Dynamic Capital Structure of Value and Growth Firms*

Jaehoon Lee†

University of New South Wales

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Abstract

This paper develops the dynamic equilibrium model of capital structure and investment decisions of value and growth firms. Although Q ratio monotonically increases with the share of growth options in total firm value, it is yet to be a perfect proxy of growth options since Q ratio is also affected by a firm's production technology. Capital-intensive firms not only have higher returns from investments and thus larger growth options but also likely have invested in more capital in the past. Thus, capital-intensive firms may have lower Q ratio even when they have higher growth options. Understanding the intra- and inter-firm variations of Q ratio can explain why leverage and profitability are important determinants of investment policy. The simulation also assures that the model can successfully match the moments of riskfree interest rates, stock market excess returns, value premium and credit spreads.

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†Email address: jaehoon.lee@unsw.edu.au. Phone: +61 2 9385 6013

1 Introduction

Tobin's Q and the book-to-market equity (B/M) ratios are the most popular measures to distinguish value and growth firms. The rationale behind these measures is simple. Firm value consists of two components. One is assets-in-place, whose cashflows are generated by the vested capital, and the other is growth options. Value firms are lacking in growth options, thus they exhibit lower Q and higher B/M ratios.

However, this rationale overlooks one important mechanism. It implicitly assumes that the assets-in-place and growth options are independent entities. The independence may be true in a static model in which growth options can be exercised only once in a firm's lifetime. Static models assume that only young and small firms can exercise growth options to become mature and large firms, and in this framework the growth options can be modelled as an separate entity from assets-in-place.

In a dynamic model, however, growth options should be exercisable repeatedly. As a firm's productivity keeps growing, the firm should have been given growth options to make new investments every time its productivity reaches the next upper restructuring threshold. Growth firms do not have to be young. Furthermore, the structure of growth options would also determine the relation between the value of assets-in-place and the vested capital. Therefore, the growth options and the assets-in-place are not independent anymore.

Accounting for this mechanism, I find that neither the Tobin's Q nor the B/M ratio is a perfect proxy for growth options. For example, suppose there are two companies. Firm A has no need to make investment. Its earnings grow by an exogenous process and thus has no growth option. In comparison, Firm B is able to double production by investing in capital, either in tangible or intangible assets. It has growth options and has repeated investments during its lifetime. If we compare the Q ratio of the two firms, Firm A's Q would be infinitely large as it has not invested in capital. However, Firm B's Q would be much lower due to its repeated investments in the past.

This paper develops the dynamic equilibrium model of capital structure and investment decisions of value and growth firms. Although Q ratio monotonically increases with the share of growth options in total firm value, it is yet to be a perfect proxy of growth options since Q ratio is also affected by a firm's production technology. Capital-intensive firms not only have higher returns from investments and thus larger growth options but also likely have invested in more capital in the past. Thus, capital-intensive firms may have lower Q ratio even when they have higher growth options. Understanding the intra- and inter-firm variations of Q ratio can explain why leverage and profitability are important determinants of investment policy since the leverage and profitability can be used to adjust for the inter-firm variations of the Q ratio.

To build a dynamic investment and capital structure model, I assume that firms make refinancing and investment decisions altogether at the upper restructuring boundary. This assumption is motivated by the empirical observation that firms, on average, issue debts not to pay for cash dividends or share repurchases but to make investments in tangible and intangible assets. Thanks to the tractability that is added to the model by the assumption, I could derive closed-form solutions.

This paper makes three sets of contributions. First, the paper illustrates how a firm's production technology affects its debt policy. Capital-intensive firms grow through investments, thus they are more incentivized to protect investment options by taking on a more conservative debt policy. The model shows that there is a monotonic relation between capital intensity and default probability. More capital-intensive firms are less likely to default. For example, let α denote an inverse capital intensity. If $\alpha = 0.4$, i.e., firms are capital-intensive, the probability of default within the next 10 years after refinancing is 0.53%. In comparison, the default probability increases to 1.55% for $\alpha = 0.8$, i.e., firms have low returns from investments.

However, the relation between capital intensity and leverage is not monotonic. As Jermann (1998) and Kogan (2004) explain, capital-intensive firms can utilize investment decisions to smooth earnings growth and thus reduce the risk of cashflows. The reduced risk motivates the firms to raise leverage to take advantage of tax shields. Therefore, the firms need to compare the trade-off between increasing tax shields due to the reduced risk of cashflows and the need to protect investment options. The optimal leverage is determined by the combined effect of these two channels. As a result, the optimal leverage ratio is derived to have a U-shaped relation with capital intensity.

Moreover, the level of the model-implied optimal leverage ratio is between 40 and 50 percents, which is comparable to its empirical counterpart. In fact, the literature including Leland (1994) and Graham (2000) has conceived it a puzzle for firms to opt for such low leverages given the high potential of increasing tax benefits. This puzzle is resolved by using the same mechanism as in Almeida and Philippon (2007) that firms are more likely to default and incur high financial distress costs during a recession. The covariation of distress costs with the macroeconomic condition leads firms to opt for conservative debt policy.

The second contribution is to show why profitability and leverage are important determinants of investments. According to rational equilibrium models, firms make investments when the cost of capital is cheaper than the return from investments, or the marginal Q ratio. As implied by Modigliani and Miller (1958), investment decisions need to be separated from financing decisions, thus the marginal Q is expected to be the sole determinant of investments. However, numerous studies such as Fazzari, Hubbard, Petersen, Blinder, and Poterba (1988) show that profitability and leverage are as important determinants of investments as the Q ratio. Those studies attribute the finding to financial constraints meanwhile Erickson and Whited (2000) attribute it to the measurement error of the Q ratio. In this paper, I show that the Tobin's Q ratio is not a perfect proxy of investments due to the heterogeneity in firms' production technologies, and that profitability and leverage are effective to adjust the inter-firm variations of the Q ratio. Consistent with the empirical evidence, the regressions on simulated samples show that high profitability and low leverage ratio predict more investments even when the Q ratio is controlled.

Lastly, the third contribution is to match the moments of asset returns such as riskfree interest rates, stock market excess returns, value premium, and credit spreads. The equity premium puzzle is resolved by the long-run risk framework that combines the time-varying macroeconomic risk with the Epstein and Zin (1989) preference. The representative agent cares about the intertemporal distribution of risk, and thus pays high risk premium even for a small but persistent long-run risk in consumption.

The value premium is explained by combining two channels in the literature. One is the costly reversibility of investments (e.g., Berk, Green, and Naik (1999), Gomes, Kogan, and Zhang (2003), Carlson, Fisher, and Giammarino (2004), and Zhang (2005)), and the other is the high leverage of value firms (e.g., Ozdagli (2012), and Choi (2013)).

My model also generates the average credit spreads of 139.6 bps, which is comparable to 126.6 bps of the spreads of the BofA Merrill Lynch US Corporate BBB and AAA option-adjusted bond yields. As Huang and Huang (2012) point out, standard models of credit risk such as Merton (1974) and Leland (1994) are not able to generate realistic credit spreads. The difficulty of matching the level of credit spreads has also been considered a puzzle until Hackbarth, Miao, and Morellec (2006), Chen, Collin-Dufresne, and Goldstein (2009), Bhamra, Kuehn, and Strebulaev (2010), and Chen (2010) show that the credit risk premium can be understood as a compensation for the covariation of default risk with macroeconomic conditions.

The paper by Kuehn and Schmid (2014) is especially close to my work as they also develop a joint dynamic equilibrium model of leverage and investments. Both of our models are also based on Bansal and Yaron (2004)'s long-run risk framework. In contrast to my study, however, they do not account for the heterogeneity in firms' production technologies. In my paper, firms span a two-dimensional space of productivity and production technology (or, capital intensity). In comparison, they assume that all firms share identical production technology but differ in the level of vested capital. This setup adds complexity to their model. Thus, closed-form solutions do not exist and numerical solutions are derived instead. Our focuses also differ. My work is to study how the heterogeneity in firms' production technologies affects leverage, investment and asset returns meanwhile their work is to study homogenous firms with differing levels of capital on the credit spreads.

2 Theory

In this section, I borrow the structural equilibrium model of dynamic capital structure from Bhamra, Kuehn, and Strebulaev (2010, hereafter BKS) and then extend the model to incorporate an investment opportunity. It is a consumption-based asset pricing model with a representative agent of the Epstein–Zin–Weil preference. The state price density in the economy, π_t, is derived from the marginal utility of the agent's consumption,

$$\pi_t = \left(\beta\, e^{-\beta t}\right)^{\frac{1-\gamma}{1-\frac{1}{\psi}}} C_t^{-\gamma} \left(p_{C,t}\, e^{\int_0^t p_{C,s}^{-1} ds}\right)^{-\frac{\gamma-\frac{1}{\psi}}{1-\frac{1}{\psi}}}, \tag{1}$$

where β is the rate of time preference, γ is the coefficient of relative risk aversion (RRA), ψ is the elasticity of intertemporal substitution (EIS), and p_C is the ratio of price to consumption. I assume $\psi > 1/\gamma$, which implies that the agent prefers early resolution of uncertainty.

The state of the economy is denoted by ν_t, and the economy can be either in a bad state ($\nu_t = 1$) or a good state ($\nu_t = 2$). The shift between these two states is governed by the Markov chain, and λ_{ν_t} denotes the probability per unit time of the economy leaving state ν_t. The convergence rate of the Markov chain shift is given by $p = \lambda_1 + \lambda_2$, and thus the long-run distribution is derived as

$$(f_1, f_2) = (\lambda_2/p,\, \lambda_1/p)\,, \tag{2}$$

where f_i is the long-run probability of being in state i.

The aggregate consumption process, C_t, is given by

$$\frac{d\,C_t}{C_t} = g_{\nu_t}\,dt + \sigma_{C,\nu_t} dB_{C,t}, \tag{3}$$

where g_{ν_t} is expected consumption growth rate in state ν_t, σ_{C,ν_t} is consumption growth volatility, and $B_{C,t}$ is a standard Brownian motion. The first and second moments of the consumption growth process depend on the state of the economy, ν_t, and the uncertainty of being in either state is priced since the representative agent cares about the intertemporal distribution of risk. Thus, this model can be considered a revised form of Bansal and Yaron (2004)'s long-run risk framework.

