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Discretionary Credit Limits, Correlation-Driven Leverages, and Unanticipated Shocks in Dynamic Equilibrium [1]

Dong Chul Won[2]
School of Business Adminstration
Ajou University, South Korea

Abstract

Surge in return correlation means a deterioration in investment opportunities in that it may reduce diversification benefits and impose excessively large leverage on the optimal portfolios of agents. Correlation-driven leverage triggers large trades without any informational reason or any contagion. Since unanticipated payoff shocks are fed into the initial wealth in the coming periods, correlation-driven trades amplify them into huge endowment shocks. Remarkably, small payoff shocks can cause market breakdown when correlation-driven leverage prevails. Correlation surge can lead rather than follow market crisis. The paper presents a simple form of credit limits on the ex-dividend capital values of portfolios which are designed to prevent the correlation-driven excessive leverage of optimal portfolios in stochastic finance models. It is illustrated that tighter discretionary credit limits can lead to Pareto improvement when they create an advantageous tradeoff between diversification benefits and risk-sharing opportunities by reducing payoff correlation. Remarkably, conventional borrowing constraints which aim at preventing doubling strategies or Ponzi schemes may fail to restrain correlation-driven leverage. Thus, they are not effective in preventing market crashes which are attributed to the correlation-amplified income shocks.

KEYWORDS: Correlation-driven trades, similar assets, discretionary credit limits, unanticipated payoff shocks, general equilibrium, incomplete asset markets.

JEL Classification: D52, C62, C63, C02

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[2] Ajou University, College of Business Adminstration, San 5 Wonchun-Dong, Yeongtong-Gu, Suwon, KOREA 443-749, Phone: +82-31-219-2708, Email: dcwon@ajou.ac.kr

1. INTRODUCTION

The market value of large-scale portfolios is very susceptible to changes in asset prices. Leverage via short sales is an easy way of inflating portfolios. As witnessed in the banckrupcy of Lehman Brothers on September of 2008, adverse changes in asset price may cause big losses to financial institutions with leverage-inflated portfolios, which may trigger their bankruptcy and lead to the destabilization of the financial system.[1] A simple driver for inflating the size of portfolios indefinitely is a zero-cost portfolio.[2] Zero-cost portfolios enable investors to inflate the size of portfolios as much as they want without additional wealth. A typical example is derivative positions which cost nothing at the initiation. A large zero-cost portfolio may incur big loss if the value of long positions drops significantly without counterbalancing price changes for the short positions.[3] Thus, to inflate portfolios with zero-cost portfolios amounts to raising risk without risk-sharing benefit. Investors can also hold large portfolios by investing in 'similar assets'. For example, hedge funds like Long-Term Capital Management were used to do convergence trading strategies which involve two fixed-income securities with similar payoff structures and maturities. A convergence trading strategy usually involve large long and short positions to profit from wider-than-normal return or yield spreads which are expected to get narrower before the maturities of fixed-income securities.

To understand the role of similar assets in making portfolio choices, we introduce the notion of endogenous payoff similarity. Two distinct risky assets (or portfolios) are called *endogenously similar* (in short, ES) at a price if their payoffs (or returns) are near perfect correlation.[4] As two distinct assets become endogenously similar in equilibrium, the spanning quality of the asset structure 'deteriorates' in that agents may need to hold a large size of optimal portfolios to meet a desired level of intertemporal income transfers. Thus, the presence of endogenously similar assets may lead to explosion of trading volumes without any surprise news on asset payoffs or any contagion.[5] Correlation-driven large portfolios inevitably involve excessive leverage which is accounted for by short positions.

Unusually high correlation can jeopardize the economy because it may cause excessively huge leverage which in turn destabilizes financial markets in the presence of small payoff shocks. As

[1]Lehman Brothers maintained high leverage before its bankruptcy. For instance, the leverage raito of the firm (the ratio of total assets to equity) amounted to 30.7 on November, 2007. For details, see Wiggins et al. (2014).

[2]A zero-cost portfolio does not entail any cost at the time of initiation.

[3]For example, a cash-and-carry trade is financed by selling bonds. Thus, it will lose much if interest rate alone falls much after the trade has occurred.

[4]Let X and Y denote two payoffs and X_0 and Y_0 their current value, respectively. Since $Corr((X - X_0)/X_0, (Y - Y_0)/Y_0) = Corr(X, Y)$, payoff and return correlations are identical. As shown later, asset redundancy is virtually identical to perfect correlation when risk-free assets are available. Thus, endogenous payoff similarity occurs in a well-defined manner when nearly perfect correlation occurs. If there are only two assets, risk-free and risky, the risky asset is endogenously similar if the variance of the payoff is sufficiently close to zero.

[5]The literature uses distinct definitions of contagion. For instance, Forbes and Rigobon (2002) define contagion as "a siginificant increase in cross-market linkages after a shock to one country (or group of countries)." Bakaert et al. (2005) define it as "excess correlation, that is correlation over and above what one would expect from economic fundamentals."

demonstrated in Section 4, unanticipated payoff shocks are fed into the initial wealth in the coming periods. The payoff shocks are amplified into huge endowment shocks in the presence of correlation-driven large trades. Thus, correlation-driven excessive leverage can degenerate into unbearable indebtedness at the outbreak of unexpected payoff shocks whether they are positive or negative. In this case, the post-shock economy may fail to attain equilibrium when asset markets are unable to make enough income transfers to repay the unanticipated huge debts. Correlation-driven trades are not related to any informational motives and thus, must be differentiated from information-driven trading activities.[6]

Correlations between asset returns are dynamically changing in the real world. International equity correlations tend to exhibit large swings during the period of financial market turbulence. Many empirical studies report that international equity returns are more correlated in bear markets than in bull markets.[7] Buraschi, et al. (2010) show that the average correlation between S&P 500 and Nikkei (FTSE) weekly stock market returns increased dramatically since summer 2007 and reached near 0.8 in April 2008. Bloomberg reported on February 18, 2016 that the correlation between S&P 500 and oil recorded a 15-year high value of 0.96 on January 21, 2016, and correlation between the S&P 500 and the Shanghai Composite Index reached 0.95 on January 25, 2016.[8] From the global perspective, high correlation between national stock indices implies the deterioration of global market spanning.

This paper presents a simple form of credit limits on the ex-dividend capital values of portfolios which are designed to prevent the correlation-driven excessive leverage of optimal portfolios in stochastic finance models. Specifically, the credit limits are imposed on the exdividend value of leveraged portfolios carried from the previous period. They become a binding constraint when some of the assets are so endogenously similar that optimal intertemporal consumptions can be financed only by correlation-driven trades with excessive leverage. The credit limits exert discretion over capital losses in that they can prevent large capital losses with correlation-driven trades without losing access to normal risk-sharing opportunities. When the discretionary credit limits are properly instituted, they can serve as an institutional stabilizer by reducing the size and likelihood of individual or institutional bankruptcy (or default) in the advent of unexpected worst scenarios like black swan events.[9] The discretionary property of the credit limits at hand are not shared by conventional borrowing constraints such as short-selling restrictions and wealth constraints. As

[6]Wang (1994) studies the equilibrium behavior of stock trading volumes in an economy with distinct private investment opportunities and heterogeneous information on dividends. Blume et al. (1994) show that trading volume can provide information which are not deduced from the price statistic. Gervais et al. (2001) find that unusually high trading volume contains information on future stock prices which leads to high-volume return premium. When the correlation effect on trading volume is not properly filtered out, it could distort the result of empirical studies on the relationship between information-driven trading volumes and asset returns.

[7]See Long and Solnik (2001), Ang and Bekaert (2002), Goetzmann, et al. (2005), Driessen, et al. (2009) and references cited therein.

[8]See the article of Bloomberg "Global Markets Are Falling Out of Lockstep, and That's a Good Thing."

[9]For discussion of a black swan, see Taleb (2010).

discussed later, borrowing constraints and wealth constraints which aim at preventing doubling strategies and Ponzi schemes, respectively, may fail to restrain correlation-driven leverage. Short-selling constraints indiscriminately regulate portfolios wether they are excessively leveraged or not. Thus, the well-known borrowing constraints are not effective in preventing market crashes which are attributed to the correlation-amplified income shocks.

The discretionary credit limits have important implications for the existence of equilibrium. They allow us to build the sure existence of equilibrium in stochastic finance models because they preclude the occurrence of the Hart problem.[10] The result is in contrast to the generic existence of equilibrium in the literature to be reviewed later.

To see how an unanticipated payoff shock can destabilize asset markets, we consider the case that correlation-driven leverage is unrestrained in equilibrium. The presence of correlation-driven leverage means some agents have a huge long position and others have a huge short position for assets. When the payoff shock arrives, the portfolios with correlation-driven leverage are exposed to large value swing in the post-shock economy even though the shock size is very small. Specifically, the negative (positive) unanticipated shock becomes a huge blessing (curse) for the short position and a huge curse (blessing, respectively) for the long position. Huge loss of the cursed positions is built into the initial wealth with which the cursed agents are endowed in the post-shock economy. The unbearable indebtedness of the cursed agents can deprive the post-shock economy of an opportunity to have equilibrium. As illustrated in Section 5, an unregulated economy with correlation-driven trades fails to attain equilibrium in the post-shock stage when it is hit by a small payoff shock. Correlation-driven leverage can be restrained by imposing discretionary credit limits. When they are fairly tightly set, equilibrium is attained robustly in the post-shock subeconomies regardless of shock sizes in wide spectrum.

Correlation-driven leverage has an interesting implication to the casuality between correlation surge and market crisis. The empirical literature reports that return correlation tends to spike during the period of financial crisis and market crashes.[11] The literature views surge in asset correlation as a consequence rather than a cause of market turmoil or crisis. The paper provides a distinct view for the relationship between surged asset correlation and market crisis. As explained above, unusually high correlation can amplify small payoff shocks into huge income shocks which destabilize asset markets. In this case, correlation surge can lead market crisis and thus, the causality between them is reversed.

The paper presents an interesting example with endogenously similar assets where agents get better off as the discretionary credit limits get tighter. The counterintuitive welfare implications of the discretionary credit limits may be explained in terms of a tradeoff between diversification

[10] Hart (1975) illustrates that no equilibrium exists in markets with real assets because the budget set may lose continuity at prices which induce perfect correlation between the payoffs of two risky assets.

[11] For instance, see Longin and Solnik (2001), Goetzmann, et al. (2005) and Driessen, et al. (2009).

benefits and risk-sharing opportunities. Specifically, the example illustrates that tightened discretionary credit limits can create the tradeoff in a desired way. As the discretionary credit limits are tightened, assets get less similar in equilibrium (or the return correlation gets smaller) while risk-sharing opportunities are shrunk. The reduced return correlation can positively affect individual welfare by increasing diversification benefits. In the example, the welfare advantage from increased diversification benefits dominates the welfare disadvantage from shrunk risk-sharing opportunities under the discretionary credit limits.

Here agents are assumed to have symmetric information on the future event tree, i.e., they share the same available information on uncertainty at the time of decision making. However, it does not mean that they have complete knowledge of all the possible events to occur in the future. Some information may be intentionally missing due to enormous processing cost or simply unavailable due to ignorance about the future. Unexpected events can crop up any time in the future.[12] For example, even a financial guru who had an early hunch for the possible outbreak of the 2008 financial crisis would not have full information on the timing, duration, and extent of the crisis.[13] Moreover, investors cannot be properly hedged against unexpected macro-level events. As witnessed in the recent financial crisis, portfolios with large financial leverage may pose serious negative externality to the stability of asset markets when they are not properly hedged against unexpected adverse events. Many countries hurried to impose stricter regulations on stock markets in 2008 by restricting or prohibiting short selling in an attempt to reduce excessive price movements during the financial crisis. Financial regulations are imposed on the financial industry to prevent the adverse effect of the portfolio scale on the stability of the economy. Basel III requires stricter capital standards for the banking sector to alleviate the excess leverage problem which was responsible for the outbreak of the recent global crisis. The debt-to-income ratio (DTI ratio) is imposed on the mortgage contracts as an institutional stabilizer that aims at keeping households repayable. Financial regulations designed to restore market stability in the crisis era may be too much strict to apply in normal states of the economy. Discretionary credit limits can be a good alternative to existing regulatory schemes because they are binding at the moments when the invisible hand forces investors to hold excessively leveraged portfolios.

[12]Market participants paid attention to the 2015 FOMC meetings to see whether the federal fund rate is raised from the near-zero range for the first time since June 2006. Yellen, the U.S. Fed Chair, announced in the June press conference of the FOMC that the decision to raise the target range for the federal fund rate would depend on the assessment of economic conditions such as labor market conditions, inflation outlook, and international developments. The announcement had no mention of concerns over the Chinese economy. In the mid of August of 2015, however, the People's Bank of China (PBOC) shocked global financial markets by unexpectedly depreciating the value of the yuan over 4% via three consecutive policy moves to revitalize the sluggish Chinese economy. Unexpected developments of the Chinese economy were not in the information set for the June decision making of the FOMC. The FOMC revised the information set in the September statement by expressing that "heightened concerns about growth in China and other emerging market economies have led to notable volatility in financial markets."

