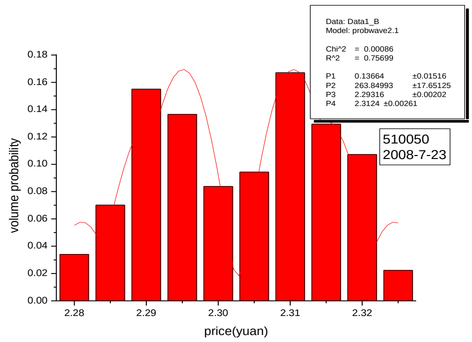


Figure 4: Trading frequency or preferences in expectation of return in a time series.



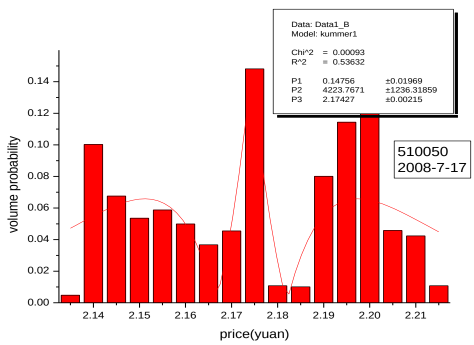
(a) The sample is fitted by equation (3)

($R^2 = 0.697 > R^2_{crit} = 0.50$)



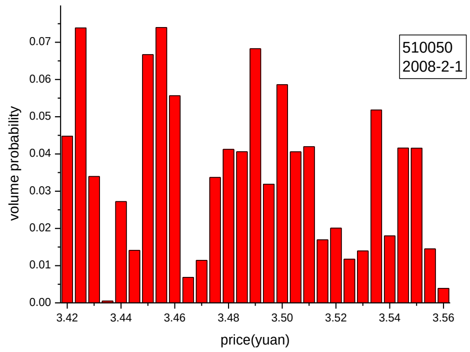
(b) The sample is fitted by equation (8)

($R^2_2 = 0.757 > R^2_{2crit} = 0.575$)



(c) The sample is fitted by equation (11)

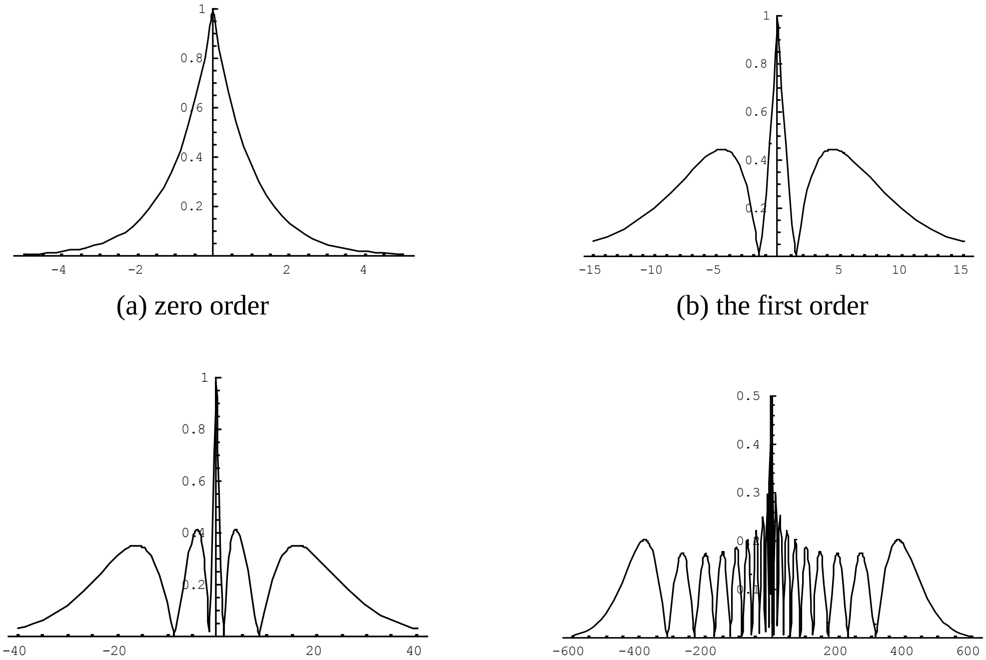
($R^2 = 0.536 > R^2_{crit} = 0.232$)



(d) One of five insignificant samples

Figure 5: The volume distribution test reports in samples[9]

[9] In Figure 5, P1, P2, and P3 are a normalized constant, an eigenvalue, and a reference point, respectively. P4 is also a reference point.



(a) zero order

(b) the first order

(c) the second order

(d) the tenth order

Figure 6: The absolute of the multi-order eigenfunctions