Firm n makes earnings by combining productivity with capital,

$$\Pi_{n,t} = X_{n,t}^{\alpha}\,K_{\nu_0}^{1-\alpha} - m\,K_{\nu_0}, \tag{4}$$

where $\Pi_{n,t}$ is the firm's earnings, $X_{n,t}$ is productivity, K_{ν_0} is the capital stock vested at time 0, α is the share of productivity in production, and m denotes the operating cost of capital. The optimized level of capital K_{ν_0} depends on the state at time 0, ν_0. m can be considered depreciation cost that the firm pays for the maintenance of the capital. Firms with low α can be considered to have capital-intensive production technology.

The firm's productivity process, $X_{n,t}$, is given by

$$\frac{dX_{n,t}}{X_{n,t}} = \theta_{n,\nu_t}\,dt + \sigma_{X,n}^{id}\,dB_{X,n,t}^{id} + \sigma_{X,n,\nu_t}^{s}\,dB_{X,t}^{s}, \tag{5}$$

where θ_{n,ν_t} is the expected productivity growth rate of firm n, and $\sigma_{X,n}^{id}$ and σ_{X,n,ν_t}^{s} are the idiosyncratic and systematic volatilities of the firm's productivity growth. The standard Brownian motion $B_{X,t}^{s}$ is the systematic shock and thus correlated with the consumption growth shock,

$$dB_{X,t}^{s}\,dB_{C,t} = \rho_{XC}\,dt. \tag{6}$$

However, the idiosyncratic shock, $B_{X,n,t}^{id}$, is not correlated with any other uncertainty. I will suppress the subscript n hereafter for the simplicity of notation. Since both consumption and productivity processes have procyclical mean growth rates and countercyclical volatilities, it can be assumed that $g_1 < g_2$, $\theta_1 < \theta_2$, $\sigma_{C,1} > \sigma_{C,2}$, and $\sigma_{X,1}^{s} > \sigma_{X,2}^{s}$.

This specification generalises BKS since the two models become equivalent for $\alpha = 1$ and $m = 0$. Throughout this paper, I assume zero operating cost of capital, $m = 0$, to assure that the effect of an investment opportunity on leverage is not due to the operating leverage induced by this variable.

2.1 Default and restructuring boundaries

The firm does not continuously adjust capital structure since it is costly to issue new debts. Instead, the firm sets an inaction region with two boundaries: default boundaries at the lower bound and restructuring boundaries at the upper bound. At the default boundaries, the firm declares bankruptcy and debt holders receive a recovered portion of its liquidated unlevered assets. At the restructuring boundaries, the firm issues new debts. Between these two boundaries, the firm distributes earnings to pay for coupons to debt holders and dividends to shareholders.

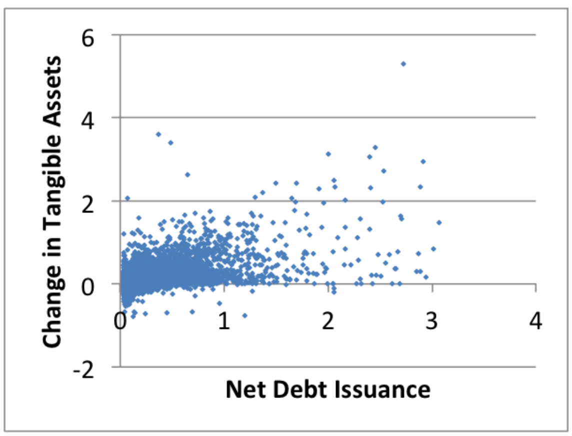
Many papers in the literature[1] consider a firm's earnings or its present value as an exogenous state variable, and thus implicitly assume that all proceeds from new debt issuance are paid to shareholders as lump-sum cash dividends. However, the empirical evidence shows that it is not the case. For example, Figure 1 shows where firms spend the proceeds from debt issuance. The figure covers only the firms whose net debt issuance is bigger than 3% of their lagged market assets so that we can focus on the firms in their restructuring years. The figure shows strong positive correlations between net debt issuance and the investments in both tangible and intangible assets. In comparison, the correlation with the change in cash holdings is positive but its magnitude is substantially smaller relative to those with the investments. Moreover, net equity payout actually turns out to have negative correlation with net debt issuance, implying that firms issue equity and debt simultaneously.

Table 1 shows the results of regressing those variables—investments in tangible and intangible assets, increase in cash holdings, and net equity payout—on net debt issuance. The table again confirms the implications from the figure. For every dollar from net debt issuance, 45.9 cents and 39.8 cents are likely to be invested in tangible and intangible assets, respectively. Cash holdings are likely to increase by 7.7 cents, still significant but much smaller than the coefficients on investments. The net equity payout has negative coefficient, which implies that one dollar from debt issue is on average matched with another 9 cents of equity issue. Instead of making cash payments to shareholders at refinancing, firms issue debt and equity simultaneously to make new investments in tangible and intangible assets.

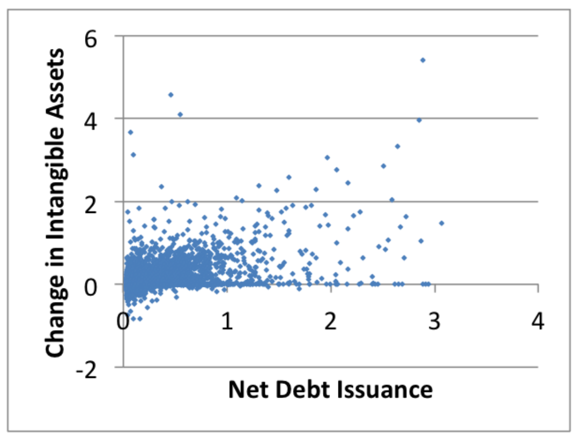
[1]See Leland (1994), Chen, Collin-Dufresne, and Goldstein (2009), Chen (2010), and Bhamra, Kuehn, and Strebulaev (2010) among many others.

Figure 1: Debt issuance with corporate policy variables

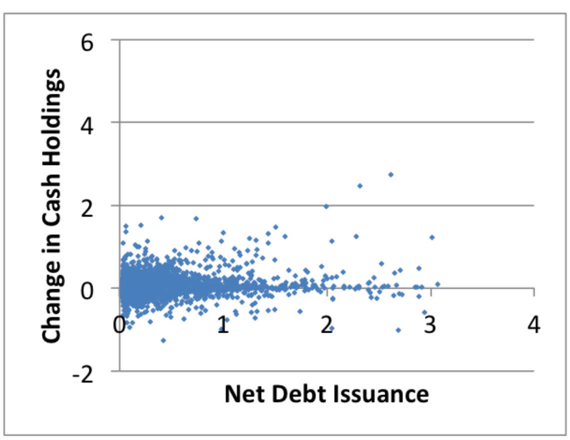
This scatter plot compares net debt issuance with four corporate policy variables. Net debt issuance is defined as the changes in the sum of total long-term debt (Compustat code: *dltt*) and total debt in current liabilities (*dlc*). The policy variable in subfigure (a) is the change in total property, plant and equipment (*ppent*), and the variable in (b) is the change in total intangible assets (*intan*). The variable in (c) is the change in cash equivalents (*che*), and the one in (d) is net equity payout, which is the sum of total dividends (*dvt*) and purchase of shares (*prstkc*) less sales of shares (*sstk*). All variables are scaled by lagged market assets. This figure includes only the samples of their refinancing years in which the net debt issuance is bigger than 3% of the lagged market assets.



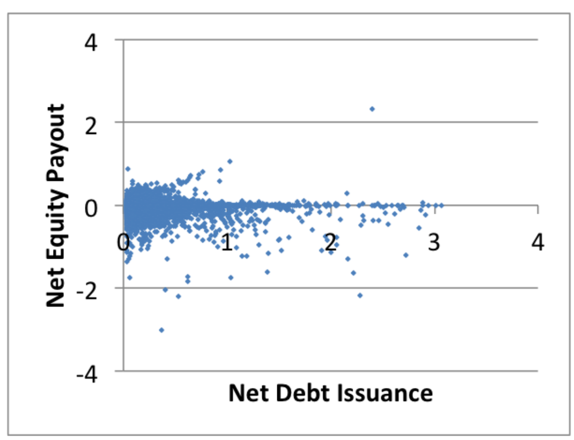
(a) Change in tangible assets



(b) Change in intangible assets



(c) Change in cash holdings



(d) Net equity payout

Table 1: Regression of corporate policy variables on debt issuance

Corporate policy variables are regressed on the net debt issuance. Net debt issuance is defined as the changes in the sum of total long-term debt (Compustat code: *dltt*) and total debt in current liabilities (*dlc*). The policy variable in column (1) is the change in total property, plant and equipment (*ppent*), and the variable in (2) is the change in total intangible assets (*intan*). The variable in (3) is the change in cash equivalents (*che*), and the one in (4) is net equity payout, which is the sum of total dividends (*dvt*) and purchase of shares (*prstkc*) less sales of shares (*sstk*). All variables are scaled by lagged market assets. This table runs regressions using only the samples of their refinancing years in which the net debt issuance is bigger than 3% of the lagged market assets. Numbers in parentheses are OLS t statistics. ***, **, and * denote significance at the 1%, 5%, and 10% level, respectively.

	(1)	(2)	(3)	(4)
dep. var.	ΔTangible Assets	ΔIntangible Assets	ΔCash Holdings	Net Equity Payout
Net debt Issuance	0.459***	0.398***	0.077***	-0.090***
	(112.67)	(87.02)	(26.59)	(-32.09)
obs	28,707	24,805	28,756	28,761
R^2	0.307	0.234	0.024	0.035

Thus, I assume that firms adjust not only capital structure but also their investment level at the restructuring boundaries. The investment cost can be either larger or smaller than the amount of new debt issuance. When the investment costs more than the debt issue, the firm is implied to issue equity to fund the investment. Otherwise, the leftover of newly issued debts after investment will be paid to shareholders.

This assumption is consistent with Bachmann and Bayer (2014)'s finding that the cross-sectional dispersion of firm-level investment rates is procyclical even though the dispersion of firm-level productivity shocks is countercyclical. The authors explain the observation using the fixed cost of investment. The so-called "spike-adjusters" make investments when they reach the boundary at which the return from investment becomes higher than fixed cost, and this boundary is more likely to be reached during a good time. I combine this investment boundary with the refinancing boundary for the tractability of the model, and this setup is supported by the empirical data.