[13]The renewal process of information on future events is comparable to weather forecasting. Daily, weakly or longer-term weather is forecasted on the basis of currently available meteorological data and information. They are frequently updated as meteorological conditions change.

The paper is related to the literature on portfolio constraints, the generic existence of equilibrium, and financial innovation. There are abundant studies of portfolio constraints in the literature. However, they do not share the discretionary property of the credit limits at hand which are designed to control correlation-driven excessive leverage by imposing restrictions on the ex-dividends capital value of optimal portfolios. For instance, borrowing or short-selling constraints of the literature lack the discretionary property. Wealth constraints discussed in the literature such as Dybvig and Huang (1988) and Duffie (2001) fail to preclude correlation-driven trades and thus, trading volumes can be exploded at prices which make some of the assets endogenously similar. The discretionary credit limits are also distinct from portfolio constraints and liquidity constraints which restrict so rigidly the size of asset holdings and borrowing, respectively, that they cannot hold the discretionary property. A typical example of portfolio constraints are restrictions on short sales (to take a few, Radner (1972), Heaton and Lucas (1996), Saffi and Sigurdsson (2011), Beber and Pagano (2013)). Especially, Saffi and Sigurdsson (2011) and Beber and Pagano (2013) include an empirical study of the effect of short-selling restrictions on stock market efficiency and market liquidity which were enacted around the world during the 2008-2009 financial crisis. Luttmer (1996) analyzes the empirical implications of portfolio constraints for asset returns in the Hansen-Jagannathan framework which is concerned about bounds on means and variances of stochastic discount factors. Liquidity (or borrowing) constraints are also widely examined in the literature (For instance, Aiyagari (1994) and Heaton and Lucas (1996)). Cornet and Gopalan (2010) build the existence of equilibrium in a multi-period economy with nominal assets where individual portfolio choices are subject to exogenously given restrictions. Hoelle et al (2016) study the effect of borrowing constraints on equilibrium in a two-period model with real assets.

The paper provides the sure existence of equilibrium under the discretionary credit limits which is in contrast to the generic existence of equilibrium in the literature. Hart (1975) illustrates that equilibrium may fail to exist in frictionless incomplete markets with real assets. The existential failure of the exemplary economy in Hart (1975) is attributed to the presence of prices which induce the endogenous collinearity of two cum-dividend returns. Duffie and Shafer (1985,1986), Brown et al. (1996), and Bottazzi (1995,2002) show that equilibrium exists generically when the payoff of every asset depends linearly on commodity or asset prices. Ku and Polemarchakis (1990) illustrate that the generic existence theorem fails in the presence of securities with nonlinear payoffs such as options. In particular, options make the existential failure robust to a perturbation of economies. Momi (2001) demonstrates that a production economy with incomplete stock markets may fail to have equilibrium in a set of endowments with positive-measure. Momi (2003, 2010) characterizes the properties of excess demand function in incomplete markets with real assets. Hoelle et al (2016) show that equilibrium exists generically under borrowing constraints. As illustrated later, the borrowing constraints of Hoelle et al (2016) do not prevent correlation-driven trades.

Financial regulation affects individual welfare through restrictions on risk-sharing opportunities. The paper illustrate that the discretionary credit limits can work in a welfare-improving way by preventing assets from being so much similar that can reduce diversification benefits. In the example, loosening portfolio restrictions increases risk-sharing opportunities but decreases diversification benefits by increasing payoff correlation. The example is related to the literature which shows the negative effect of financial innovation on welfare. Hart (1975), Elul (1995), Cass and Citanna (1998) among others show that the introduction of new assets may make every agent worse off in incomplete markets. Hoelle et al. (2016) provide an example where tighter borrowing constraints lead to Pareto improvement. Elul (1995) and Cass and Citanna (1998) examine the welfare effect of introducing new assets that are not traded in pre-innovation equilibrium by slightly perturbing the asset structure. Hoelle et al. (2016) deal with local changes in borrowing constraint. The current paper and the literature have a common ground in that increases in risk-sharing opportunities may lead to overall reduction in individual welfare. The example of the paper is distinct from the literature in two respects. First, the effect of loosened credit limits on welfare can be explained in terms of the tradeoff between increased risk-sharing opportunities and reduction in diversification benefits. Second, the example of the paper examine the welfare effect of large-scale changes in credit limits while the aforementioned except for Hart (1975) studies the welfare effect of infinitesimal or local changes in risk sharing opportunities.

2. THE ECONOMY

The single-good economy under study persists in three periods with finitely many agents and assets. The assets are long-lived so that they can be retraded in the second period. The reason that we restrict ourselves to the three-period economy is it is the simplest one that admits assets to be retraded and captures the essence of the dynamic income-spanning property of the asset markets that is not shared with the two-period model. Uncertainty is assumed to be resolved in the following simple way. Nature is assumed to choose an event in a finite set $\mathcal{S} = \{1, \ldots, S\}$ in the second and third periods. Thus, agents in each event of the second period face the same uncertainty as they does in the first period. If events $s \in \mathcal{S}$ and $(s, s') \in \mathcal{S}^2$ occur in the second and third periods, respectively, then the economy takes the state $(0, s, (s, s'))$, where 0 indicates the first period. We define sets

$$\mathcal{S}_3 = \{0\} \times \mathcal{S} \times \mathcal{S}^2 \quad \text{and} \quad \mathcal{S}_A = \{0\} \cup \mathcal{S} \cup \mathcal{S}^2.$$

A point in $\mathcal{S}_3$ denotes a state that consists of the events to be revealed over the three periods. The set $\mathcal{S}_A$ indicates the collection of possible events in each period. For an event $\omega \in \mathcal{S}_A$, let ω^+ denote the set of immediately succeeding events $\{\omega' \in \mathcal{S}_A : \omega' = (\omega, s') \text{ for some } s' \in \mathcal{S}\}$.

A single consumption good is available in each event in $\mathcal{S}_A$. Thus, a consumption plan is denoted by a point in the space of consumption plans $\mathbb{R} \times \mathbb{R}^S \times \mathbb{R}^{S^2}$. Let $\ell = 1 + S + S^2$ and $\ell_1 = S + S^2$.

Then $\mathbb{R}^\ell$ indicates the space of consumption plans. The following notation is used.

$$X = \mathbb{R}^\ell_+,\ X^\circ = \mathbb{R}^\ell_{++},\ X_1 = \mathbb{R}^{\ell_1}_+,\ \text{ and }\ X^\circ_1 = \mathbb{R}^{\ell_1}_{++}.$$

Each agent $i \in \mathcal{I}$ has the consumption set X, the initial endowment $e^i \in X$ of goods, and the preferences represented by a utility function $u_i : X \to \mathbb{R}$. We set $e = (e^1, \dots, e^I)$. A point $y \in \mathbb{R}^\ell$ is decomposed into $y = (y_0, (y_s), (y_{s,s'}))$ where $y_0 \in \mathbb{R}$, $(y_s) \in \mathbb{R}^S$ and $(y_{s,s'}) \in \mathbb{R}^{S^2}$. Let $\tilde{y} = ((y_s), (y_{s,s'}))$ denote the vector y with y_0 deleted.

Let $\mathcal{I} = \{1, 2, \dots, I\}$ denote the set of agents, $\mathcal{J} = \{1, 2, \dots, J\}$ the set of long-lived financial assets that are introduced in the first period. No new assets are added in the second period. Thus, the J assets are traded repeatedly in the second period. Assets make payoffs in units of the consumption good in the second and third periods. It is assumed that $S < J$, i.e., asset markets are incomplete.

Asset j pays r^j_s units of the commmodity in event s of the second period and $r^j_{s,s'}$ in event (s, s') of the third period. Let r_s and $r_{s,s'}$ denote the collection of the payoffs of the J assets in events $s \in \mathcal{S}$ and $(s, s') \in \mathcal{S} \times \mathcal{S}$, respectively. We define $S \times J$ matrices

$$R_0 = \begin{bmatrix} r_1 \\ \vdots \\ r_S \end{bmatrix} \quad \text{and} \quad R_s = \begin{bmatrix} r_{s,1} \\ \vdots \\ r_{s,S} \end{bmatrix} \ \text{each } s \in \mathcal{S}.$$

Asset j has the payoff $r^j = (r^j_1, \dots, r^j_S, r^j_{1,1}, \dots, r^j_{S,S}) \in \mathbb{R}^{\ell_1}$. The asset structure $r = (r^1, \dots, r^J)$ is identified as a point in $\mathbb{R}^{J\ell_1}$.

When a portfolio θ_0 is chosen in the initial period, the net debt to be repaid in state s of the second period is $q_s \cdot \theta_0$ at a price $q_s \in \mathbb{R}^J$. For each i, we introduce a credit limit $b^i = (b^i_1, \dots, b^i_S) \in \mathbb{R}^S$ with each $b^i_s < 0$ such that

$$q_s \cdot \theta_0 \geq b^i_s \quad \text{for each } s \in \mathcal{S}. \tag{1}$$

The credit constraints (1) aim at restraining the exdividend value of leveraged portfolios carried from the previous period. As discussed in Sections 3 and 5, they have the discretionary property that they tends to bind at leverage-inflated portfolios when R_0 has full rank. The payoff matrix R_0 can be assumed to have full rank without loss of generality. If the column vectors of R_0 are linearly dependent, we introduce an artificial payoff matrix R'_0 with rank J and an artificial price q'_s where R'_0 consists of S vectors r'_s in $\mathbb{R}^J$ and $q'_s = q_s + r_s - r'_s$ for each $s \in \mathcal{S}$. When the single good is taken as a numeraire, we can proceed by replacing R_0 and q_s by R'_0 and q'_s, respectively. Without loss of generality, we can assume that R_0 has full rank.

Definition 1 : The set of restrictions in (1) is called *discretionary credit constraints* of the economy.

Each agent is assumed to face the discretionary credit constraint (1). For a price $(p,q) \in \mathbb{R}^{S+1}_{+} \times \mathbb{R}^{J(S+1)}$, agent i has the following choice problem in the three-period economy.

$$\begin{aligned}
\max_{(x,\theta)\in X\times\mathbb{R}^{J(S+1)}} \; & u_i(x) \\
\text{s.t.} \quad & p_0(x_0 - e^i_0) + q^t_0 \cdot \theta_0 \leq 0, \\
& \begin{bmatrix} p_1(x_1 - e^i_1) \\ \vdots \\ p_S(x_S - e^i_S) \end{bmatrix} \leq \begin{bmatrix} p_1 r^t_1 + q^t_1 \\ \vdots \\ p_S r^t_S + q^t_S \end{bmatrix} \cdot \theta_0 - \begin{bmatrix} q^t_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & q^t_S \end{bmatrix} \begin{bmatrix} \theta_1 \\ \vdots \\ \theta_S \end{bmatrix}, \\
& x_{s,s'} - e^i_{s,s'} \leq r^t_{s,s'} \cdot \theta_s, \quad \text{for each } (s,s') \in \mathcal{S}\times\mathcal{S}, \quad \text{and} \\
& q^t_s \cdot \theta_0 \geq b^i_s, \quad \text{for all } s \in \mathcal{S},
\end{aligned} \tag{2}$$

where the superscript t indicates the matrix transpose. (It will be omitted below if no confusion arises in the context.) When one unit of asset j is bought at a cost q^j_0 in the first period, it pays $q^j_s + p_s r^j_s$ at the event s of the second period and $r^j_{s,s'}$ at the event (s,s') of the final period. Let $\mathcal{B}_i(p,q,e^i,r)$ denote the set of (x,θ)'s in $X \times \mathbb{R}^{J(S+1)}$ that satisfies the intertemporal budget constraint of agent i specified in the optimization problem (2), and $\xi_i(p,q,e^i,r)$ denote the demand correspondence of agent i, i.e., the set of optimal choices (x^*,θ^*) of agent i.[14]

Equilibrium of the economy is defined as follows.

Definition 2 : A list $(q,x,\theta) \in \mathbb{R}^{J(S+1)} \times X^I \times \mathbb{R}^{IJ(S+1)}$ is an *equilibrium* of the economy if it satisfies the conditions

(*i*) $(x_i,\theta_i) \in \xi_i(p,q,e^i,r)$ for every $i \in \mathcal{I}$,

(*ii*) $\sum_{i\in\mathcal{I}}(x_i - e^i) = 0$, and

(*iii*) $\sum_{i\in\mathcal{I}} \theta_i = 0$.

The following is a list of assumptions imposed on the economy.

Assumption 1 : Each u_i is continuous, strictly increasing, and quasiconcave in X.[15]
Assumption 2 : Each e^i is in X°.
Assumption 3 : Matrices R_0 and R_s for each $s \in \mathcal{S}$ have rank J.