Since the economy is in either one of two states, the default and restructuring boundaries have four different values, respectively, depending on the state at time 0 and the state at the time of default/restructuring, ν_0 and ν_D/ν_U,

$$X_{D,\nu_0\,\nu_D} \Rightarrow \{X_{D,11}, X_{D,12}, X_{D,21}, X_{D,22}\}\,, \tag{7}$$

and

$$X_{U,\nu_0\,\nu_U} \Rightarrow \{X_{U,11}, X_{U,12}, X_{U,21}, X_{U,22}\}\,. \tag{8}$$

Firms are more likely to default during a bad time, and this counter-cyclicality of default implies $X_{D,\nu_0\,1} > X_{D,\nu_0\,2}$ for $\nu_0 = 1, 2$. Similarly, firms are more likely to make investments during a good time, and this pro-cyclicality of investments implies $X_{U,\nu_0\,1} > X_{U,\nu_0\,2}$

2.2 Debt and equity valuation

When the firm goes bankrupt and then liquidated, its asset is priced as the after-tax value of the unlevered firm's future earnings. Let A_{ν_t} denote the liquidation value in state ν_t.

$$A_{\nu_t}(X_t, K_{\nu_0}) = (1-\tau)\, E_t\left[\int_t^\infty \frac{\pi_s}{\pi_t}\,\Pi_s\,ds\,\middle|\,\nu_t\right] \tag{9}$$

$$= (1-\tau)\left(\frac{X_t^\alpha\,K_{\nu_0}^{1-\alpha}}{r_{A,\nu_t}} - \frac{m\,K_{\nu_0}}{r_{P,\nu_t}}\right), \tag{10}$$

where τ is tax rate, r_{A,ν_t} is the discount rate in the standard Gordon growth model, and r_{P,ν_t} is the discount rate of perpetuity.

$$r_{A,\nu_t} = \bar{\mu}_{\nu_t} - \alpha\,\theta_{\nu_t} + \frac{(\bar{\mu}_j - \alpha\,\theta_j) - (\bar{\mu}_{\nu_t} - \alpha\,\theta_{\nu_t})}{\hat{p} + \bar{\mu}_j - \alpha\,\theta_j}\,\hat{p}\,\hat{f}_j, \quad j \neq \nu_t, \tag{11}$$

$$\bar{\mu}_{\nu_t} = r_{\nu_t} + \gamma\,\alpha\,\rho_{XC}\,\sigma^s_{X,\nu_t}\,\sigma_{C,\nu_t}, \tag{12}$$

and

$$r_{P,\nu_t} = r_{\nu_t} + \frac{r_j - r_{\nu_t}}{\hat{p} + r_j}\,\hat{p}\,\hat{f}_j, \quad j \neq \nu_t, \tag{13}$$

where r_{ν_t} is the one-period riskfree interest rate in state ν_t, $\hat{p}$ is the risk-neutral convergence rate of the Markov chain, and $\hat{f}_j$ is the risk-neutral long-run probability of being in state j. The derivation of the riskfree interest rate and the risk-neutral probabilities is explained in Appendix A.

Debtholders are entitled to three different types of cashflows. The first is the coupon payment until either boundary is reached. The second is the liquidation value of the firm's assets when it goes bankrupt. The third is the continuation value of the debt when the restructuring boundary is reached. Thus, the value of debt can be derived as

$$B_{\nu_t}\left(X_t, K_{\nu_0}, c_{\nu_0}, \nu_0\right) = \frac{c_{\nu_0}}{r_{P,\nu_t}} + \sum_{\nu_D=1}^{2} q_{D,\nu_t\nu_D}(X_t, \nu_0)\left(\phi_{\nu_D}\, A_{\nu_D}(X_{D,\nu_0\nu_D}, K_{\nu_0}) - \frac{c_{\nu_0}}{r_{P,\nu_D}}\right)$$
$$+ \sum_{\nu_U=1}^{2} q_{U,\nu_t\nu_U}(X_t, \nu_0)\left(R_{\nu_0\nu_U} - \frac{c_{\nu_0}}{r_{P,\nu_U}}\right), \tag{14}$$

where $q_{D,\nu_t\nu_D}$ and $q_{U,\nu_t\nu_U}$ denote Arrow-Debreu securities that pay one at the default and restructuring boundaries. ϕ_{ν_D} is the recovery ratio after bankruptcy, and $R_{\nu_0\nu_U}$ is the continuation value of the debt,

$$R_{\nu_0\nu_U} = \frac{c_{\nu_0}}{c_{\nu_U}(c_{\nu_0})}\, B_{\nu_U}\left(X_{U,\nu_0\,\nu_U}, K_{\nu_U}(K_{\nu_0}), c_{\nu_U}(c_{\nu_0}), \nu_U\right), \tag{15}$$

where $c_{\nu_U}(c_{\nu_0})$ and $K_{\nu_U}(K_{\nu_0})$ denote the new coupon payment and capital stock in the next restructuring cycle. All debts are assumed to have equal seniority, thus the debt is diluted during the restructuring on a per-coupon basis.

Similarly, shareholders receive dividend payments within a cycle and the restructured equity at the upper boundary. Thus, the value of equity can be derived as

$$S_{\nu_t}(X_t, K_{\nu_0}, c_{\nu_0}, \nu_0) = Div_{\nu_t}(X_t, K_{\nu_0}, c_{\nu_0}, \nu_0) + \sum_{\nu_U=1}^{2} q_{U,\nu_t\nu_U}(X_t, \nu_0)\, E_{\nu_0\nu_U} \tag{16}$$

where Div_{ν_t} is the present value of dividends paid to equity holders during the current cycle,

$$Div_{\nu_t}(X_t, K_{\nu_0}, c_{\nu_0}, \nu_0) = A_{\nu_t}(X_t, K_{\nu_0}) - (1-\tau)\frac{c_{\nu_0}}{r_{P,\nu_t}}$$
$$+ \sum_{\nu_D=1}^{2} q_{D,\nu_t\nu_D}(X_t, \nu_0)\left[(1-\tau)\frac{c_{\nu_0}}{r_{P,\nu_D}} - A_{\nu_D}(X_{D,\nu_0\nu_D}, K_{\nu_0})\right]$$
$$+ \sum_{\nu_U=1}^{2} q_{U,\nu_t\nu_U}(X_t, \nu_0)\left[(1-\tau)\frac{c_{\nu_0}}{r_{P,\nu_U}} - A_{\nu_U}(X_{U,\nu_0\nu_U}, K_{\nu_0})\right]. \tag{17}$$

$E_{\nu_0\nu_U}$ is the value of equity right before the restructuring occurs, which is the sum of net debt issuance and stock price less investment costs,

$$E_{\nu_0 \nu_U} = [\,(1-\iota_{\nu_U})\,B_{\nu_U}\,(X_{U,\nu_0\,\nu_U}, K_{\nu_U}(K_{\nu_0}), c_{\nu_U}(c_{\nu_0}), \nu_U) - R_{\nu_0 \nu_U}\,]$$
$$- \psi\,(K_{\nu_U}(K_{\nu_0}) - K_{\nu_0}) + S_{\nu_U}\,(X_{U,\nu_0\,\nu_U}, K_{\nu_U}(K_{\nu_0}), c_{\nu_U}(c_{\nu_0}), \nu_U)\,, \quad (18)$$

where ι_{ν_U} is the debt issuance cost, and $\psi(I)$ denotes the investment adjustment costs,

$$\psi(I) = I + \frac{\kappa}{2}\left(\frac{I}{K_{\nu_0}}\right)^2 K_{\nu_0}. \quad (19)$$

I assume $\kappa = 0$ to show that the implications of this model are not due to the convexity of investment adjustment costs.

2.3 Homogeneity

Homogeneity is a very effective tool to keep the tractability of the model. Let $\xi_{\nu_0 \nu_U}$ denote a scaling factor that depends on the initial state (ν_0) and the restructuring state (ν_U), then one could easily see

$$\xi_{\nu_0 \nu_U} = \frac{X_{U,\nu_0 \nu_U}}{X_0}, \quad (20)$$

since the firm restarts the next cycle at $X_{U,\nu_0 \nu_U}$ and all values are to be scaled by the same constant.

BKS show that two conditions are required for the homogeneity property to hold. First, $\xi_{\nu_0\,\nu_U}$ is time-invariant and level-invariant quantity. Second, the following condition needs to be satisfied:

$$\frac{\xi_{2i}}{\xi_{1i}} = \frac{c_2}{c_1} = \frac{K_2}{K_1}, \quad i = 1, 2. \quad (21)$$

The scalability due to the homogeneity condition implies that the optimal coupon and capital for the next cycle are equal to the scaled values of those in the current cycle,

$$\frac{c_{\nu_U}(c_{\nu_0})}{c_{\nu_0}} = \frac{K_{\nu_U}(K_{\nu_0})}{K_{\nu_0}} = \xi_{\nu_U \nu_U}. \quad (22)$$

Combining with equation (21), it is straightforward to derive

$$\frac{c_{\nu_U}(c_{\nu_0})}{c_{\nu_U}} = \frac{K_{\nu_U}(K_{\nu_0})}{K_{\nu_U}} = \xi_{\nu_0 \nu_U}. \quad (23)$$

The value of debt and equity at the restructuring boundaries can be simplified by applying the homogeneity conditions, (22) and (23), as follows:

$$B_{\nu_U}\left(X_{U,\nu_0\,\nu_U}, K_{\nu_U}(K_{\nu_0}), c_{\nu_U}(c_{\nu_0}), \nu_U\right) = \xi_{\nu_0\nu_U}\, B_{\nu_U}\left(X_0, K_{\nu_U}, c_{\nu_U}, \nu_U\right), \tag{24}$$

$$S_{\nu_U}\left(X_{U,\nu_0\,\nu_U}, K_{\nu_U}(K_{\nu_0}), c_{\nu_U}(c_{\nu_0}), \nu_U\right) = \xi_{\nu_0\nu_U}\, S_{\nu_U}\left(X_0, K_{\nu_U}, c_{\nu_U}, \nu_U\right), \tag{25}$$

$$R_{\nu_0\nu_U} = \frac{c_{\nu_0}}{c_{\nu_U}} B_{\nu_U}\left(X_0, K_{\nu_U}, c_{\nu_U}, \nu_U\right), \tag{26}$$

$$\begin{aligned} E_{\nu_0\nu_U} = &\left[(1-\iota_{\nu_U})\,\xi_{\nu_0\nu_U} - \frac{c_{\nu_0}}{c_{\nu_U}}\right] B_{\nu_U}\left(X_0, K_{\nu_U}, c_{\nu_U}, \nu_U\right) \\ &- K_{\nu_0}\left(\xi_{\nu_U\nu_U} - 1\right)\left[1 + \frac{\kappa}{2}\left(\xi_{\nu_U\nu_U} - 1\right)\right] + \xi_{\nu_0\nu_U}\, S_{\nu_U}\left(X_0, K_{\nu_U}, c_{\nu_U}, \nu_U\right). \end{aligned} \tag{27}$$

2.4 Optimization of coupons and investments

The model has 12 policy variables in total : $X_{D,ij}$, $X_{U,ij}$, K_i and c_i for $i, j = 1, 2$. Among these twelve variables, five of them are determined by the homogeneity condition.

$$\frac{X_{U,2i}}{X_{U,1i}} = \frac{X_{D,2i}}{X_{D,1i}} = \frac{K_2}{K_1} = \frac{c_2}{c_1} \quad \text{for} \quad i = 1, 2. \tag{28}$$

Default boundaries are chosen by the following smooth pasting conditions:

$$\left.\frac{\partial S_1(X_t, K_1, c_1, 1)}{\partial X_t}\right|_{X_t = X_{D,11}} = 0, \qquad \left.\frac{\partial S_2(X_t, K_1, c_1, 1)}{\partial X_t}\right|_{X_t = X_{D,12}} = 0. \tag{29}$$

The smooth pasting conditions implicitly assume convexity of S at X_D and thus infer two important implications. One is the limited liability of equity as $S\left(X_t\,; X_D\right) > 0$ for $X_t > X_D$, and the other is to guarantee that shareholders do not have any incentive to delay default further from X_D since $S\left(X_t\,; X_D - \Delta\right)|_{X_t \in [X_D - \Delta, X_D]} < 0$ for a small Δ. In addition, the value of equity is always zero at X_D, $S\left(X_D\,; X_D\right) = 0$, due to the definition of the Arrow–Debrew securities and equation (16) and (17). Note that X_D cannot be chosen by firm value maximization because firm value is maximized when $X_D = 0$, the firm never defaults, and thus the tax shield becomes a perpetuity. Of course, the limited liability of equity prevents X_D from being arbitrarily small.

Now only five policy variables are left undetermined: $X_{U,11}$, $X_{U,12}$, K_1, c_1 and c_2. For a given coupon c_2, other policy variables are chosen to maximise the levered firm value in state 1,

$$\left(X_{U,11}, X_{U,12}, K_1, c_1\right) = \arg\max\, F_1(X_{U,11}, X_{U,12}, K_1, c_1) \tag{30}$$

where $F_1 = B_1(X_0, K_1, c_1, 1)(1-\iota_1) - K_1 + S_1(X_0, K_1, c_1, 1)$. Finally, coupon c_2 is determined by maximising the levered firm value in state 2,

$$c_2 = \arg\max\ F_2(c_2) \tag{31}$$

where $F_2 = B_2(X_0, K_2, c_2, 2)(1 - \iota_2) - K_2 + S_2(X_0, K_2, c_2, 2)$.

3 Model's implication about the investment policy

Table 2 reports the estimates of model parameters. Panel A shows the parameters for the preference function of the representative agent. RRA and EIS are taken from Bansal and Yaron (2004)'s calibration. $\psi > 1/\gamma$ implies that the agent prefers the early resolution of

Table 2: Parameter estimates

This table reports the estimates of model parameters. The parameters for the representative agent's preference in panel A are taken from the calibration of Bansal and Yaron (2004), and the parameters for the economy and the firm in panel B and C are from Bhamra, Kuehn, and Strebulaev (2010).

Parameter	Symbol	State 1	State 2
Panel A. Representative agent's preference			
Relative risk aversion (RRA)	γ	10	
Elasticity of intertemporal substitution (EIS)	ψ	1.5	
Annual discount rate	β	0.01	
Panel B. Economy			
Consumption growth rate	g_{ν_t}	0.0141	0.0420
Consumption growth volatility	σ_{C,ν_t}	0.0114	0.0094
Actual long-run probabilities	f_{ν_t}	0.3555	0.6445
Actual convergence rate to long run	p	0.7646	0.7646
Tax rate	τ	0.35	0.35
Panel C. Firm			
Productivity growth rate	θ_{ν_t}	-0.0401	0.0782
Productivity growth systematic volatility	σ^s_{X,ν_t}	0.1334	0.0834
Productivity growth idiosyncratic volatility	σ^{id}_{X,ν_t}	0.2258	0.2258
Correlation	ρ_{XC}	0.1998	0.1998
Bankruptcy costs	$1 - \phi_{\nu_t}$	0.30	0.10
Debt issuance costs	ι_{ν_t}	0.03	0.01

uncertainty, and $\psi > 1$ is to ensure that the price-to-consumption ratio is procyclical.

Panel B shows the parameters for the aggregate economy. I borrowed the parameters from Bhamra, Kuehn, and Strebulaev (2010), who calibrate them using the consumption expenditure data from the Bureau of Economic Analysis. According to their calibration, consumption growth rate is procyclical but its volatility is countercyclical. The average duration of the bad state ($\nu_t = 1$) is 2.029 years, and the duration of the good state ($\nu_t = 2$) is 3.679 years. In the literature, tax rate varies from 15 percent in BKS to 35 percent in Leland (1994), and I use 35 percent in this paper.[2]

Panel C shows the parameters for a firm. Productivity growth rates are pro-cyclical meanwhile its volatilities, bankruptcy costs and debt issuance costs are couter-cyclical. Productivity growth rates and its volatilities are yet to be calibrated to data. They are temporarily matched to BKS' estimates of earnings growth rate parameters.

Table 3: Earnings growth rate moments

This table shows the moments of earnings growth rates. The first row shows their empirical moments which are estimated by Bhamra, Kuehn, and Strebulaev (2010), and the rows below show the model-implied moments from the simulation of 1,000 firms for 10,000 years for each value of α, the share of productivity in production. The simulation is run on a monthly basis and then the moments are annualised. The column "overall" shows the weighted average of moments in bad and good states by the long-run probabilities of each state, f_{ν_t}.

	mean			standard deviation			kurtosis		
	bad state	good state	overall	bad state	good state	overall	bad state	good state	overall
empirical	-0.0401	0.0782	0.0361	0.2623	0.2407	0.2484			
$\alpha = .99$	-0.0379	0.0782	0.0369	0.2595	0.2385	0.2460	3.00	3.00	3.00
$\alpha = .90$	-0.0328	0.0770	0.0379	0.2368	0.2202	0.2261	3.04	3.17	3.12
$\alpha = .80$	-0.0271	0.0754	0.0389	0.2123	0.2022	0.2058	3.28	4.30	3.94
$\alpha = .70$	-0.0214	0.0734	0.0397	0.1887	0.1873	0.1878	4.23	8.41	6.93
$\alpha = .60$	-0.0158	0.0714	0.0404	0.1664	0.1762	0.1727	7.5	19.0	14.9
$\alpha = .50$	-0.0098	0.0691	0.0411	0.1462	0.1696	0.1613	18.1	40.4	32.5
$\alpha = .40$	-0.0041	0.0662	0.0412	0.1289	0.1680	0.1541	49.9	74.1	65.5
$\alpha = .30$	0.0013	0.0626	0.0408	0.1167	0.1718	0.1522	136	118	124
$\alpha = .20$	0.0067	0.0574	0.0394	0.1137	0.1817	0.1575	330	173	229

Table 3 shows the moments of earnings growth rates from the model's simulations and

[2]The model-implied optimal leverage level falls from 40–50% when tax rate is 35 percent to 25–35% when tax rate is 15 percent due to the decreasing benefits of tax shields.

compare them to the empirical moments which are estimated by BKS. For each value of α, the share of productivity in production, 1,000 firms are simulated for 10,000 years and then the moments are estimated conditional on the state of the economy. The column "overall" shows the average of the moments in each state weighted by its long-run probability, f_{ν_t}.

According to the table, the case of $\alpha = .99$ closely matches BKS' estimates since the model converges to BKS as $\alpha \to 1$. Its kurtosis implies that its earnings growth shocks are driven by normal distribution.

As the firm becomes more capital-intensive and thus α decreases, however, not only the average growth rates and its volatilities get closer to zero but also its kurtosis increases significantly. The change in the distribution of earnings growth is due to the fact that capital-intensive firms are more likely to grow through investments rather than a gradual rise in productivity. The decreasing standard deviation implies that capital-intensive firms have less volatile cashflows and thus are incentivised to borrow more and opt for higher leverage. This implication will be reassured by the next figure.

Figure 2 plots model-implied firm characteristics at the initial refinancing, $t = 0$, with regard to α. Each subfigure corresponds to log capital, log firm size, Tobin's Q, leverage, and the probability of default and refinancing within the next 10 years after refinancing, respectively. Productivity at $t = 0$ is normalised to one, $X_0 = 1$. These characteristics are the outcomes of the optimal policy variables that are derived in Section 2.4. The figure plots the weighted average of firm characteristics at each initial state, $\nu_0 = 1, 2$.

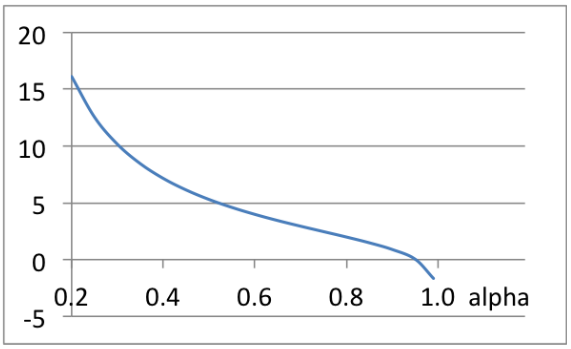
Panel (a) shows the log optimal investment in capital, $\log K_{\nu_0}$. The optimal investment decreases with α since high-α firms earn less marginal returns from the investment in capital. For example, the amount of capital at $\nu_0 = 1$ rapidly decreases from 1,108 for $\alpha = 0.4$ to 172 for $\alpha = 0.5$ and 46 for $\alpha = 0.6$.