Assumption 1 is a standard condition. Assumption 2 imposes a strong survival condition on the initial endowments for analytical simplicity. Assumption 3 is made only for simplicity. As discussed

[14] It is convenient to take the single good as a numeraire in many contexts. For technical flexibility in proving the existence of equilibrium, the budget constraints in (2) do not yet reflect such price normalization. Since price normalization has no real effect on equilibrium outcomes, the single good can be taken as a numeraire anytime in the relevant context.
[15] The function u_i is strictly increasing if for any x, x' in X with $x - x' \in X$ and $x \neq x'$, $u_i(x) > u_i(x')$.

before, R_0 is assumed to have full rank without loss of generality. So is each R_s.[16]

3. CORRELATION-DRIVEN TRADES AND CREDIT CONSTRAINTS

In this section, the discretionary credit limits (1) is comparatively examined with familiar borrowing constraints of the literature from the perspective of correlation-driven leverage. Specifically, we provide an intuition that the discretionary credit limits (1) can prevent portfolios from being excessively inflated through leverage at asset prices which make some of the assets endogenously similar. As demonstrated below, the borrowing constraints of the literature are not effective in deterring correlation-driven excessive leverage.

As a preliminary step we explain the relation between payoff perfect correlation and asset redundancy. Let $\tilde{x}$ and $\tilde{y}$ denote the random payoff of two risky assets X and Y, and x and y denote the vector of their payoffs to be realized in each state, respectively. Let μ_x and μ_y be the average payoffs of X and Y assets, respectively. The payoffs $\tilde{x}$ and $\tilde{y}$ are perfectly correlated if

$$E[(\tilde{x}-\mu_x)(\tilde{y}-\mu_y)]^2 = E[(\tilde{x}-\mu_x)^2]E[(\tilde{y}-\mu_y)^2].$$

Let $\pi_s > 0$ denote the objective probability that state s occurs. By the Cauchy-Schwarz inequality, $\tilde{x}$ and $\tilde{y}$ are perfectly correlated if and only if there exists a nonzero α such that for each $s = 1, \ldots, S$,

$$\sqrt{\pi_s}(x_s - \mu_x) = \alpha\sqrt{\pi_s}(y_s - \mu_s).$$

i.e., $x - \mu_x \mathbf{1} = \alpha(y - \mu_y \mathbf{1})$ where $\mathbf{1}$ indicates the vector of 1's with the same dimension as x and y. Therefore $\tilde{x}$ and $\tilde{y}$ are perfectly correlated if and only if $x - \mu_x \mathbf{1}$ and $y - \mu_y \mathbf{1}$ are linearly dependent. The relation is rearranged as $x = \alpha y + (\mu_x - \alpha\mu_y)\mathbf{1}$, implying that the payoff of asset X is replicated as the payoff of a portfolio of α units of asset Y and $(\mu_x - \alpha\mu_y)$ units of a risk-free asset that pays one unit of the underlying currency in the next period. Consequently, when risk-free assets are available, one of the two risky assets are redundant (or collinear) if and only if they are perfectly correlated.[17] These results are summarized in the following lemma.

Lemma 1 : Suppose that risk-free assets are marketed. The payoffs $\tilde{x}$ and $\tilde{y}$ are perfectly correlated if and only if x and y are collinear.

[16] If R_s has rank $J' < J$ for some s, then some of the assets are redundant at the state s, which causes the portfolio multiplicity problem. To see this, let θ_s be a portfolio in $\mathbb{R}^J$. Let $\hat{\theta}_s$ denote the project of θ_s onto the J'-dimensional subspace $\langle R_s \rangle$ of $\mathbb{R}^J$ spanned by the rows of R_s. Then $R_s \cdot \theta_s = R_s \cdot \hat{\theta}_s$ and moreover, $\hat{\theta}_s$ is the unique point in $\langle R_s \rangle$ that yields the income $R_s \cdot \hat{\theta}_s$. Thus, the portfolio multiplicity problem with redundant assets can be avoided by adopting $\langle R_s \rangle$ as the choice set for portfolios at state s instead of $\mathbb{R}^J$.

[17] When there are only two assets where the first asset is risk-free, the second asset is redundant if its payoff has zero variance.

Now we provide an intuition into how portfolios can be inflated through leverage as assets become endogenously similar. (A full-fledged example where an ES asset arises in equilibrium is given in Section 5.) We consider an economy with two agents and two assets where uncertainty consists of three events in each period. For expositional simplicity, the first asset is assumed to be risk-free and short-lived, which pays one unit of the numeraire asset and expires in the second period. The second asset is risky and long-lived, and thus, pays capital value plus dividends d_s at event s. For a parameter $\epsilon > 0$, let $q_s(\epsilon)$ denote an asset price in state s. The next-period payoff matrix is summarized as

$$\Lambda(q(\epsilon), r) = \begin{bmatrix} 1 & q_1(\epsilon) + d_1 \\ 1 & q_2(\epsilon) + d_2 \\ 1 & q_3(\epsilon) + d_3 \end{bmatrix}.$$

We suppose that the payoff of the second asset satisfies

$$d_1 + q_1(\epsilon) = d_3 + q_3(\epsilon) = 2(1 + \epsilon') \quad \text{and} \quad d_2 + q_2(\epsilon) = 2 + \epsilon'',$$

where ϵ' and ϵ'' satisfy the following conditions: i) $\epsilon' \to 0$ and $\epsilon'' \to 0$ as $\epsilon \to 0$ and ii) $\epsilon'/\epsilon'' \to 1$ as $\epsilon \to 0$. When ϵ' and ϵ'' equal zero, the second asset becomes risk-free and thus, redundant. As ϵ is close to zero, the asset becomes endogenously similar to the risk-free asset. Suppose that agent 1 need to finance net expenditure $\{z_1(\epsilon), z_2(\epsilon), z_1(\epsilon)\}$ in the second period which satisfies $\lim_{\epsilon \to 0} z_s(\epsilon) = z_s$ for each s with $z_1 \neq z_2$. Let $(\theta_0^1(\epsilon), \theta_0^2(\epsilon))$ denote the optimal portfolio of assets that agent 1 holds to attain the expenditure $\{z_1(\epsilon), z_2(\epsilon), z_1(\epsilon)\}$, i.e.,

$$\begin{bmatrix} z_1(\epsilon) \\ z_2(\epsilon) \\ z_1(\epsilon) \end{bmatrix} = \begin{bmatrix} 1 & 2(1+\epsilon') \\ 1 & 2+\epsilon'' \\ 1 & 2(1+\epsilon') \end{bmatrix} \begin{bmatrix} \theta_0^1(\epsilon) \\ \theta_0^2(\epsilon) \end{bmatrix}. \tag{3}$$

The first and second equations in (3) are solved to obtain the portfolio $(\theta_0^1(\epsilon), \theta_0^2(\epsilon))$ where

$$\theta_0^1(\epsilon) = \frac{z_1(\epsilon) - 2z_2(\epsilon)\frac{\epsilon'}{\epsilon''}}{1 - 2\frac{\epsilon'}{\epsilon''}} + 2\frac{z_1(\epsilon) - z_2(\epsilon)}{\epsilon'' - 2\epsilon'} \quad \text{and} \quad \theta_0^2(\epsilon) = \frac{z_2(\epsilon) - z_1(\epsilon)}{\epsilon'' - 2\epsilon'}. \tag{4}$$

Since $z_1 \neq z_2$, both $\theta_0^1(\epsilon)$ and $\theta_0^2(\epsilon)$ go unbounded as $\epsilon \to 0$. The portfolio $(\theta_0^1(\epsilon), \theta_0^2(\epsilon))$ has the relation

$$\theta_0^1(\epsilon) = \frac{z_1(\epsilon) - 2z_2(\epsilon)\frac{\epsilon'}{\epsilon''}}{1 - 2\frac{\epsilon'}{\epsilon''}} - 2\,\theta_0^2(\epsilon),$$

where the first term of $\theta_0^1(\epsilon)$ is bounded. As ϵ gets close to zero, the two payoffs become perfectly correlated and thus, the correlation-driven trades $\theta_0^1(\epsilon)$ and $\theta_0^2(\epsilon)$ become a huge number with opposite signs where the trade with negative sign provides a source of excessive leverage.

Let $b_s^i < 0$ denote the credit limit at state s for agent i. The discretionary credit limits (1) require $(\theta_0^1(\epsilon), \theta_0^2(\epsilon))$ to satisfy

$$q_s(\epsilon)\theta_0^2(\epsilon) \geq b_s^1 \quad \text{for all } s = 1, 2, 3. \tag{5}$$

By the market clearing condition, agent 2 holds the optimal portfolio $(-\theta_0^1(\epsilon), -\theta_0^2(\epsilon))$ that satisfies

$$-q_s(\epsilon)\theta_0^2(\epsilon) \geq b_s^2 \quad \text{for all } s = 1, 2, 3. \tag{6}$$

As ϵ get close to zero, one of the two constraints in (5) and (6) must bind. This example gives an intuition into how the discretionary credit limits prevent excessively inflated leverage at asset prices which make some of the assets endogenously similar.

Now we turn to well-known credit constraints of the literature to see that they are effective in managing the problem of leveraged portfolio inflation. Hernadez and Santos (1996) and Santos and Woodford (1997) impose the following form of limits on portfolios to eliminate Ponzi schemes in infinite-horizon models; for some $b_s^i < 0$,

$$q_s(\epsilon) \cdot \theta_s \geq b_s^i. \tag{7}$$

The borrowing constraints (7) do not impose any limit on the correlation-driven leverage of $(\theta_0^1(\epsilon), \theta_0^2(\epsilon))$. Thus, they fail to prevent the Hart problem.[18] As illustrated in Section 5, correlation-driven trades may occur under the constraints (7). The constraints (7) aim at restricting the leverage of current portfolios while the discretionary credit limits (5) aim at restricting the exdividend value of leveraged portfolios carried from the previous period.

One of familiar borrowing restrictions in the finance literature is the wealth constraints as following; for each $s = 1, 2, 3$,

$$(r_s + q_s(\epsilon)) \cdot \theta_0(\epsilon) \geq b_s^i. \tag{8}$$

Wealth constraints are introduced to prohibit doubling strategies in the continuous-time finance literature such as Dybvig and Huang (1988) and Duffie (2001). The wealth constraints (8) differ from the discretionary credit limit (5) in that they aim at restricting the cum-dividend value of leveraged portfolios carried from the previous period. Note that by the relation (3), (8) is not binding at $(\theta_0^1(\epsilon), \theta_0^2(\epsilon))$ if $b_s^1 < \min\{z_1(\epsilon), z_2(\epsilon)\}$ for each $s = 1, 2, 3$. Since $(z_1(\epsilon), z_2(\epsilon)) \to (z_1, z_2)$, there exists a small number δ such that for each ϵ sufficiently close to 0, $\min\{z_1(\epsilon), z_1(\epsilon)\} < \min\{z_1, z_2\} + \delta$. Thus, the wealth constraints may fail to restrain correlation-driven trades if the wealth limit $\{b_s\}$ is not tightly set. (This fact is illustrated in Section 5.) Hoelle et al. (2016) study borrowing constraints in a two-period economy with real assets whose payouts consist of multiple commodities. They can be specialized as following in the current one-good framework; for each $s = 1, 2, 3$,

$$(r_s + q_s(\epsilon)) \cdot \theta_0(\epsilon) \geq -\delta_s^i e_s^i. \tag{9}$$

[18]The Hart problem can be illustrated with the relation (3). As far as $\epsilon > 0$, the leveraged portfolio in (3) is well defined. However, the portfolio $(\theta_0^1(\epsilon), \theta_0^2(\epsilon))$ is not defined at $\epsilon = 0$ because $z_1 \neq z_2$. In other words, the limit expenditure (z_1, z_2, z_1) is not financed by the limit of $(\theta_0^1(\epsilon), \theta_0^2(\epsilon))$. In this case, the budget set loses continuity at $\epsilon = 0$. The Hart problem represents an extreme form of correlation-driven leverage. The borrowing limits b_s is endogenously determined in Hernadez and Santos (1996). That does not matter to the Hart problem.

where δ_s^i indicates an agent-specific magnitude of debt capacity. The borrowing constraints (9) are wealth constraints with $b_s^i = -\delta_s^i e_s^i$. It is illustrated later that portfolios can be excessively inflated in an unrestricted way in the face of tight borrowing constraints with small b_s^i and δ_s^i. In particular, the Hart problem can occur under the borrowing constraints (8) and (9).[19] However, it is bypassed under the discretionary credit limits (1).

4. EQUILIBRIUM WITH UNANTICIPATED PAYOFF SHOCKS

This section discusses the existence of equilibrium for two types of economies, the original three-period economy and its subeconomies. The first part of the section is concerned about the three-period stochastic finance economy with the discretionary credit limits. It is assumed that as the period ends, an unexpected payoff shock arrives before uncertainty is resolved in the second period. The second part deals with the subeconomy which emerges in the second period after the payoff shock is realized.

The pre-shock consumption and portfolio plans of agents may not be sustainable in the post-shock subeconomy because the unexpected payoff shock affects the wealth of agents through the portfolios carried from the previous period. Moreover, when agents carry portfolios with correlation-driven leverage, the unanticipated shock may make them vulnerable to the risk of unbearable huge loss.

4.1 The Full Economy

The subsection provides the sure existence of the full stochastic finance economy that begins in the first period. The result of sure existence relies on the fact that the discretionary credit limits (1) bind whenever asset payoffs fall in nearly perfect correlation which triggers correlation-driven trades. Especially, they bind in the case that asset payoffs are perfectly correlated so that agents could be forced to possess an indefinite size of portfolios.