Panel (b) shows the log of total firm value, $\log(B_{\nu_0}(X_0) + S_{\nu_0}(X_0))$. The log firm size also decreases with α since the productivity at $t = 0$ is normalised and thus earnings monotonically increase with capital, $\Pi_0 = K_{\nu_0}^{1-\alpha}$. Capital-intensive firms with low α make bigger investments, generate higher earnings, and thus lead to larger market values.

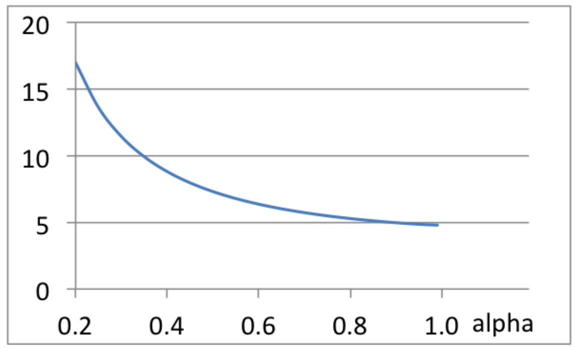
Panel (c) exhibits Tobin's Q, $\frac{B_{\nu_0}(X_0)+S_{\nu_0}(X_0)}{K_{\nu_0}}$, which is the ratio of the previous two characteristics. As shown earlier, the optimal investment in the denominator decreases with α much faster than the firm size in the numerator, thus the Q ratio monotonically increases with α. In particular, Q explodes to infinity as $\alpha \to 1$. Although all firms are at $t = 0$ and thus have little difference in terms of the share of growth options in total firm

Figure 2: Firm characteristics at the initial refinancing

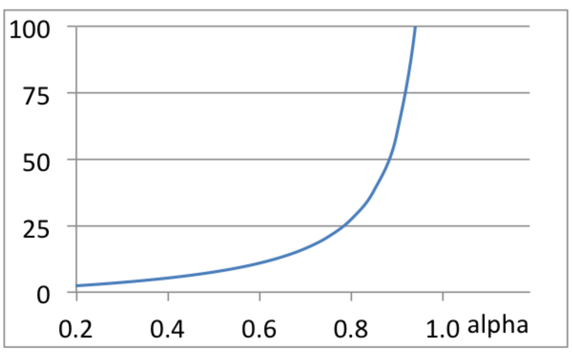
The figure shows model-implied firm characteristics at the initial refinancing, $t = 0$, for each value of α. In panel (a) to (d), the characteristics are derived for each state of $\nu_0 = 1, 2$ and their weighted averages are plotted. Panel (e) and (f) show the probabilities of default and refinancing within the next 10 years after refinancing. The probabilities are estimated from the simulation of 10,000 firms for 120 months twice, one to start in a bad state and the other to start in a good state, and the whole simulation is repeated for 1,000 times.



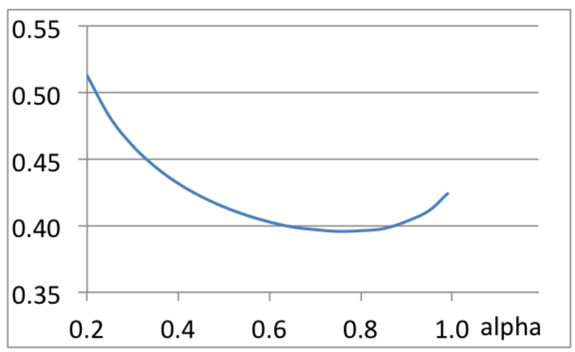
(a) Log capital ($\log K_{\nu_0}$)



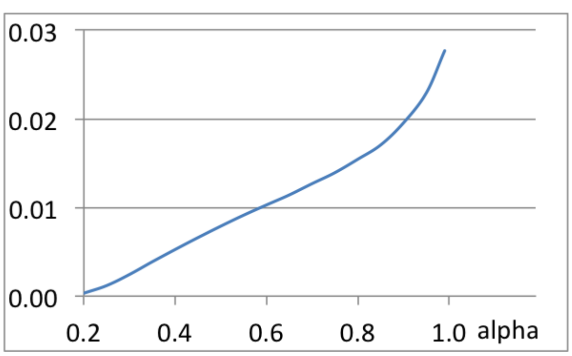
(b) Log firm size ($\log\{B_{\nu_0}(X_0) + S_{\nu_0}(X_0)\}$)



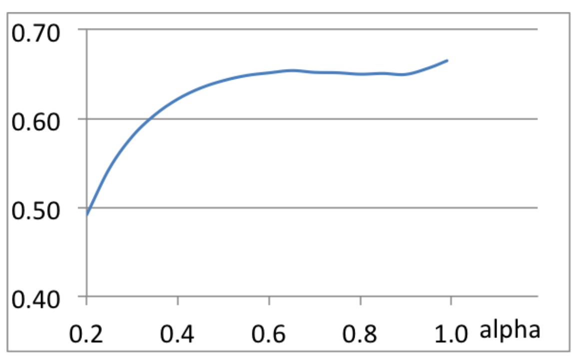
(c) Q ratio $\left(\frac{B_{\nu_0}(X_0)+S_{\nu_0}(X_0)}{K_{\nu_0}}\right)$



(d) Leverage $\left(\frac{B_{\nu_0}(X_0)}{B_{\nu_0}(X_0)+S_{\nu_0}(X_0)}\right)$



(e) Probability of default



(f) Probability of refinancing

value, the Q ratio shows huge variation depending on their reliance on capital. In other words, value firms with low α may have higher Q ratio than growth firms with high α.

Panel (d) shows the optimal leverage, $\frac{B_{\nu_0}(X_0)}{B_{\nu_0}(X_0)+S_{\nu_0}(X_0)}$. As it was previously implied by Table 3, low-α firms have less volatile earnings and are thus incentivised to take on higher leverage. However, the figure shows that leverage does not monotonically decrease with α. As implied by the liquidation value in equation (9), $A_{\nu_t}(X_t, K_{\nu_0})$, firms lose further investment opportunities at default. Since low-α firms grow through investment, they adopt conservative debt policy to prevent the prospective loss of investment opportunity. In contrast, high-α firms are less concerned with the loss and thus take on more aggressive debt policy. This is consistent with George and Hwang (2010)'s observation that firms with low distress costs are likely to have higher leverage ratio at refinancing. Therefore, the optimal leverage shows U-shape due to the combined effect of these two channels. Moreover, the magnitude of the model-implied leverage is well matched with the level observed from empirical data.

Panel (e) and (f) show the probabilities of default and refinancing within the next 10 years after the initial refinancing. The probabilities are estimated from simulations. I simulate 10,000 firms for 120 months twice, one to start in a bad state and the other to start in a good state, count the number of defaults and refinancing, and repeat the whole process for 1,000 times.

In panel (e), the probability of default monotonically increases with α. Although low-α firms take on high leverage, their leverage ratio is not high enough to raise the default probability since the firms are afraid of losing future investment opportunities. In contrast, high-α firms gain more from tax shields than investments, and thus issue more debts at the risk of raising the chance of default.

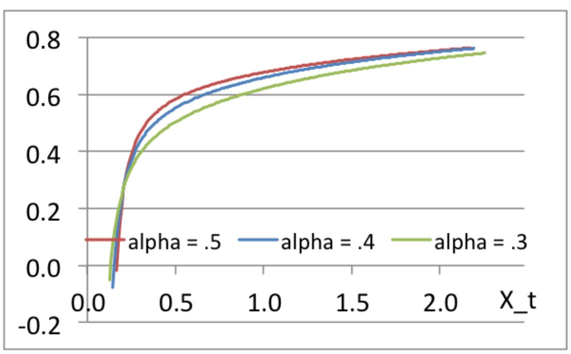
Panel (f) shows that the probability of refinancing also generally increases with α, implying that high-α firms refinance more frequently than low-α firms. In a static model such as Leland (1994), firms compare the tradeoff between tax shields and financial distress costs. In a dynamic model such as BKS, however, firms can defer a portion of tax shields until the next refinancing cycle in order to reduce the distress costs. Since high-α firms have higher chance of default, they are incentivised to temporarily defer some tax shields instead of refinancing more frequently.

To summarise, the figure provides the following implications. First, $Q \to \infty$ as $\alpha \to 1$ not because high-α firms have higher growth options but because they have low returns from investments and thus the denominator of their Q ratio converges to zero. Second, high-α firms gain more from tax shields than investment opportunities. With these two implications combined, one may conclude that firms with higher Q ratio are actually less concerned with future investment opportunities. This conclusion seemingly challenges the traditional approach to use the Q ratio as a proxy for growth options.

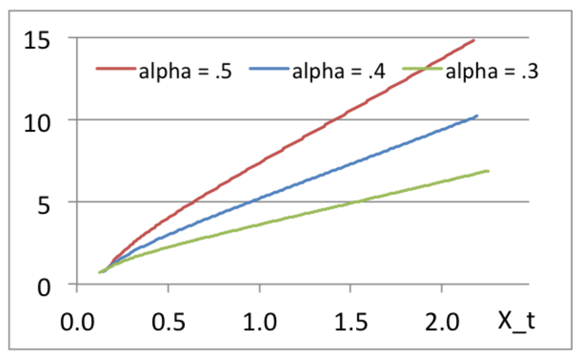
So far, the figure has covered the cross-sectional firm characteristics at the initial refinancing point ($t = 0$) and shows that the Q ratio is affected by a firm's production technology, α. Now the next figure will cover the characteristics **within** a cycle depending on the state variable of productivity, X_t. It assures that the Q ratio indeed increases with growth options within a cycle but still posits the problem that **inter-firm variations** of Q may be as large as, or even larger than, its **intra-firm variations**.