Theorem 1: Suppose that Assumptions 1–3 hold. Then there exists an equilibrium of the economy with the discretionary credit limits.

PROOF : See Appendix.

The sure existence result of Theorem 1 is in contrast to the generic existence theorem presented in Duffie and Shafer (1986) and Hoelle et al. (2016) which have no regulatory device to preclude the Hart problem.

4.2 The Post-Shock Subeconomy with Possible Credit Failure

[19]Due to the Hart problem, Hernadez and Santos (1996) and Hoelle et al. (2016) show the generic existence of equilibrium.

To explain the impact of unanticipated payoff shocks on the post-shock economy, we consider the unregulated stochastic finance economy which has equilibrium $(\hat{q}, \hat{x}, \hat{\theta})$ where some $\hat{\theta}_i$ involves correlation-driven leverage. As the first period is over, the unregulated economy restarts at some state, say $\bar{s}$, in the second period. The optimal choice $(\hat{x}_i, \hat{\theta}_i)$ satisfies the budget constraint of the subeconomy.

$$\hat{x}_{i,\bar{s}} - e^i_{\bar{s}} = (r_{\bar{s}} + \hat{q}_{\bar{s}}) \cdot \hat{\theta}^i_0 - \hat{q}_{\bar{s}} \cdot \hat{\theta}^i_{\bar{s}},$$
$$\hat{x}_{i,\bar{s},s} - e^i_{\bar{s},s} = r_{\bar{s},s} \cdot \hat{\theta}^i_{\bar{s}}, \quad s = 1, \ldots, S.$$

Since agent i carries $\hat{\theta}_{i,0}$ from the first period, $\hat{\theta}_{i,0}$ can be considered the initial asset ownership of agent i in the second period. Thus, he is virtually endowed with the mixture of asset ownerships and goods $((e^i_{\bar{s}}, \hat{\theta}_{i,0}), e^i_{\bar{s},1}, \ldots, e^i_{\bar{s},S})$ in the subeconomy. The above budget relation is rewritten as

$$\hat{x}_{i,\bar{s}} - (e^i_{\bar{s}} + r_{\bar{s}} \cdot \hat{\theta}^i_0) = -\hat{q}_{\bar{s}} \cdot (\hat{\theta}^i_{\bar{s}} - \hat{\theta}^i_0),$$
$$\hat{x}_{i,\bar{s},s} - (e^i_{\bar{s},s} + r_{\bar{s},s} \cdot \hat{\theta}_{i,0}) = r_{\bar{s},s} \cdot (\hat{\theta}^i_{\bar{s}} - \hat{\theta}^i_0), \quad s = 1, \ldots, S.$$

We define augmented endowment $(\tilde{e}^i_{\bar{s}}(r), \tilde{e}^i_{\bar{s},1}(r), \ldots, \tilde{e}^i_{\bar{s},S}(r))$ where

$$\tilde{e}^i_{\bar{s}}(r) = e^i_{\bar{s}} + r_{\bar{s}} \cdot \hat{\theta}_{i,0} \quad \text{and} \quad \tilde{e}^i_{\bar{s},s}(r) = e^i_{\bar{s},s} + r_{\bar{s},s} \cdot \hat{\theta}_{i,0}, \quad s = 1, \ldots, S. \tag{10}$$

Note that the augmented endowment $\tilde{e}^i_{\bar{s},s}(r)$ depends on the payoff $r_{\bar{s},s}$ which is exposed to unanticipated shocks. When no payoff shocks occur, agent i makes the optimal choice $(\hat{x}_{i,\bar{s}}, (\hat{x}_{i,\bar{s},s}), \hat{\theta}^i_{\bar{s}} - \hat{\theta}^i_0)$ at the price $\hat{q}_{\bar{s}}$ in the subeconomy where he possesses the augmented endowment (10). In particular, he chooses the same consumption as planned in equilibrium $(\hat{q}, \hat{x}, \hat{\theta})$.

Suppose that before the asset markets reopen in the subeconomy, a surprise news on the payoff has arrived in which $r_{\bar{s},s}$ is shifted to $\tilde{r}_{\bar{s},s}$ for each $s = 1, \ldots, S$. The payoff shock is built into the augmented endowment so that agent i is expected to have the post-shock endowment $(\tilde{e}^i_{\bar{s},1}(\tilde{r}), \ldots, \tilde{e}^i_{\bar{s},S}(\tilde{r}))$ in the final period where

$$\tilde{e}^i_{\bar{s},s}(\tilde{r}) = e^i_{\bar{s},s} + \tilde{r}_{\bar{s},s} \cdot \hat{\theta}_{i,0} = e^i_{\bar{s},s}(r) + (\tilde{r}_{\bar{s},s} - r_{\bar{s},s}) \cdot \hat{\theta}_{i,0}, \quad s = 1, \ldots, S.$$

The post-shock endowment consists of two components, anticipated $(e^i_{\bar{s},s}(r))$ and unanticipated $((\tilde{r}_{\bar{s},s} - r_{\bar{s},s}) \cdot \hat{\theta}_{i,0})$. The original equilibrium $(\hat{q}, \hat{x}, \hat{\theta})$ is not fulfilled any more in the post-shock subeconomy because of the unanticipated income shock $(\tilde{r}_{\bar{s},s} - r_{\bar{s},s}) \cdot \hat{\theta}_{i,0}$. Agents need to revise the pre-shock consumption and portfolio plans in the subeconomy in response to unanticipated new information. The post-shock economy may fail to attain equilibrium because of correlation-driven leverage with $\hat{\theta}_{i,0}$. Since $\hat{\theta}_{i,0}$ is excessively leveraged, the unanticipated income $(\tilde{r}_{\bar{s},s} - r_{\bar{s},s}) \cdot \hat{\theta}_{i,0}$ is so sensitive to slight changes in the payoffs that the post-shock endowment $\tilde{e}^i_{\bar{s},s}(\tilde{r})$ may end up carrying unanticipated huge indebtedness in some states. It is illustrated in the next section that the post-shock subeconomy fails to attain equilibrium in the face of a tiny payoff shock, say 1% drop or rise in payoffs. The market failure takes place because asset markets fail to generate enough income transfers to repay unanticipated huge debts with $\tilde{r}_{\bar{s},s} \cdot \hat{\theta}_{i,0}$.

5. EXAMPLES WITH CORRELATION-DRIVEN LEVERAGES AND UNANTICIPATED PAYOFF SHOCKS

We provide an example with two assets in which optimal portfolios are excessively leverage-inflated at a price which makes the assets endogenously similar. The first asset pays constant dividends while the second asset pays distinct dividends in each state of the future. Their dividend payoffs are far from being collinear but the asset resale prices make the total payoffs nearly collinear in equilibrium. For comparative purpose, we first discuss the unregulated economy where agents are forced to hold an excessively leveraged portfolio. Excessive leverage is necessitated to finance the next-period net expenditures which display a different pattern from the dividend payoffs. Then discretionary credit limits are introduced to see how they restrict the excessive leverage of optimal portfolios in equilibrium. The discretionary credit limits improve the efficiency of market equilibrium as far as they are not too much tight. Specifically agents get better off under discretionary credit limits than in the unrestricted case, and tightened credit limits lead to greater Pareto improvement. As explained below, the counterintuitive result arises because the binding discretionary credit limits bring about the tradeoff between diversification benefits and risk-sharing opportunities. In contrast, the correlation-driven trades are unrestrained in equilibrium under tight wealth constraints. Finally, we are concerned about unexpected payoff shocks to see what happens to the post-shock subeconomy. When the payoff shocks arrive after uncertainty is revealed in the second period, the excessive leverage of portfolios carried from the previous period makes agents vulnerable to unbearable indebtedness which can deprive them of any consumption opportunities in the post-shock subeconomy.

In this example, the economy persists in three periods with two agents, three states in each period, and two long-lived assets. Agents have a utility function as following

$$u_1(x) = 6\log x_0 + \sum_{s=1}^{3} \alpha_s \log x_s + \sum_{s=1}^{3}\sum_{s'=1}^{3} \alpha_{s,s'} \log x_{s,s'},$$
$$u_2(y) = 6\log y_0 + \sum_{s=1}^{3} \beta_s \log y_s + \sum_{s=1}^{3}\sum_{s'=1}^{3} \beta_{s,s'} \log y_{s,s'},$$

where $\alpha_1 = 1, \alpha_2 = 2, \alpha_3 = 3$ and $\alpha_{s,1} = 1$, $\alpha_{s,2} = 2$, $\alpha_{s,3} = 3$ for all $s = 1,2,3$, and $\beta_1 = 3, \beta_2 = 2, \beta_3 = 1$ and $\beta_{s,1} = 3$, $\beta_{s,2} = 2$, $\beta_{s,3} = 1$ for all $s = 1,2,3$. It is assumed here that agents have state-dependent preferences and the occurrence of each state is equally probable, i.e., $P(1) = P(2) = P(3) = 1/3$ where P is the objective probability distribution over the three states. The first asset is risk-free which pays one unit of the numeraire commodity in each state $\omega \in \mathcal{S}_A \setminus \{0\}$, and the second asset has the payoff r^2 given by

$$\begin{bmatrix} r_1^2 \\ r_2^2 \\ r_3^2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad \begin{bmatrix} r_{1,1}^2 \\ r_{1,2}^2 \\ r_{1,3}^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix}, \quad \begin{bmatrix} r_{2,1}^2 \\ r_{2,2}^2 \\ r_{2,3}^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} r_{3,1}^2 \\ r_{3,2}^2 \\ r_{3,3}^2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}.$$

Each agent has the following endowments.

$$
\begin{array}{ll}
e_0^1 = 2.30995; & e_0^2 = 0.19664 \\
(e_1^1, e_2^1, e_3^1) = (1.82931, 0.57776, 2); & (e_1^2, e_2^2, e_3^2) = (0.62069, 0.75558, 0.2) \\
(e_{1,1}^1, e_{1,2}^1, e_{1,3}^1) = (4, 16, 24); & (e_{1,1}^2, e_{1,2}^2, e_{1,3}^2) = (1, 1, \frac{3}{5}) \\
(e_{2,1}^1, e_{2,2}^1, e_{2,3}^1) = (3, 3, 1); & (e_{2,1}^2, e_{2,2}^2, e_{2,3}^2) = (1, 1, \frac{1}{7}) \\
(e_{3,1}^1, e_{3,2}^1, e_{3,3}^1) = (2, 2, 1); & (e_{3,1}^2, e_{3,2}^2, e_{3,3}^2) = (1, 1, \frac{1}{10})
\end{array}
$$

5.1 The Unregulated Economy

The economy without credit limits has an equilibrium with the following profile.[20]

Asset price :
$(q_0^1, q_0^2) = (4.333495, 4.333492)$, $(q_1^1, q_1^2) = (0.999999, 0.999997)$,
$(q_2^1, q_2^2) = (2.000004, 1)$, $(q_3^1, q_3^2) = (3.000033, 1.000033)$.

Consumption allocation :

$$
\begin{array}{ll}
x_0 = 2.00005; & y_0 = 0.50653 \\
(x_1, x_2, x_3) = (1.99999, 1.00001, 1.99998); & (y_1, y_2, y_3) = (0.45001, 0.33333, 0.20002) \\
(x_{1,1}, x_{1,2}, x_{1,3}) = (3.99999, 15.99999, 23.99999); & (y_{1,1}, y_{1,2}, y_{1,3}) = (1.00001, 1.00001, 0.60001) \\
(x_{2,1}, x_{2,2}, x_{2,3}) = (3.00000, 3.00002, 1.00000); & (y_{2,1}, y_{2,2}, y_{2,3}) = (1, 00000, 0.99998, 0.14285) \\
(x_{3,1}, x_{3,2}, x_{3,3}) = (1.99991, 1.99991, 0.99999); & (y_{3,1}, y_{3,2}, y_{3,3}) = (1.00009, 1.00009, 0.10001)
\end{array}
$$

Portfolio allocation :

$$
\begin{array}{ll}
(\theta_0^1, \theta_0^2) = (100029.64564, -100029.64568) & (\eta_0^1, \eta_0^2) = (-100029.64564, 100029.64568) \\
(\theta_1^1, \theta_1^2) = (-0.00001, 0.00000) & (\eta_1^1, \eta_1^2) = (0.00001, 0.00000) \\
(\theta_2^1, \theta_2^2) = (0.00000, 0.00000) & (\eta_2^1, \eta_2^2) = (-0.00000, -0.00000) \\
(\theta_3^1, \theta_3^2) = (-0.00001, -0.00008) & (\eta_3^1, \eta_3^2) = (0.00001, 0.00008)
\end{array}
$$

Here (x, θ) indicates the optimal choice of agent 1 and (y, η) the optimal choice of agent 2. In the unregulated economy, the optimal asset holdings of agent 1 and 2 are huge in the first period, i.e., more than 100,000 in absolute value, compared to the negligible size of asset holdings close to zero in the second period. Both asset holdings are excessively leverage-inflated. Agent 1 finances the large long position of the risk-free asset with the correspondingly large short position of the risky asset and agent 2 does the other way around. As shown below, the reason why they are forced to inflate their portfolios through excessive leverage lies in the striking distinction between the cash flow patterns of the total asset payoff and the net expenditure: the two assets are so endogenously

[20] Numbers in consumption and portfolio allocations are rounded to the 5th decimal place. In contrast, numbers in asset prices are rounded to the 6th decimal place to clarify the endogenous payoff similarity arising in the second period.

similar at the second-period equilibrium price that agents cannot meet the need to finance the strikingly different pattern of the net expenditure on a small-scale leverage.

The following shows that the two assets are sufficiently endogenously similar in equilibrium to be almost collinear in the second period.

$$\begin{bmatrix} r_1^1 + q_1^1 & r_1^2 + q_1^2 \\ r_2^1 + q_2^1 & r_2^2 + q_2^2 \\ r_3^1 + q_3^1 & r_3^2 + q_3^2 \end{bmatrix} = \begin{bmatrix} 1 + 0.999999 & 1 + 0.999997 \\ 1 + 2.000004 & 2 + 1 \\ 1 + 3.000033 & 3 + 1.000033 \end{bmatrix} = \begin{bmatrix} 1.999999 & 1.999997 \\ 3.000004 & 3 \\ 4.000033 & 4.000033 \end{bmatrix}$$

The two assets make distinct dividend payoffs but the equilibrium asset prices make the total payoffs endogenously similar in the second period. On the other hand, the net income of agent 1 in the second period to be financed by asset holdings is

$$\begin{bmatrix} x_1 - e_1^1 \\ x_2 - e_2^1 \\ x_3 - e_3^1 \end{bmatrix} = \begin{bmatrix} 1.99999 - 1.82931 \\ 1.00001 - 0.57776 \\ 1.99998 - 2 \end{bmatrix} = \begin{bmatrix} 0.17068 \\ 0.42225 \\ -0.00002 \end{bmatrix}.$$

The pattern of net incomes for agent 1 is quite different from the payoffs of the assets in the second period. Thus, the endogenous payoff similarity forces agent 1 to hold the excessively leveraged portfolio (θ_0^1, θ_0^2) to finance the expenditure on consumptions in the second period.

5.2 Tightened Discretionary Credit Limits and Pareto Improvement

Now we examine the effect of discretionary credit limits on optimal choices and individual welfare. Especially we see how equilibrium outcomes behave in response to drastic changes in the credit limits. An interesting point is both agents get better off under credit limits than in unregulated markets. Moreover, tightened discretionary credit limits lead to higher individual welfare as far as they are not too much tight.[21] The following table shows four distinct levels of discretionary credit limits which are uniformly imposed on agents, i.e., $b_s^1 = b_s^2 = \bar{b}$ for each $i = 1, 2$ and $s = 1, 2, 3$.

	$\bar{b} = -100$	$\bar{b} = -500$	$\bar{b} = -1000$	$\bar{b} = -10,000$	$\bar{b} = -\infty$
agent 1's utility	28.7852	28.7746	28.7735	28.7725	28.7724
agent 2's utility	-14.8923	-15.0060	-15.0236	-15.0400	-15.0418

In this example, severe credit restrictions on risk-sharing opportunities improve the efficiency of market equilibrium. The counterintuitive results can be explained by the tradeoff between diversification benefits and risk-sharing opportunities. As the credit limits are tightened from $-10,000$ to -100, risk-sharing opportunities are shrunk but the correlation between the cum-dividend returns

[21] If they are too much tight, agents will get worse than in the unregulated markets. For example, they get worse in the extreme case where they are prohibited from trading the assets.

of the two assets is slightly reduced.[22] The reduced return correlation can positively affect individual welfare by increasing diversification benefits. The tradeoff acts in a positive direction as far as the credit tightening is not so severe.

Details on equilibrium profiles are provided in Appendix. Specifically, the equilibrium asset holding (η_0^1, η_0^2) of agent 2 in the unregulated markets violates the credit limits of the table. In other words, the credit limits are a binding constraint on optimal asset holdings. In fact, the four discretionary credit limits bind at the optimal asset holding of agent 2 only in state 3 of the second period.

The example shows that individual welfare can increase even when the discretionary credit limits are drastically tightened from -10,000 to -100. This result complements the literature such as Elul (1995), Cass and Citanna (1998) and Hoelle et al. (2016) which study the welfare effect of small changes in risk sharing opportunities.

5.3 Correlation-Driven Leverages are Unrestrained under Tight Wealth Limits

Now we check the wealth constraints (8) are effective in curbing leveraged portfolio inflation the unregulated economy suffers. The following shows the state-contingent values of the leveraged portfolios θ_0 and η_0 in the second period.

$$\begin{bmatrix} r_1^1 + q_1^1 & r_1^2 + q_1^2 \\ r_2^1 + q_2^1 & r_2^2 + q_2^2 \\ r_3^1 + q_3^1 & r_3^2 + q_3^2 \end{bmatrix} \cdot \begin{bmatrix} \theta_0^1 \\ \theta_0^2 \end{bmatrix} = \begin{bmatrix} 1.999999 & 1.999997 \\ 3.000004 & 3 \\ 4.000033 & 4.000033 \end{bmatrix} \cdot \begin{bmatrix} 100029.64639 \\ -100029,64643 \end{bmatrix} = \begin{bmatrix} 0.17067 \\ 0.42226 \\ -0.00014 \end{bmatrix}$$

$$\begin{bmatrix} r_1^1 + q_1^1 & r_1^2 + q_1^2 \\ r_2^1 + q_2^1 & r_2^2 + q_2^2 \\ r_3^1 + q_3^1 & r_3^2 + q_3^2 \end{bmatrix} \cdot \begin{bmatrix} \eta_0^1 \\ \eta_0^2 \end{bmatrix} = \begin{bmatrix} -0.17067 \\ -0.42226 \\ 0.00014 \end{bmatrix}$$

The wealth constraints (8) fail to restrain the excessive leverage of θ_0 and η_0 even under a very strict uniform wealth limit $\bar{b} = -1$.

The borrowing constraints (7) with $\bar{b} = -1$ which limit current leverage are not binding at the leverage-inflated portfolios θ_0 and η_0 either;

$$(q_0^1, q_0^2) \cdot (\theta_0^1, \theta_0^2) = (4.333495, 4.333492) \cdot (100029.64639, -100029.64643) = 0.30989 > -1$$
$$(q_0^1, q_0^2) \cdot (\eta_0^1, \eta_0^2) = -0.30989 > -1.$$

5.4 Unexpected Payoff Shocks and Market Failure in the Unregulated Economy

A good news to one agent is a bad news to the other agent in the unregulated economy where both agents possess highly leveraged portfolios in the first period. To see this, we consider the case

[22] As the credit limits are changed from $-10,000$ to -100, the correlation is reduced from 0.9999999992 to 0.9999883077.

that the third state is realized in the second period. The subeconomy that emerges in the third state of the second period is a two-period economy with incomplete markets. In the subeconomy, agents 1 and 2 possess the initial ownership of assets θ_0 and η_0, respectively, in addition to the initial endowments e_3^1 and e_3^2 of the good. Suppose that an unexpected payoff shock arrives before markets reopen in the second period so that equilibrium price of the subeconomy would differ from the expected equilibrium price q_3. In this case, one of the excessively leveraged portfolios θ_0 and η_0 would lead to huge gain or loss in the face of the unexpected payoff shock. Capital loss would be intolerably huge to the agent for whom price changes are a bad news because the size of the initial endowments of the good is quite small relative to the size of leverage in θ_0 and η_0. In this case, the agent would face negative initial wealth, which is not sustainable in equilibrium.

Before going into general equilibrium analysis, we do comparative analysis to reckon the sensitivity of the excessively leveraged initial portfolios to price changes in a partial equilibrium framework. If the payoff shock drops the price of the second asset by 1% from 1.00 to 0.99 without changing the price of the first asset, the wealth of the initial portfolio undergoes unsustainable changes. This is a good news to agent 1 but a bad news to agent 2. The price change would increase agent 1's wealth by 100.03 but do unbearable loss of -99.97 to agent 2 which leaves him in default. If the shock drops the price of the first asset alone from 3.00 to 2.97, agent 2 would gain 299.91 and agent 1 lose 300.09 which bankrupts him.

It is worth noting that no default is allowed in equilibrium of the economy. The previous episode couched in the partial equilibrium context cannot happen in the general equilibrium setup. However, the subeconomy fails to attain equilibrium if an agent with excessively leveraged portfolios cannot make enough income transfer through asset markets to repay huge debts in coming periods. To see this, we introduce an unanticipated payoff shock to the second asset which affects the payoff only in the second state of the third period, i.e., for some d,

$$(\tilde{r}_{3,1}^2, \tilde{r}_{3,2}^2, \tilde{r}_{3,3}^2) = (1, 1+d, 0),$$

where d measures the size of the shock. The following tables show the post-shock endowments $(\tilde{e}_{3,1}^1(\tilde{r}), \tilde{e}_{3,2}^1(\tilde{r}), \tilde{e}_{3,3}^1(\tilde{r}))$ and $(\tilde{e}_{3,1}^2(\tilde{r}), \tilde{e}_{3,2}^2(\tilde{r}), \tilde{e}_{3,3}^2(\tilde{r}))$ with distinct d values in the unregulated economy. By the relation (10), the shock d affects only $\tilde{e}_{3,2}^1(\tilde{r})$ and $\tilde{e}_{3,2}^2(\tilde{r})$.

	$d=-1$	$d=-0.1$	$d=-0.01$
$\tilde{e}_{3,2}^1(\tilde{r})$	100031.646391	10004.964608	1002.296429
$\tilde{e}_{3,2}^2(\tilde{r})$	-100028.646391	-10001.964608	-999.296429
	$d=1$	$d=0.1$	$d=0.01$
$\tilde{e}_{3,2}^1(\tilde{r})$	-100027.646462	-10000.964678	-998.296499
$\tilde{e}_{3,2}^2(\tilde{r})$	100030.646462	10003.964678	1001.296499

The above table shows that one of the agents is endowed with huge indebtedness in the second state. This is in contrast to the case that they would be endowed with positive income ($\tilde{e}^1_{3,2}(r) = 1.999965$ and $\tilde{e}^2_{3,2}(r) = 1.000035$) if there were no payoff shock. The sharp divergence between the shock-ridden and shock-free endowments leads to the failure of asset markets to attain equilibrium in the post-shock subeconomy of the unregulated economy.

There exists no equilibrium in the post-shock subeconomies with the three distinct shocks described in the above table. Remarkably, the small payoff shock with $d = \pm 0.01$ deprives the post-shock subeconomy of the opportunity to attain equilibrium. For illustrative purpose, we focus on the subeconomy with the large shock $d = -1$.[23] To see the large-shock subeconomy has no equilibrium, we rely on Proposition 12.4 of Magill and Quinzii (1996) which characterize constrained Pareto optimality in the two-period economy. The initial endowments are assumed to be composed of goods alone in Magill and Quinzii (1996). The initial-endowment structure of the post-shock subeconomy is given by

$$\{(\tilde{e}^1_3, \tilde{e}^1_{3,1}, \tilde{e}^1_{3,2}, \tilde{e}^1_{3,3}), (\tilde{e}^2_3, \tilde{e}^2_{3,1}, \tilde{e}^2_{3,2}, \tilde{e}^2_{3,3})\},$$

where

$$(\tilde{e}^1_3, \tilde{e}^1_{3,1}, \tilde{e}^1_{3,2}, \tilde{e}^1_{3,3}) = (-200057.292888, 1.999965, 100031.646391, 100030.646391),$$
$$(\tilde{e}^2_3, \tilde{e}^2_{3,1}, \tilde{e}^2_{3,2}, \tilde{e}^2_{3,3}) = (200059.492888, 1.000035, -100028.646391, -100029.546391).$$

We set $\tilde{x} = (x_3, x_{3,1}, x_{3,2}, x_{3,3})$ and $\tilde{y} = (y_3, y_{3,1}, y_{3,2}, y_{3,3})$. Let $\pi^1(\tilde{x})$ and $\pi^2(\tilde{y})$ denote the present-value vectors of agents 1 and 2, respectively. It holds that

$$\pi^1(\tilde{x}) = \left(\frac{x_3}{3x_{3,1}}, \frac{2x_3}{3x_{3,2}}, \frac{x_3}{x_{3,3}}\right), \quad \pi^2(\tilde{y}) = \left(\frac{3y_3}{y_{3,1}}, \frac{2y_3}{y_{3,2}}, \frac{y_3}{y_{3,3}}\right).$$

The set of constrained Pareto optimal allocations for the post-shock subeconomy is given as

$$\begin{aligned} A = \{(\tilde{x}, \tilde{y}) \in \mathbb{R}^6_+ : \pi^1(x)R'_3 = \pi^2(x)R'_3 = q_3 \text{ for some } q \in \mathbb{R}^2, \\ \tilde{x} - \tilde{e}^1 = R'_3 \cdot \theta_3, \quad \tilde{y} - \tilde{e}^2 = R'_3 \cdot \eta_3, \text{ and} \\ \tilde{x} + \tilde{y} = \tilde{e}^1 + \tilde{e}^2 \text{ for some } (q_3, \theta_3, \eta_3) \in \mathbb{R}^6\} \end{aligned}$$

As shown in Appendix, the set A is a 1-dimensional set of points where $(\tilde{x}, \tilde{y}) \in A$ can be expressed as a function of $y_{3,3}$.