Figure 3: Firm characteristics within a cycle

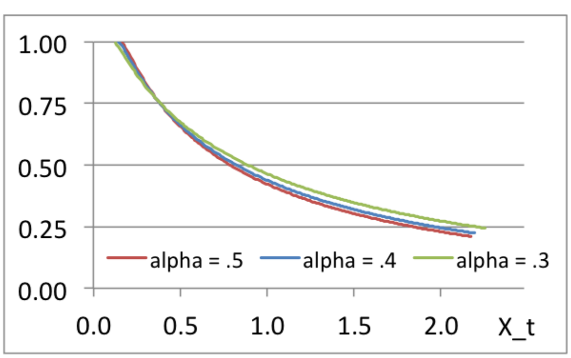
This figure shows model-implied firm characteristics within a refinancing cycle for $\alpha = 0.5$, 0.4 and 0.3, respectively. The horizontal axis in all four subfigures denotes the productivity state variable, X_t. The characteristics are derived for the economy to be in a good state, $\nu_0 = \nu_t = 2$.



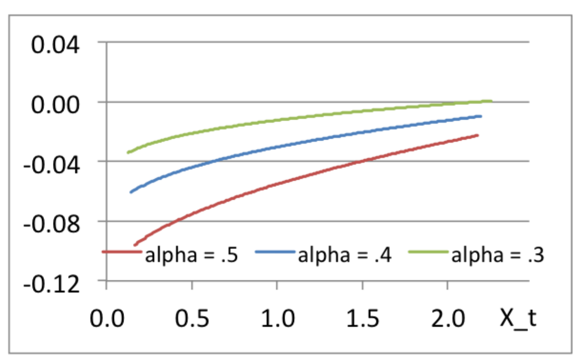
(a) Share of growth option in total firm value



(b) Q ratio



(c) Leverage



(d) Profitability

Figure 3 shows model-implied firm characteristics within a refinancing cycle for $\alpha = 0.5$, 0.4 and 0.3, respectively. The horizontal axis in all four subfigures denotes the productivity state variable, X_t. The figure is plotted for the inaction region of X_t between the maximum of default boundaries, $X_{D,\nu_0\nu_t}$, and the minimum of refinancing boundaries, $X_{U,\nu_0\nu_t}$. The characteristics are derived for the economy to be in a good state, $\nu_0 = \nu_t = 2$. Although not reported in this article, the relations of firm characteristics to X_t are equivalent to the other states of the economy except a little difference in magnitudes.

To begin with, panel (a) shows the share of growth options in the total firm value, $1 - \frac{A_{\nu_t}(X_t,\nu_0)}{B_{\nu_t}(X_t,\nu_0)+S_{\nu_t}(X_t,\nu_0)}$. It is obvious that the share of growth options monotonically increases with X_t as firms get closer to the exercise of the next refinancing and investment options. Also, the share turns negative when X_t falls near to the default boundary due to financial distress costs $(1 - \phi_{\nu_t})$. What is interesting, however, is the limited effects of α. There is virtually no difference between the firms of different α's. Thus, the share of growth options appears to be determined by X_t relative to its boundaries regardless of the cross-sectional difference of firms' production technologies.

Next, panel (b) revisits the Q ratio, $\frac{B_{\nu_t}(X_t,\nu_0)+S_{\nu_t}(X_t,\nu_0)}{K_{\nu_0}}$. The Q ratio monotonically increases with X_t as firms with high X_t bear bigger growth options. In contrast to the previous panel, however, the Q ratio is also significantly affected by α, the cross-sectional difference of firms. Therefore, one can conclude that the Q ratio is not perfect as a proxy of growth options since it is affected by inter-firm variations although the share of growth options is not. Growth firms with low α possibly have lower Q ratio than value firms with high α.

The figure shows two more characteristics: leverage in panel (c), $\frac{B_{\nu_t}(X_t,\nu_0)}{B_{\nu_t}(X_t,\nu_0)+S_{\nu_t}(X_t,\nu_0)}$, and profitability in panel (d), which is defined as earnings less coupons divided by the vested capital, $\frac{X_t^\alpha K_{\nu_0}^{1-\alpha} - c_{\nu_0}}{K_{\nu_0}}$. These two characteristics also have monotonic relations to X_t but in different directions from the Q ratio. The leverage ratio decreases with X_t but with limited effects by α, and the profitability increases with X_t but is negatively related to α. Thus, low leverage and high profitability predict more investments in the next time period.

Note that the model assumes two independent dimensions of firm heterogeneity. One is the productivity state variable, X_t, and the other is a firm's production technology, α. X_t indicates the timing of next investments meanwhile α determines the magnitude of investments to be made. Therefore, although the Q ratio alone is not a perfect proxy for growth options, the two-dimensional space of X_t and α can be spanned by the combination of Q ratio with either leverage or profitability. In other words, the leverage and profitability can be used to adjust for the inter-firm variations of the Q ratio and thus play an important role as the determinants of investments.

Panel (c) and (d) also reassure Danis, Rettl, and Whited (2014) and Korteweg and Strebulaev (2013)'s finding that the cross-sectional correlation between profitability and leverage is positive when firms are at their optimal leverage but negative otherwise. According to the figure, high-α firms have lower leverage and lower profitability at the initial refinancing point when $X_0 = 1$, thus implying a positive relation between these two variables. Within a cycle, however, leverage and profitability are negatively correlated since leverage decreases with X_t while profitability increases. Moreover, their unconditional correlation is expected to be negative since most variations in leverage and profitability are driven by X_t rather than α. This implication can resolve Myers (1993), who states that "the most telling evidence against the static trade-off theory is the strong inverse correlation between profitability and leverage."

Table 4: Regression of net investments on Q, profitability and leverage

Net investments are regressed on lagged Q ratio, profitability and leverage. Panel A runs the regression on simulated samples, and Panel B on the Compustat database. In Panel A, 200 firms for each group of $\alpha = .3$, .4 and .5 are simulated for 100 years, of which the first 50 years are discarded as burn-in periods and the latter 50 years are used to run the regression. This whole process is repeated 8,845 times to derive the .05 and .95 confidence intervals, which are shown in brackets. In Panel B, the net investments are defined as the change in the sum of tangible and intangible assets ($ppent + intan$) scaled by the sum's lagged value. Numbers in parentheses are OLS t statistics. ***, **, and * denote significance at the 1%, 5%, and 10% level, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)
	Panel A. Simulation			**Panel B. Empirical**		
Q	0.035***	0.035***	0.023***	0.023***	0.021***	0.018***
	[0.029, 0.041]	[0.028, 0.043]	[0.021, 0.026]	(55.05)	(48.67)	(37.74)
profitability		2.629***			0.118***	
		[1.800, 3.688]			(21.27)	
leverage			-0.215***			-0.188***
			[-0.390, -0.097]			(-22.96)
R^2	0.110	0.153	0.117	0.048	0.055	0.056
	[0.100, 0.119]	[0.131, 0.172]	[0.103, 0.129]			
obs				60,598	60,598	60,598

Table 4 reports the results of regressing net investments on the lagged Q ratio, profitability and leverage ratio. The regression is run on the simulated samples in Panel A and on the Compustat database in Panel B. In Panel A, 200 firms for each group of $\alpha = .3$, .4 and .5 are simulated for 100 years, of which the first 50 years are discarded as burn-in periods and the latter 50 years are used to run the regression. This whole process is repeated 8,845 times to derive the .05 and .95 confidence intervals, which are shown in brackets. In Panel B, capital stock is first defined as the sum of tangible and intangible assets ($ppent + intan$ in the Compustat item codes). Net investments are the change in the capital stock scaled by its lagged value, Q ratio is the market value of assets divided by the capital stock, profitability is net incomes scaled by the capital stock, and leverage is the book value of debts divided by the market value of assets. Firms whose total book assets are less than 10 million dollars are dropped, and the outliers of the characteristics are filtered at 1% and 99% levels.

As expected, the characteristics predict investments with correct signs. Q and profitability predict high investments while leverage does low ones. However, I could not put the three characteristics altogether since it creates multicollinearity among variables. The table reassures that profitability and leverage can raise the predictability of investments even in the presence of the Q ratio.

Moreover, the magnitudes of regression coefficients from the simulations in Panel A are comparable to those from the Compustat database in Panel B except the coefficient of profitability. The coefficient of leverage is within the .05 and .95 confidence interval, and the coefficients of Q ratio are also close to the confidence interval boundaries. Only the profitability shows a large difference due to the unconditional second moments of its values. For example, the unconditional standard deviation of profitability from the simulations is 0.015 while its standard deviation from the Compustat is 0.343. The simulated standard deviation is 22.9 (= 0.343/0.015) times smaller, thus its coefficient becomes 22.3 (= 2.629/0.118) times bigger. This comparison reminds of Table 3 which shows that the model underestimates the second moments of earnings growth rates compared to the empirical estimates by BKS.

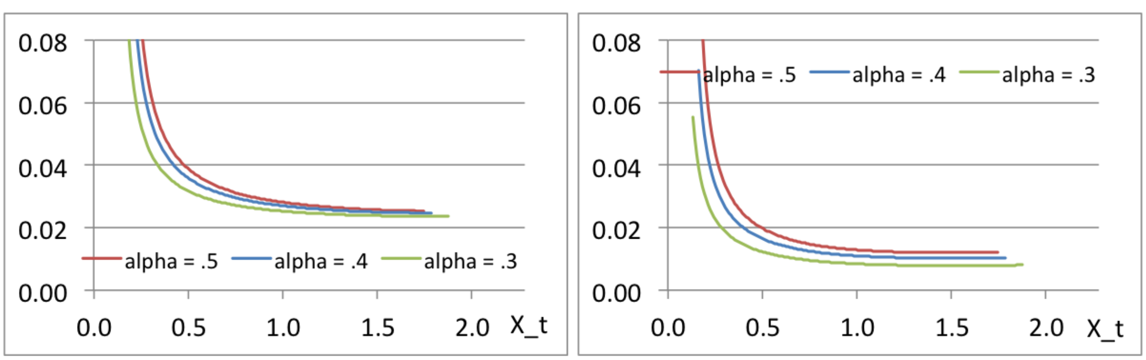
In sum, this section shows that the variations of the Q ratio are driven by not only the likelihood of the exercise of growth options but also the cross-sectional difference of a firm's production technology. Thus, capital-intensive firms can have lower Q ratio even when they have larger growth options. The inter-firm variations of the Q ratio can be adjusted by adding leverage and profitability as the explanatory variables of investment decisions. Thus, the model can explain why leverage and profitability are important determinants of investments without the help of the pecking order theory.

4 Risk premium and asset returns

One long-time yardstick to test a structural model of investments is to see whether the model is able to recreate the value premium. For example, Carlson, Fisher, and Giammarino (2004) and Zhang (2005) explain the value premium using the costly reversibility of investments and its covariation with the stochastic discount factor. In comparison, Ozdagli (2012) and Choi (2013) find that value firms tend to take on higher leverages and thus yield higher stock returns than growth firms but their unlevered asset returns have little variations across book-to-market portfolios and can be explained by a single-factor CAPM model. My model is related to both approaches since (1) it rules out the disinvestment of capital and (2) the growth options and leverage are negatively correlated with each other due to their relation to the state variable, X_t.

Figure 4: Equity risk premium and credit spreads

The figure shows the model-implied moments of equity risk premium and credit spreads. The risk premiums are estimated for each state of $(\nu_0, \nu_t) \in \{(1,1), (1,2), (2,1), (2,2)\}$, and their weighted averages are plotted. The horizontal axis denotes the productivity state variable, X_t.



(a) Equity risk premium (b) Credit spreads

Figure 4 shows the model-implied moments of equity risk premium and credit spreads with regard to the productivity state variable, X_t. Panel (a) compares equity risk premium, which is derived as

$$\text{equity risk premium} = \gamma\, \rho_{XC}\, \frac{\partial \ln S_{\nu_t}(X_t, \nu_0)}{\partial \ln X_t}\, \sigma^s_{X,\nu_t}\, \sigma_{C,\nu_t} + \Gamma_{\nu_t}, \tag{32}$$

where the first term of its right-hand side is the risk premium due to the covariation with the Browninan systematic shock and the second term, Γ_{ν_t}, is the premium to the Markov regime switch,

$$\Gamma_{\nu_t} = (1 - \omega_{\nu_t}) \left\{ \frac{S_j(X_t, \nu_0)}{S_{\nu_t}(X_t, \nu_0)} - 1 \right\} \lambda_{\nu_t}, \qquad j \neq \nu_t. \tag{33}$$

The figure suggests two implications. First, the term $\frac{\partial \ln S_{\nu_t}(X_t, \nu_0)}{\partial \ln X_t}$ in (32) captures the leverage effect, thus the equity risk premium monotonically decreases with X_t. Second, high-α firms have slightly higher risk premium than low-α firms since their earnings growth rates have higher systematic risk.

As discussed in the previous section, the Q ratio is positively related to X_t and α. If firms have low Q ratio due to low productivity (X_t), their equity would have been more levered and thus offer higher equity risk premium. If the low Q were accounted by low α, instead, the firms would bear lower systematic risk and thus yield lower unlevered asset returns. This implication is consistent with Choi (2013)'s finding that high book-to-market portfolios have higher leverage and stock returns but their unlevered asset returns have lower systematic risk.

Panel (b) compares the credit spreads of corporate debts, which is derived as

$$\text{credit spread} = \frac{c}{B_{\nu_t}\left(X_t, c_{\nu_0}, \nu_0\right)} - r_{P,\nu_t}. \tag{34}$$

Since the corporate debts are assumed to pay coupons indefinitely, their yields are subtracted by the riskfree perpetuity rates. The figure shows that the credit spreads explode as X_t approaches its default boundary due to the increasing chance of default. Moreover, high-α firms show higher credit spreads than low-α firms since they adopt more aggressive debt policy as explained in the previous section.

In sum, the figure shows the risk premiums conditional on the state variables. The risk premiums in both the stock and corporate bond markets decrease with X_t due to a leverage effect but increase with α because of the increasing systematic risk and more aggressive debt policy. Now the next table is to compare the unconditional moments of asset returns from simulations to those from the empirical data.

Table 5 shows the unconditional first and second moments of riskfree interest rates, stock market excess returns, value premium and the bond market credit spreads. In Panel A, 200 firms for each group of $\alpha = .3$, .4 and .5 are simulated for 100 years, of which the first 20 years are discarded as burn-in periods and the latter 80 years are used to estimate

Table 5: Asset return moments

This table shows the first and second moments of asset returns from the simulations in Panel A and from the empirical data in Panel B. In Panel A, 200 firms for each group of $\alpha = .3$, .4 and .5 are simulated for 100 years, of which the first 20 years are discarded as burn-in periods and the latter 80 years are used to estimate the asset return moments. This whole process is repeated 9,130 times to derive the .05 and .95 confidence intervals, which are shown in brackets. In Panel B, the moments of riskfree interest rates, stock market excess returns and value premium are estimated from the monthly Fama–French factors, and the moments of credit spreads are estimated from the BofA Merrill Lynch US Corporate BBB/AAA Option-Adjusted Spreads.

	Panel A. Simulation		**Panel B. Empirical**	
	mean	stdev	mean	stdev
Riskfree rates	0.248 [0.225, 0.269]	0.076 [0.066, 0.079]	0.284	0.254
Stock market excess returns	0.379 [0.168, 0.529]	4.133 [3.432, 4.782]	0.651	5.391
Value premium	0.177 [0.003, 0.289]	1.448 [1.173, 1.743]	0.394	3.489
Credit spreads	1.396 [1.286, 1.531]	0.156 [0.098, 0.228]	1.266	0.699

the asset return moments. This whole process is repeated 9,130 times to derive the .05 and .95 confidence intervals, which are shown in brackets. In Panel B, the moments of riskfree interest rates, stock market excess returns and value premium are estimated from the monthly Fama–French factors, and the moments of credit spreads are estimated from the spread of the BofA Merrill Lynch US Corporate BBB and AAA option-adjusted bond yields.[3] All data are based on monthly observations. The first three variables are the monthly realised values while the last variable—credit spreads—is annualised.

The table shows that the mean value of riskfree interest rates from the simulation is well matched to its empirical counterpart although the simulated standard deviation is far short of the empirical standard deviation. The difference in the second moment of riskfree rates is because the model does not take into account the variations in inflation rates. In other words, the model-implied riskfree rates are in real terms while the empirical riskfree rates are nominal.

[3] `http://research.stlouisfed.org/fred2/categories/32297`

The model-implied mean and volatility of the stock market excess returns are 0.349 and 0.4133, which are smaller than their empirical counterparts of 0.651 and 5.391 but not completely off the mark. The model underestimates the stock market return moments since it also underestimates the volatility of earnings growth rates.

To derive the value premium from the simulation, I follow Fama and French (1993)'s portfolio formation method. Book-to-market (BM) portfolios are formed every July conditional on the BM ratios in the last December, and the portfolios are maintained for 12 months until the next June. Again, the model-implied moments of value premium turn out to be smaller than the empirical moments due to the underestimation of the systematic risk of earnings growth but still stay within a reasonable range.

Lastly, the model-implied average credit spreads are 139.6 bps, which is very close to the empirical average of 126.6 bps. Thus, one can conclude that the credit spread puzzle, which was first introduced by Huang and Huang (2002) as the difficulty of matching the level of credit spreads using a structural model, is now fully resolved.[4]

5 Conclusion

A Derivation of riskfree rate and risk-neutral measures

B Arrow-Debreu Securities

[4] One can refer to Chen, Collin-Dufresne, and Goldstein (2009), Chen (2010) and Bhamra, Kuehn, and Strebulaev (2010) to study how the literature of the credit spread puzzle has developed since Huang and Huang (2002).

B.1 Arrow-Debreu restructuring claim if $X_{U,2} < X_{U,1}$

	$X \leq X_{D,2}$	$X_{D,2} < X \leq X_{D,1}$	$X_{D,1} < X \leq X_{U,2}$	$X_{U,2} < X \leq X_{U,1}$	$X_{U,1} < X$
$q_{U,11}$	0	0	$\sum_{m=1}^{4} h_{11,m} X^{k_m}$	$\sum_{m=1}^{2} g_{11,m} X^{l_m}$	1
$q_{U,12}$	0	0	$\sum_{m=1}^{4} h_{12,m} X^{k_m}$	$\sum_{m=1}^{2} g_{12,m} X^{l_m} + \frac{\hat{\lambda}_1}{r_1+\hat{\lambda}_1}$	0
$q_{U,21}$	0	$\sum_{m=1}^{2} s_{21,m} X^{j_m}$	$\sum_{m=1}^{4} h_{11,m}\, \epsilon(k_m) X^{k_m}$	0	0
$q_{U,22}$	0	$\sum_{m=1}^{2} s_{22,m} X^{j_m}$	$\sum_{m=1}^{4} h_{12,m}\, \epsilon(k_m) X^{k_m}$	1	1