Suppose that the post-shock subeconomy has an equilibrium $(q_3, (\tilde{x}, \tilde{y}), (\theta_3, \eta_3))$. Then the point $(\tilde{x}, \tilde{y})$ is constrained Pareto optimal (by Theorem 12.3 of Magill and Quinzii (1996)) such that it

[23] When $d = -1$, $\tilde{r}^2_{3,2}$ has 0. The case with $\tilde{r}^2_{3,2} = 0$ has the simpler structural equations which determine equilibrium relative to the case with $\tilde{r}^2_{3,2} \neq 0$. I have also shown that no equilibrium exists with a small positive or negative shock whereby $r^2_{3,2} = 1$ changes by 10% and 1%, i.e., $\tilde{r}^2_{3,2} = 1.1$ or $\tilde{r}^2_{3,2} = 0.9$, and $\tilde{r}^2_{3,2} = 1.01$ or $\tilde{r}^2_{3,2} = 0.99$, respectively. The small-shock cases are omitted from the paper because the structural equations with these shocks are too tediously complicated to write down in the limited space. Mathematica program provides a full set of solutions for the structural equations but they are not qualified as an equilibrium. The results are available upon request.

satisfies the relations specified in the set A for the point (q_3, θ_3, η_3). Thus, as shown in Appendix, $(q_3, (\tilde{x}, \tilde{y}), (\theta_3, \eta_3))$ is expressed as a function of $y_{3,3}$. In particular, by plugging the results into the budget constraint $\tilde{e}_3^2 = y_3 + q_3^1\eta_3^1 + q_3^2\eta_3^2$ of agent 2, we obtain the polynomial equation

$$145,241,561,321,571 + 220,083,811,129,600\, y_{3,3} + 373,472,913,945,700\, y_{3,3}^2 \\ - 266,768,723,748,000\, y_{3,3}^3 = 0. \tag{11}$$

This equation has three solutions, one real and two complex numbers. The real solution is $y_{3,3} = 1.96193489$. Since $\tilde{e}_{3,3}^1 + \tilde{e}_{3,3}^2 = 1.1$, it gives $x_{3,3} = -0.86193489$, which is impossible. Thus, the post-shock subeconomy has no equilibrium.

5.5 The Effect of Unexpected Payoff Shocks under Discretionary Credit Limits

The previous subsection shows that small unanticipated payoff shocks may prevent the post-shock subeconomy from attaining equilibrium when correlation-driven leverages are unrestrained. This subsection examines how the post-shock subeconomy responds to distinct sizes of payoff shocks under various discretionary credit limits. As before, let d denote the size of the payoff shock to the second asset in the second state of the final period. The following table summarizes the performance of the post-shock subeconomy in terms of the existential failure (denote by X) and success (denote by O).

	$d = 0.1$	$d = -0.1$	$d = 0.05$	$d = -0.05$	$d = 0.01$	$d = -0.01$
$\bar{b} = -1,000$	X	X	X	X	O	X
$\bar{b} = -500$	X	X	X	X	O	X
$\bar{b} = -100$	O	X	O	X	O	O
$\bar{b} = -50$	O	X	O	O	O	O
$\bar{b} = -30$	O	O	O	O	O	O

As discretionary credit limits get tighter, the post-shock subeconomy tends to perform better in attaining equilibrium. For instance, the subeconomy succeeds in attaining equilibrium only in the case with the small positive shock $d = 0.01$ under the discretionary credit limit $\bar{b} = -1,000$. When $\bar{b}$ is tightly set to -30, equilibrium exists robustly regardless of the shock sizes counted in the table. The market failures under the loosely-set credit limit are attributed to the large negative effect of excessive leverage on income as discussed in the unregulated case of the previous subsection. The above table shows that the post-shock subeconomy tends to be less susceptible to the payoff shocks as discretionary credit limits get tighter.

APPENDIX

A.1 Equilibrium in Abstract Economies

(This part heavily relies on Appendix of Won and Yannelis (2008).) The appendix need the following topological properties of correspondences. For two nonempty subsets Z and W in $\mathbb{R}^k$, consider a correspondence $\varphi : Z \to 2^W$. Let $cl\,\varphi$, $int\,\varphi$ and $co\,\varphi$ denote the correspondence from Z to 2^W which has the value $cl\,\varphi(z)$, $int\,\varphi(z)$ and $co\,\varphi(z)$ for all $z \in Z$, respectively. The correspondence φ is said to have an *open graph* if $G_\varphi \equiv \{(z, w) \in Z \times W : w \in \varphi(z)\}$ is open in $Z \times W$. The correspondence φ is said to have *open lower sections* if the set $\varphi^{-1}(w) = \{z \in Z : w \in \varphi(z)\}$ is open in Z for every $w \in W$ and φ is said to have *open upper sections* (or *open-valued*) if $\varphi(z)$ is open in W for every $z \in Z$. The correspondence φ is said to be *lower semi-continuous* if for every open set V of W, $\{z \in Z : \varphi(z) \cap V \neq \emptyset\}$ is open in Z and φ is said to be *upper semi-continuous* if for every open set V of W, $\{z \in Z : \varphi(z) \subset V\}$ is open in Z.

The proof of the existence of equilibrium to be done below will exploit the results of an abstract economy presented in Won and Yannelis (2008). Let k denote a positive integer. For each $i \in \mathcal{I}$, let Y_i be a nonempty set in $\mathbb{R}^k$. We set $Y = \prod_{i \in \mathcal{I}} Y_i$. An *abstract economy* $\Gamma = \{(Y_i, A_i, P_i) : i \in \mathcal{I}\}$ is a set of ordered triples (Y_i, A_i, P_i) where $A_i : Y \to 2^{Y_i}$ and $P_i : Y \to 2^{Y_i}$ are correspondences. The abstract economy provides a simple but powerful conceptual framework for studying an exchange economy in a general setting.

Definition A.1: A *quasi-equilibrium* for Γ is a point $y \in Y$ such that for all $i \in \mathcal{I}$,

(i) $y_i \in cl\,A_i(y)$

(ii) $P_i(y) \cap A_i(y) = \emptyset$.

The point $y \in Y$ is an *equilibrium* for Γ if it satisfies (i) and the following condition

(ii′) $P_i(y) \cap cl\,A_i(y) = \emptyset$.

The following theorem will be used in proving the existence of quasi-equilibrium of the economy with credit constraints.

Theorem A.1: Let $\Gamma = \{(Y_i, A_i, P_i) : i \in \mathcal{I}\}$ be an abstract economy satisfying the following conditions for each $i \in \mathcal{I}$

A1. Each Y_i is convex, compact and nonempty in $\mathbb{R}^\ell$.

A2. Each P_i is lower semi-continuous.

A3. A_i is convex-valued, nonempty-valued and has an open graph.

A4. $cl\,A_i$ is upper semi-continuous.

A5. $y_i \notin co\,P_i(y)$ for all $y \in Y$.

Then Γ has a quasi-equilibrium, i.e., there exists $y^* \in Y$ such that for all $i \in \mathcal{I}$,

(i) $y^*_i \in cl\,A_i(y^*)$, and

(ii) $P_i(y^*) \cap A_i(y^*) = \emptyset$.

PROOF : See Appendix of Won and Yannelis (2008).

A.2 Proof of Theorem 1

To prove Theorem 1, we need to express the budget constraint $\mathcal{B}_i(p, q, e^i, r)$ in a compact mathematical form. To do this, let $V(p, q, r)$ denote the $\ell \times (J(S+1))$ payoff matrix of the J assets in $\mathcal{B}_i(p, q, e^i, r)$ where the row coordinate is indexed in the order $0, 1, \ldots, S, (1, 1), \ldots, (1, S), (2.1), \ldots, (S, S)$ and the column coordinate is indexed in the lexical order (s, j) with $s = 0, 1, \ldots, S$ and $j = 1, \ldots k$. For notational convenience, we introduce matrices

$$\Lambda(p, q, r) = \begin{bmatrix} p_1 r_1^t + q_1^t \\ \vdots \\ p_S r_S^t + q_S^t \end{bmatrix} \quad \text{and} \quad \Lambda(q) = \begin{bmatrix} q_1^t \\ \vdots \\ q_S^t \end{bmatrix}.$$

The matrix $\Lambda(p, q, r)$ indicates the contingent payoff in the second period from investment in each unit of the assets. The budget set of agent i is expressed in a condensed form

$$\mathcal{B}_i(p, q, e^i, r) = \left\{ \ (x, \theta) \in X \times \mathbb{R}^{J(S+1)} \ \middle| \ \hat{p} \,\square\, (x - e^i) \leq V(p, q, r) \cdot \theta \text{ and } \Lambda(q) \cdot \theta_0 \geq b^i \ \right\},$$

where $\hat{p}$ is the extension of $p \in \mathbb{R}^{S+1}$ to $\mathbb{R}^\ell$ such that $\hat{p}_\omega = p_\omega$ for each $\omega \in \{0\} \cup \mathcal{S}$ and $\hat{p}_\omega = 1$ for $\omega \in \mathcal{S} \times \mathcal{S}$, and for two vectors x, y in $\mathbb{R}^\ell$, $x \,\square\, y$ indicates the column vector $\{x_\omega y_\omega, \omega \in \mathcal{S}_3\}$ in $\mathbb{R}^\ell$.

For each $i \in \mathcal{I}$ and each $x_i \in X$, we define a set $P_i(x_i) = \{x_i' \in X : u_i(x_i') > u_i(x_i)\}$. By Assumption 1, it holds that $cl\, P_i(x_i) = \{x_i' \in X : u_i(x_i') \geq u_i(x_i)\}$. By Assumption 1, P_i is nonempty-valued and satisfies the conditions A2 and A5 of Theorem A.1. We introduce sets

$$F = \{(x_1, \ldots, x_I) \in X^I : \sum_{i \in \mathcal{I}} (x_i - e^i) = 0\} \quad \text{and} \quad H = F \cap \prod_{i \in \mathcal{I}} cl\, P_i(e^i),$$

where F indicates the set of feasible consumption allocations and H the set of feasible and individually rational consumption allocations. Let $\hat{H}$ denote the projection of H onto X^I. Since H is bounded, so is $cl\, H$ and therefore, $\hat{H}$ is bounded. Hence, we can choose a closed and bounded ball K centered at the origin in $\mathbb{R}^\ell$ which contains $\hat{H}$ and e^i in its interior for all $i \in \mathcal{I}$. We introduce the truncated economy $\hat{E} = \{(\hat{X}, e^i, \hat{P}_i), i \in \mathcal{I}\}$ where for all $i \in \mathcal{I}$,

$$\hat{X} = X \cap K, \ \ \hat{X}^I = \prod_{i \in I} \hat{X} \ \text{ and } \ \hat{P}_i(x_i) = P_i(x_i) \cap K \ \text{ for all } x \in \hat{X}.$$

To truncate the portfolio set $\mathbb{R}^{J(S+1)}$, we take an increasing sequence $\{M_n\}$ of compact convex cubes with center 0 in $\mathbb{R}^{J(S+1)}$ such that $0 \in int\, M_1$ and $\bigcup_n M_n = \mathbb{R}^{J(S+1)}$. We set

$$\begin{aligned} \Theta_i^n &= \mathbb{R}^{J(S+1)} \cap M_n \quad \text{for all } n, \\ \Theta^n &= \textstyle\prod_{i \in \mathcal{I}} \Theta_i^n \quad \text{for all } n. \end{aligned}$$

For a m-dimensional vector y, let $\|y\|$ indicate the Euclidean norm, i.e.,$\|y\| = \sqrt{\sum_{h=1}^m y_h^2}$. We introduce sets Δ and Δ_1 in $\mathbb{R}^{(J+1)(S+1)}$ defined by

$$\begin{aligned} \Delta &= \textstyle\prod_{\omega \in S \cup \{0\}} \Delta_\omega \ \text{ where } \Delta_\omega = \{(p_\omega, q_\omega) \in \mathbb{R}_+ \times \mathbb{R}^J : p_\omega + \|q_\omega\| \leq 1\}, \\ \Delta_1 &= \textstyle\prod_{\omega \in S \cup \{0\}} \Delta_{1,\omega} \ \text{ where } \Delta_\omega = \{(p_\omega, q_\omega) \in \mathbb{R}_+ \times \mathbb{R}^J : p_\omega + \|q_\omega\| = 1\}. \end{aligned}$$

The following notion is used in the proof.

- For a vector $p \in \mathbb{R}^{S+1}$, $\hat{p}$ is the extension of p to $\mathbb{R}^\ell$ such that $\hat{p}_\omega = p_\omega$ if $\omega \in \{0\} \cup \mathcal{S}$, and $\hat{p}_\omega = 1$ if $\omega \in \mathcal{S} \times \mathcal{S}$.
- For a vector $y \in \mathbb{R}^\ell$, $\breve{y}$ is the subvector of y in $\mathbb{R}^{S+1}$ such that $\breve{y}_\omega = y_\omega$ for each $\omega \in \{0\} \cup \mathcal{S}$.

The proof is broken into several steps.

STEP 1: To apply Theorem A.1, we introduce the augmented set $\mathcal{I}' \equiv \{0\} \cup \mathcal{I}$ of agents by adding agent 0. For each n, we consider an abstract economy $\Gamma^n = (\hat{X} \times \Theta_i^n, A_i^n, G_i^n)_{i \in \mathcal{I}'}$ where for all $(p, q, x, \theta) \in \Delta \times \hat{X}^I \times \Theta^n$, we make the following definitions; for $i = 0$, we define

$$\begin{array}{rcl} G_0^n(p,q,x,\theta) & = & \left\{(p',q') \in \Delta : \ (p'-p) \cdot \sum_{i \in \mathcal{I}} (\breve{x}_i - \breve{e}_i) + (q'-q) \cdot \sum_{i \in \mathcal{I}} \theta_i < 0 \right\}, \\ A_0^n(q,x,\theta) & = & \Delta, \end{array}$$

and for each $i \in \mathcal{I}$, we define

$$\begin{array}{rcl} G_i^n(p,q,x,\theta) & = & \hat{P}_i(x_i) \times \Theta_i^n, \ \text{ and} \\ A_i^n(p,q,x,\theta) & = & \{(x_i', \theta_i') \in \hat{X} \times \Theta_i^n : \hat{p} \,\square\, (x' - e^i) \ll V(p,q,r) \cdot \theta_i' \text{ and } \Lambda(q) \cdot \theta_{i,0}' \gg b^i\}. \end{array}$$

Noting that for each $i \in \mathcal{I}$, $(x_i', 0) \in A_i^n(p,q,x,\theta)$ with $x_i' \ll e^i$ for all $(p,q,x,\theta) \in \Delta \times \hat{X}^I \times \Theta^n$, we see that A_i^n is nonempty-valued, convex-valued, and has an open graph. For each $i \in \mathcal{I}$, it also holds that

$$c\ell\, A_i^n(p,q,x,\theta) = \mathcal{B}_i(p,q,e^i,r) \cap (\hat{X} \times \Theta_i^n).$$

On the other hand, $cl\, A_i^n : \Delta \times \hat{X}^I \times \Theta^n \to 2^{\hat{X} \times \Theta_i^n}$ has a closed graph and $\hat{X} \times \Theta_i^n$ is compact. These imply that the correspondence $cl\, A_i^n$ is upper semi-continuous. Thus, Γ^n satisfies A1-A5 of Theorem A.1, and consequently, Γ^n has a quasi-equilibrium, i.e., there exists $(p^n, q^n, x^n, \theta^n) \in \Delta \times \hat{X}^I \times \Theta^n$ such that

(a) $(p^n, q^n) \in cl\, A_0^n(q^n, x^n, \theta^n) = \Delta$ and $G_0^n(p^n, q^n, x^n, \theta^n) \cap \Delta = \emptyset$,

and for all $i \in \mathcal{I}$,

(b) $(x_I^n, \theta_I^n) \in cl\, A_i^n(p^n, q^n, x^n, \theta^n)$, i.e.,

$$\hat{p}^n \,\square\, (x_i^n - e^i) \leq V(p^n, q^n, r) \cdot \theta_i^n,\ \Lambda(q^n) \cdot \theta_{i,0}^n \geq b^i, \text{ and } \theta_i^n \in \Theta_i^n,$$

(c) $G_i^n(p^n, q^n, x^n, \theta^n) \cap A_i^n(p^n, q^n, x^n, \theta^n) = \emptyset$, i.e., $(\hat{P}_i(x_i^n) \times \Theta_i^n) \cap A_i^n(p^n, q^n, x^n, \theta^n) = \emptyset$.

STEP 2: We show that for each n, $\sum_{i \in \mathcal{I}} \theta_i^n = 0$ and $\sum_{i \in \mathcal{I}} (x_i^n - e^i) \leq 0$. Since $(p^n, q^n) \in \Delta$ and $G_0^n(q^n, x^n, \theta^n) \cap \Delta = \emptyset$, we see that for all $(p,q) \in \Delta$ and for all $\omega \in \{0\} \cup \mathcal{S}$,

$$p_\omega^n \sum_{i \in \mathcal{I}} (x_{i,\omega}^n - e_\omega^i) \geq p_\omega \sum_{i \in \mathcal{I}} (x_{i,\omega}^n - e_\omega^i) \quad \text{and} \quad q_\omega^n \cdot \sum_{i \in \mathcal{I}} \theta_{i,\omega}^n \geq q_\omega \cdot \sum_{i \in \mathcal{I}} \theta_{i,\omega}^n. \tag{12}$$

In particular, the first part of (12) gives $p_\omega^n \sum_{i \in \mathcal{I}} (x_{i,\omega}^n - e_\omega^i) \geq 0$ for all $\omega \in \{0\} \cup \mathcal{S}$. First, we show that $\sum_{i \in \mathcal{I}} \theta_{i,0}^n = 0$. Otherwise, we could take a point $(p_0', q_0') \in \Delta_0$ that satisfies $q_0' = (1 -$

$p_0') \sum_{i\in\mathcal{I}} \theta^n_{i,0} / \| \sum_{i\in\mathcal{I}} \theta^n_{i,0} \|$ and $0 < p_0' < 1$. It holds that

$$q_0^n \cdot \sum_{i\in\mathcal{I}} \theta^n_{i,0} \geq q_0' \cdot \sum_{i\in\mathcal{I}} \theta^n_{i,0} = (1 - p_0') \| \sum_{i\in\mathcal{I}} \theta^n_{i,0} \| > 0.$$

Recalling that $p_0^n \sum_{i\in\mathcal{I}} (x^n_{i,0} - e^i_0) \geq 0$ and $p_0^n (x^n_{i,0} - e^i_0) \leq -q_0^n \cdot \theta^n_{i,0}$ for all $i \in \mathcal{I}$, we must have $q_0^n \cdot \sum_{i\in\mathcal{I}} \theta^n_{i,0} \leq 0$, which is impossible. Therefore, we have $\sum_{i\in\mathcal{I}} \theta^n_{i,0} = 0$.

Suppose that $\sum_{i\in\mathcal{I}} \theta^n_{i,s} \neq 0$ for some $s \in \mathcal{S}$. By the same arguments made above, we have $q_s^n \cdot \sum_{i\in\mathcal{I}} \theta^n_{i,s} > 0$. On the other hand, we know that $p_s^n (x^n_{i,s} - e^i_s) \leq (r_s + q_s^n) \cdot \theta^n_{i,0} - q_s^n \cdot \theta^n_{i,s}$ for all $i \in \mathcal{I}$. By aggregating it over $\mathcal{I}$, we must have $q_s^n \cdot \sum_{i\in\mathcal{I}} \theta^n_{i,s} \leq 0$, which is impossible. We conclude that $\sum_{i\in\mathcal{I}} \theta^n_i = 0$.

Now by summing the relation $\hat{p}^n \,\square\, (x^n_i - e^i) \leq V(p^n, q^n, r) \cdot \theta^n_i$ in (b) over $\mathcal{I}$, we obtain $\hat{p}^n \,\square\, \sum_{i\in\mathcal{I}} (x^n_i - e^i) \leq 0$. It follows from (12) that for each $p_\omega \in [0,1]$, $p_\omega \sum_{i\in\mathcal{I}} (x^n_{i,\omega} - e^i_\omega) \leq 0$ and thus, $\sum_{i\in\mathcal{I}} (x^n_i - e^i) \leq 0$ for each n.

STEP 3: Clearly, $\{(p^n, q^n)\}$ is bounded. The sequence $\{x^n\}$ is also bounded because $0 \leq x^n_i \leq \sum_{i\in\mathcal{I}} e^i$ for all i and n. Without loss of generality, we will assume that $(p^n, q^n, x^n) \to (p, q, x) \in \Delta \times X^I$.

STEP 4: We claim that $\{\theta^n\}$ is bounded and thus, $\theta^n \to \theta \in \Theta$. It follows from (b) of Step 1 that for each $i \in \mathcal{I}$ and each n,

i) $\hat{p}^n \,\square\, (x^n_i - e^i) \leq V(p^n, q^n, r) \cdot \theta^n_i$,

ii) $\Lambda(q^n) \cdot \theta^n_{i,0} \geq b^i$.

Suppose that $\{\theta^n_i\}$ is unbounded for some $i \in \mathcal{I}$. Then $a^n \equiv 1/\sum_{i\mathcal{I}} \|\theta^n_i\| \to 0$, and thus, $\{a^n \theta^n_i\}$ is bounded. Let $a^n \theta^n_i \to \phi_i$ for each $i \in \mathcal{I}$. Then it holds that $\sum_{i\mathcal{I}} \|\phi_i\| = 1$ and $\sum_{i\mathcal{I}} \phi_i = 0$. Since x^n_i and p^n are bounded, the third-period budget constraints in $i)$ and the asset market clearing condition in Step 2 give $R_s \cdot \phi_{i,s} = 0$. By Assumption 3, we have $\phi_{i,s} = 0$ for each $i \in \mathcal{I}$ and each $s \in \mathcal{S}$. Now we claim that $\phi_{i,0} = 0$ for all $i \in \mathcal{I}$. By $ii)$, $\Lambda(q) \cdot \phi_{i,0} = 0$ while the second-period budget constraints in $i)$ gives $\Lambda(p, q, r) \cdot \phi_{i,0} = 0$. Thus, we have $r_s \cdot \phi_{i,0} = 0$ for all $s \in \mathcal{S}$. Since R_0 has rank J, it imply $\phi_{i,0} = 0$ for all $i \in \mathcal{I}$. Consequently, we have $\phi_i = 0$ for all $i \in \mathcal{I}$, which contradicts the relation $\sum_{i\mathcal{I}} \|\phi_i\| = 1$. Therefore, we conclude that $\{\theta^n_i\}$ is bounded for each $i \in \mathcal{I}$

STEP 5: We claim that $p^n \gg 0$. Suppose that $p^n_\omega = 0$ for some $\omega \in \mathcal{S} \cup \{0\}$. For a sufficiently small number $\epsilon > 0$, let δ^ω be a vector in $\mathbb{R}^\ell$ such that $\delta^\omega_\omega = \epsilon$ and $\delta^\omega_{\omega'} = 0$ for $\omega' \neq \omega$. In particular, ϵ is taken to satisfy $x_i + \delta^\omega \in int\, \hat{X}_i$. Let n be a sufficiently large number such that $x^n_i + \delta^\omega$ is in the interior of $\hat{X}$. Then we have $(x^n_i + \delta^\omega, \theta^n_i) \in A^n_i(p^n, q^n, x^n, \theta^n)$. On the other hand, the strict monotonicity of u_i implies $x^n_i + \delta^\omega \in \hat{P}_i(x^n_i)$. It follows that $(x^n_i + \delta^\omega, \theta^n_i) \in (\hat{P}_i(x^n_i) \times \Theta^n_i) \cap A^n_i(p^n, q^n, x^n, \theta^n)$, which contradicts (c) of Step 1.

STEP 6: This step will verify that $p \gg 0$ and $(P_i(x_i) \times \mathbb{R}^{J(S+1)}) \cap \mathcal{B}_i(p, q, e^i, r) = \emptyset$ for each $i \in \mathcal{I}$. For sufficiently, large n, (x_i, θ_i) is in the interior of $K \times M_n$. First, we claim that $p \gg 0$. Suppose that $p_0 = 0$. Noting that $(x_i, \theta_i) \in cl\, A^n(p, q, x, \theta)$ and $p_0 = 0$, we can choose a small $\delta \in \mathbb{R}^\ell$ with $(x_i + \delta, \theta_i) \in cl\, A^n(p, q, x, \theta)$ such that $\delta_0 = \epsilon$ for some $\epsilon > 0$ and $\delta_\omega = 0$ at each $\omega \neq 0$. Since u_i is strictly increasing, $u_i(x_i + \delta) > u_i(x_i)$ and thus, $u_i(x_i^n + \delta) > u_i(x_i^n)$ for sufficiently large n. We choose $\beta \in (0, 1)$ such that $u_i(\beta x_i^n + \delta) > u_i(x_i^n)$. Recalling that $p^n \gg 0$ and $b^i \ll 0$, we see that

$$\hat{p}^n \,\square\, (\beta x_i^n + \delta - e^i) \ll V(p^n, q^n, r) \cdot (\beta\theta_i^n), \quad \Lambda(q^n) \cdot (\beta\theta_{i,0}^n) \gg b^i, \quad \text{and} \quad \beta\theta_i^n \in \Theta_i^n.$$

This implies that $(\beta x_i^n + \delta, \beta\theta_i^n) \in (\hat{P}_i(x_i^n) \times \Theta_i^n) \cap A_i^n(p^n, q^n, x^n, \theta^n)$, which contradicts (c) of Step 1. By making the same arguments, we can see that $p_s > 0$ for all $s \in \mathcal{S}$. Thus, we have $p \gg 0$.

What remains is to show that $(P_i(x_i) \times \mathbb{R}^{J(S+1)}) \cap \mathcal{B}_i(p, q, e^i, r) = \emptyset$ for each $i \in \mathcal{I}$. Suppose that there exists $(x_i', \theta_i') \in (P_i(x_i) \times \mathbb{R}^{J(S+1)}) \cap \mathcal{B}_i(p, q, e^i, r)$. Then we could choose $\beta \in (0, 1)$ such that $u_i(\beta x_i') > u_i(x_i)$, $\hat{p} \,\square\, (\beta x_i' - e^i) \ll V(p, q, r) \cdot (\beta\theta_i')$, $\Lambda(q) \cdot (\beta\theta_i') \gg b^i$ and $\beta\theta_i' \in \mathbb{R}^{J(S+1)}$. Then for sufficiently large n, it holds that $u_i(\beta x_i') > u_i(x_i^n)$, $\hat{p^n} \,\square\, (\beta x_i' - e^i) \ll V(p^n, q^n, r) \cdot (\beta\theta_i')$, $\Lambda(q^n) \cdot (\beta\theta_i') \gg b^i$ and $\beta\theta_i' \in \Theta_i^n$. In other words, $(\hat{P}_i(x_i^n) \times \Theta_i^n) \cap A_i^n(p^n, q^n, x^n, \theta^n) \neq \emptyset$, which contradicts (c) of Step 1.

STEP 7: Now we are ready to show that (x, θ) satisfies the market clearing condition. By Step 2, we have $\sum_{i\in\mathcal{I}} \theta_i = 0$. The results of Step 6 and the strict monotonicity of u_i gives $\hat{p} \,\square\, (x_i - e^i) = V(p, q, r) \cdot \theta_i$ for each $i \in \mathcal{I}$. By aggregating it over $\mathcal{I}$, we see that $\sum_{i\in\mathcal{I}}(x_i - e^i) = 0$. Hence, we conclude that (p, q, x, θ) is an equilibrium of the economy.

□

A.3 Equilibrium Profiles with Distinct Discretionary Credit Limits

i) Equilibrium profile with $\bar{b} = -10,000$

Asset price :
$(q_0^1, q_0^2) = (4.333400, 4.333338)$, $(q_1^1, q_1^2) = (0.999980, 0.999946)$,
$(q_2^1, q_2^2) = (2.000085, 1)$, $(q_3^1, q_3^2) = (3.000105, 1.000105)$.

Consumption allocation :

$x_0 = 2.00006$; $y_0 = 0.50654$
$(x_1, x_2, x_3) = (1.99987, 1.00013, 1.99992)$; $(y_1, y_2, y_3) = (0.45013, 0.33320, 0.20008)$
$(x_{1,1}, x_{1,2}, x_{1,3}) = (3.99999, 15.99999, 23.99999)$; $(y_{1,1}, y_{1,2}, y_{1,3}) = (1.00029, 1.00029, 0.60020)$
$(x_{2,1}, x_{2,2}, x_{2,3}) = (3.00008, 3.00039, 1.00008)$; $(y_{2,1}, y_{2,2}, y_{2,3}) = (1.00000, 0.99998, 0.14285)$
$(x_{3,1}, x_{3,2}, x_{3,3}) = (1.99971, 1.99971, 0.99996)$; $(y_{3,1}, y_{3,2}, y_{3,3}) = (1.00029, 1.00029, 0.10004)$

Portfolio allocation :

$(\theta_0^1, \theta_0^2) = (5000.00006, -5000.00017)$ $(\eta_0^1, \eta_0^2) = (-5000.00006, 5000.00017)$
$(\theta_1^1, \theta_1^2) = (-0.00029, 0.00002)$ $(\eta_1^1, \eta_1^2) = (0.00029, -0.00002)$
$(\theta_2^1, \theta_2^2) = (0.00008, 0.00010)$ $(\eta_2^1, \eta_2^2) = (-0.00008, -0.00010)$
$(\theta_3^1, \theta_3^2) = (-0.00004, -0.00025)$ $(\eta_3^1, \eta_3^2) = (0.00004, 0.00025)$

ii) Equilibrium profile with $\bar{b} = -1000$

Asset price :
$(q_0^1, q_0^2) = (4.332509, 4.331881)$, $(q_1^1, q_1^2) = (0.999801, 0.999464)$,
$(q_2^1, q_2^2) = (2.000858, 1)$, $(q_3^1, q_3^2) = (3.000808, 1.000808)$.

Consumption allocation :

$x_0 = 2.00002$; $y_0 = 0.50657$
$(x_1, x_2, x_3) = (1.99876, 1.00131, 1.99939)$; $(y_1, y_2, y_3) = (0.45124, 0.33202, 0.20061)$
$(x_{1,1}, x_{1,2}, x_{1,3}) = (3.99716, 15.99716, 23.99803)$; $(y_{1,1}, y_{1,2}, y_{1,3}) = (1.00283, 1.00283, 0.60197)$
$(x_{2,1}, x_{2,2}, x_{2,3}) = (3.00085, 3.00394, 1.00085)$; $(y_{2,1}, y_{2,2}, y_{2,3}) = (0.99915, 0.99606, 0.14200)$
$(x_{3,1}, x_{3,2}, x_{3,3}) = (1.99778, 1.99778, 0.99970)$; $(y_{3,1}, y_{3,2}, y_{3,3}) = (1.00222, 1.00222, 0.10030)$

Portfolio allocation :

$(\theta_0^1, \theta_0^2) = (500.00043, -500.00129)$ $(\eta_0^1, \eta_0^2) = (-500.00043, 500.00129)$
$(\theta_1^1, \theta_1^2) = (-0.00283, 0.00022)$ $(\eta_1^1, \eta_1^2) = (0.00283, -0.00022)$
$(\theta_2^1, \theta_2^2) = (0.00085, 0.00103)$ $(\eta_2^1, \eta_2^2) = (-0.00085, -0.00103)$
$(\theta_3^1, \theta_3^2) = (-0.00030, -0.00192)$ $(\eta_3^1, \eta_3^2) = (0.00030, 0.00192)$

iii) Equilibrium profile with $\bar{b} = -500$

Asset price :
$(q_0^1, q_0^2) = (4.331540, 4.330270)$, $(q_1^1, q_1^2) = (0.999607, 0.998941)$,
$(q_2^1, q_2^2) = (2.001743, 1)$, $(q_3^1, q_3^2) = (3.001627, 1.001627)$.

Consumption allocation :

$x_0 = 1.99999;$	$y_0 = 0.50660$
$(x_1, x_2, x_3) = (1.99756, 1.00266, 1.99878);$	$(y_1, y_2, y_3) = (0.45244, 0.33067, 0.20122)$
$(x_{1,1}, x_{1,2}, x_{1,3}) = (3.99440, 15.99440, 23.99610);$	$(y_{1,1}, y_{1,2}, y_{1,3}) = (1.00560, 1.00560, 0.60390)$
$(x_{2,1}, x_{2,2}, x_{2,3}) = (3.00173, 3.00799, 1.00173);$	$(y_{2,1}, y_{2,2}, y_{2,3}) = (0.99827, 0.99201, 0.14113)$
$(x_{3,1}, x_{3,2}, x_{3,3}) = (1.99553, 1.99553, 0.99939);$	$(y_{3,1}, y_{3,2}, y_{3,3}) = (1.00447, 1.00447, 0.10061)$

Portfolio allocation :

$(\theta_0^1, \theta_0^2) = (250.00087, -250.00259)$	$(\eta_0^1, \eta_0^2) = (-250.00087, 250.00259)$
$(\theta_1^1, \theta_1^2) = (-0.00560, 0.00043)$	$(\eta_1^1, \eta_1^2) = (0.00560, -0.00043)$
$(\theta_2^1, \theta_2^2) = (0.00173, 0.00209)$	$(\eta_2^1, \eta_2^2) = (-0.00173, -0.00209)$
$(\theta_3^1, \theta_3^2) = (-0.00061, -0.00386)$	$(\eta_3^1, \eta_3^2) = (0.00061, 0.00386)$

iv) Equilibrium profile with $\bar{b} = -100$

Asset price :
$(q_0^1, q_0^2) = (4.324694, 4.317614)$, $(q_1^1, q_1^2) = (0.998167, 0.995054)$,
$(q_2^1, q_2^2) = (2.009984, 1)$, $(q_3^1, q_3^2) = (3.009700, 1.009700)$.

Consumption allocation :

$x_0 = 2.00029;$	$y_0 = 0.50630$
$(x_1, x_2, x_3) = (1.98859, 1.01502, 1.99273);$	$(y_1, y_2, y_3) = (0.46141, 0.31831, 0.20727)$
$(x_{1,1}, x_{1,2}, x_{1,3}) = (3.97383, 15.97383, 23.98173);$	$(y_{1,1}, y_{1,2}, y_{1,3}) = (1.02617, 1.02617, 0.61827)$
$(x_{2,1}, x_{2,2}, x_{2,3}) = (3.00962, 3.04506, 1.00962);$	$(y_{2,1}, y_{2,2}, y_{2,3}) = (0.99038, 0.95494, 0.13324)$
$(x_{3,1}, x_{3,2}, x_{3,3}) = (1.97358, 1.97358, 0.99636);$	$(y_{3,1}, y_{3,2}, y_{3,3}) = (1.02642, 1.02641, 0.10364)$

Portfolio allocation :

$(\theta_0^1, \theta_0^2) = (50.00519, -50.01547)$	$(\eta_0^1, \eta_0^2) = (-50.00519, 50.01547)$
$(\theta_1^1, \theta_1^2) = (-0.02617, 0.00197)$	$(\eta_1^1, \eta_1^2) = (0.02617, -0.00197)$
$(\theta_2^1, \theta_2^2) = (0.00962, 0.01181)$	$(\eta_2^1, \eta_2^2) = (-0.00962, -0.01181)$
$(\theta_3^1, \theta_3^2) = (-0.00364, -0.02278)$	$(\eta_3^1, \eta_3^2) = (0.00364, 0.02278)$

A.4 The Set of Constrained Pareto Optimal Allocations

This appendix provides a characterization of constrained Pareto optimal allocations in the post-shock subeconomy discussed in subsection 5.4. The allocation $(\tilde{x}, \tilde{y})$ of theassumed equilibrium $(q_3, (\tilde{x}, \tilde{y}), (\theta_3, \eta_3))$ is constrained Pareto optimal. Thus, $(\tilde{x}, \tilde{y})$ satisfies the relations specified in the set A for the point (q_3, θ_3, η_3). The allocation $(\tilde{x}, \tilde{y})$ is solved in terms of $y_{3,3}$ as following.

$$
\begin{aligned}
x_3 &= \frac{11}{5} + \frac{110y_{3,3}(-153 - 80y_{3,3} + 100y_{3,3}^2)}{6237 + 19800y_{3,3} - 22100y_{3,3}^2 + 4000y_{3,3}^2}, \\
x_{3,1} &= 3 - \frac{150y_{3,3}(-153 - 80y_{3,3} + 100y_{3,3}^2)}{-693 - 9000y_{3,3} - 1100y_{3,3}^2 + 4000y_{3,3}^3}, \quad x_{3,2} = \frac{21}{10} - y_{3,3}, \quad x_{3,3} = \frac{11}{10} - y_{3,3}, \\
y_3 &= -\frac{110y_{3,3}(-153 - 80y_{3,3} + 100y_{3,3}^2)}{6237 + 19800y_{3,3} - 22100y_{3,3}^2 + 4000y_{3,3}^2}, \\
y_{3,1} &= \frac{150y_{3,3}(-153 - 80y_{3,3} + 100y_{3,3}^2)}{-693 - 9000y_{3,3} - 1100y_{3,3}^2 + 4000y_{3,3}^3}, \quad y_{3,2} = \frac{9}{10} - y_{3,3}.
\end{aligned}
$$

The price-portfolio pair (q_3, θ_3, η_3) is also solved as following.

$$
\begin{aligned}
q_3^1 &= -\frac{11(-8343 - 30000y_{3,3} + 13900y_{3,3}^2 + 4000y_{3,3})}{5(6237 + 19800y_{3,3} - 22100y_{3,3}^2 + 4000y_{3,3}^2)}, \\
q_3^2 &= -\frac{11(693 + 90000y_{3,3} + 1100y_{3,3}^2 - 4000y_{3,3})}{5(6237 + 19800y_{3,3} - 22100y_{3,3}^2 + 4000y_{3,3}^2)}, \\
\eta_3^1 &= 100029.546391 + y_{3,3}, \\
\eta_3^2 &= -100030.546426 - y_{3,3} + \frac{150y_{3,3}(-153 - 80y_{3,3} + 100y_{3,3}^2)}{-693 - 9000y_{3,3} - 1100y_{3,3}^2 + 4000y_{3,3}^3}.
\end{aligned}
$$

By plugging the above results into $\tilde{e}_3^2 = y_3 + q_3^1\eta_3^1 + q_3^2\eta_3^2$, we obtain the equation (11).

$$
\begin{aligned}
145,241,561,321,571 &+ 220,083,811,129,600y_{3,3} + 373,472,913,945,700y_{3,3}^2 \\
&- 266,768,723,748,000y_{3,3}^3 = 0.
\end{aligned}
$$

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