Boundary conditions

1. $q_{U,11}$ at $X_{D,1}$ $\Rightarrow$ $\sum_{m=1}^{4} h_{11,m} X_{D,1}^{k_m} = 0$
2. $q_{U,11}$ at $X_{U,2}$ $\Rightarrow$ $\sum_{m=1}^{2} g_{11,m} X_{U,2}^{l_m} - \sum_{m=1}^{4} h_{11,m} X_{U,2}^{k_m} = 0$
3. $q'_{U,11}$ at $X_{U,2}$ $\Rightarrow$ $\sum_{m=1}^{2} g_{11,m}\, l_m X_{U,2}^{l_m} - \sum_{m=1}^{4} h_{11,m}\, k_m X_{U,2}^{k_m} = 0$
4. $q_{U,11}$ at $X_{U,1}$ $\Rightarrow$ $\sum_{m=1}^{2} g_{11,m} X_{U,1}^{l_m} = 1$
5. $q_{U,12}$ at $X_{D,1}$ $\Rightarrow$ $\sum_{m=1}^{4} h_{12,m} X_{D,1}^{k_m} = 0$
6. $q_{U,12}$ at $X_{U,2}$ $\Rightarrow$ $\sum_{m=1}^{2} g_{12,m} X_{U,2}^{l_m} - \sum_{m=1}^{4} h_{12,m} X_{U,2}^{k_m} = -\frac{\hat{\lambda}_1}{r_1+\hat{\lambda}_1}$
7. $q'_{U,12}$ at $X_{U,2}$ $\Rightarrow$ $\sum_{m=1}^{2} g_{12,m}\, l_m X_{U,2}^{l_m} - \sum_{m=1}^{4} h_{12,m}\, k_m X_{U,2}^{k_m} = 0$
8. $q_{U,12}$ at $X_{U,1}$ $\Rightarrow$ $\sum_{m=1}^{2} g_{12,m} X_{U,1}^{l_m} = -\frac{\hat{\lambda}_1}{r_1+\hat{\lambda}_1}$
9. $q_{U,21}$ at $X_{D,2}$ $\Rightarrow$ $\sum_{m=1}^{2} s_{21,m} X_{D,2}^{j_m} = 0$
10. $q_{U,21}$ at $X_{D,1}$ $\Rightarrow$ $\sum_{m=1}^{4} h_{11,m}\, \epsilon(k_m) X_{D,1}^{k_m} - \sum_{m=1}^{2} s_{21,m} X_{D,1}^{j_m} = 0$
11. $q'_{U,21}$ at $X_{D,1}$ $\Rightarrow$ $\sum_{m=1}^{4} h_{11,m}\, \epsilon(k_m)\, k_m X_{D,1}^{k_m} - \sum_{m=1}^{2} s_{21,m}\, j_m X_{D,1}^{j_m} = 0$
12. $q_{U,21}$ at $X_{U,2}$ $\Rightarrow$ $\sum_{m=1}^{4} h_{11,m}\, \epsilon(k_m) X_{U,2}^{k_m} = 0$
13. $q_{U,22}$ at $X_{D,2}$ $\Rightarrow$ $\sum_{m=1}^{2} s_{22,m} X_{D,2}^{j_m} = 0$
14. $q_{U,22}$ at $X_{D,1}$ $\Rightarrow$ $\sum_{m=1}^{4} h_{12,m}\, \epsilon(k_m) X_{D,1}^{k_m} - \sum_{m=1}^{2} s_{22,m} X_{D,1}^{j_m} = 0$
15. $q'_{U,22}$ at $X_{D,1}$ $\Rightarrow$ $\sum_{m=1}^{4} h_{12,m}\, \epsilon(k_m)\, k_m X_{D,1}^{k_m} - \sum_{m=1}^{2} s_{22,m}\, j_m X_{D,1}^{j_m} = 0$
16. $q_{U,22}$ at $X_{U,2}$ $\Rightarrow$ $\sum_{m=1}^{4} h_{12,m}\, \epsilon(k_m) X_{U,2}^{k_m} = 1$

B.2 Arrow-Debreu default claim if $X_{U,2} < X_{U,1}$

	$X \leq X_{D,2}$	$X_{D,2} < X \leq X_{D,1}$	$X_{D,1} < X \leq X_{U,2}$	$X_{U,2} < X \leq X_{U,1}$	$X_{U,1} < X$
$q_{D,11}$	1	1	$\sum_{m=1}^{4} h_{11,m} X^{k_m}$	$\sum_{m=1}^{2} g_{11,m} X^{l_m}$	0
$q_{D,12}$	0	0	$\sum_{m=1}^{4} h_{12,m} X^{k_m}$	$\sum_{m=1}^{2} g_{12,m} X^{l_m}$	0
$q_{D,21}$	0	$\sum_{m=1}^{2} s_{21,m} X^{j_m} + \frac{\hat{\lambda}_2}{r_2+\hat{\lambda}_2}$	$\sum_{m=1}^{4} h_{11,m}\, \epsilon(k_m) X^{k_m}$	0	0
$q_{D,22}$	1	$\sum_{m=1}^{2} s_{22,m} X^{j_m}$	$\sum_{m=1}^{4} h_{12,m}\, \epsilon(k_m) X^{k_m}$	0	0

Boundary conditions

1. $q_{D,11}$ at $X_{D,1}$ $\Rightarrow$ $\sum_{m=1}^{4} h_{11,m} X_{D,1}^{k_m} = 1$
2. $q_{D,11}$ at $X_{U,2}$ $\Rightarrow$ $\sum_{m=1}^{2} g_{11,m} X_{U,2}^{l_m} - \sum_{m=1}^{4} h_{11,m} X_{U,2}^{k_m} = 0$
3. $q'_{D,11}$ at $X_{U,2}$ $\Rightarrow$ $\sum_{m=1}^{2} g_{11,m}\, l_m X_{U,2}^{l_m} - \sum_{m=1}^{4} h_{11,m}\, k_m X_{U,2}^{k_m} = 0$
4. $q_{D,11}$ at $X_{U,1}$ $\Rightarrow$ $\sum_{m=1}^{2} g_{11,m} X_{U,1}^{l_m} = 0$
5. $q_{D,12}$ at $X_{D,1}$ $\Rightarrow$ $\sum_{m=1}^{4} h_{12,m} X_{D,1}^{k_m} = 0$
6. $q_{D,12}$ at $X_{U,2}$ $\Rightarrow$ $\sum_{m=1}^{2} g_{12,m} X_{U,2}^{l_m} - \sum_{m=1}^{4} h_{12,m} X_{U,2}^{k_m} = 0$
7. $q'_{D,12}$ at $X_{U,2}$ $\Rightarrow$ $\sum_{m=1}^{2} g_{12,m}\, l_m X_{U,2}^{l_m} - \sum_{m=1}^{4} h_{12,m}\, k_m X_{U,2}^{k_m} = 0$
8. $q_{D,12}$ at $X_{U,1}$ $\Rightarrow$ $\sum_{m=1}^{2} g_{12,m} X_{U,1}^{l_m} = 0$
9. $q_{D,21}$ at $X_{D,2}$ $\Rightarrow$ $\sum_{m=1}^{2} s_{21,m} X_{D,2}^{j_m} = -\frac{\hat{\lambda}_2}{r_2+\hat{\lambda}_2}$
10. $q_{D,21}$ at $X_{D,1}$ $\Rightarrow$ $\sum_{m=1}^{4} h_{11,m}\, \epsilon(k_m) X_{D,1}^{k_m} - \sum_{m=1}^{2} s_{21,m} X_{D,1}^{j_m} = \frac{\hat{\lambda}_2}{r_2+\hat{\lambda}_2}$
11. $q'_{D,21}$ at $X_{D,1}$ $\Rightarrow$ $\sum_{m=1}^{4} h_{11,m}\, \epsilon(k_m)\, k_m X_{D,1}^{k_m} - \sum_{m=1}^{2} s_{21,m}\, j_m X_{D,1}^{j_m} = 0$
12. $q_{D,21}$ at $X_{U,2}$ $\Rightarrow$ $\sum_{m=1}^{4} h_{11,m}\, \epsilon(k_m) X_{U,2}^{k_m} = 0$
13. $q_{D,22}$ at $X_{D,2}$ $\Rightarrow$ $\sum_{m=1}^{2} s_{22,m} X_{D,2}^{j_m} = 1$
14. $q_{D,22}$ at $X_{D,1}$ $\Rightarrow$ $\sum_{m=1}^{4} h_{12,m}\, \epsilon(k_m) X_{D,1}^{k_m} - \sum_{m=1}^{2} s_{22,m} X_{D,1}^{j_m} = 0$
15. $q'_{D,22}$ at $X_{D,1}$ $\Rightarrow$ $\sum_{m=1}^{4} h_{12,m}\, \epsilon(k_m)\, k_m X_{D,1}^{k_m} - \sum_{m=1}^{2} s_{22,m}\, j_m X_{D,1}^{j_m} = 0$
16. $q_{D,22}$ at $X_{U,2}$ $\Rightarrow$ $\sum_{m=1}^{4} h_{12,m}\, \epsilon(k_m) X_{U,2}^{k_m} = 0$

B.3 Arrow-Debreu restructuring claim if $X_{U,1} < X_{U,2}$

	$X \leq X_{D,2}$	$X_{D,2} < X \leq X_{D,1}$	$X_{D,1} < X \leq X_{U,1}$	$X_{U,1} < X \leq X_{U,2}$	$X_{U,2} < X$
$q_{U,11}$	0	0	$\sum_{m=1}^{4} h_{11,m} X^{k_m}$	1	1
$q_{U,12}$	0	0	$\sum_{m=1}^{4} h_{12,m} X^{k_m}$	0	0
$q_{U,21}$	0	$\sum_{m=1}^{2} s_{21,m} X^{j_m}$	$\sum_{m=1}^{4} h_{11,m}\, \epsilon(k_m) X^{k_m}$	$\sum_{m=1}^{2} g_{21,m} X^{j_m} + \frac{\hat{\lambda}_2}{r_2+\hat{\lambda}_2}$	0
$q_{U,22}$	0	$\sum_{m=1}^{2} s_{22,m} X^{j_m}$	$\sum_{m=1}^{4} h_{12,m}\, \epsilon(k_m) X^{k_m}$	$\sum_{m=1}^{2} g_{22,m} X^{j_m}$	1

Boundary conditions

1. $q_{U,11}$ at $X_{D,1}$ $\Rightarrow$ $\sum_{m=1}^{4} h_{11,m} X_{D,1}^{k_m} = 0$
2. $q_{U,11}$ at $X_{U,1}$ $\Rightarrow$ $\sum_{m=1}^{4} h_{11,m} X_{U,1}^{k_m} = 1$
3. $q_{U,12}$ at $X_{D,1}$ $\Rightarrow$ $\sum_{m=1}^{4} h_{12,m} X_{D,1}^{k_m} = 0$
4. $q_{U,12}$ at $X_{U,1}$ $\Rightarrow$ $\sum_{m=1}^{4} h_{12,m} X_{U,1}^{k_m} = 0$
5. $q_{U,21}$ at $X_{D,2}$ $\Rightarrow$ $\sum_{m=1}^{2} s_{21,m} X_{D,2}^{j_m} = 0$
6. $q_{U,21}$ at $X_{D,1}$ $\Rightarrow$ $\sum_{m=1}^{4} h_{11,m}\, \epsilon(k_m) X_{D,1}^{k_m} - \sum_{m=1}^{2} s_{21,m} X_{D,1}^{j_m} = 0$
7. $q'_{U,21}$ at $X_{D,1}$ $\Rightarrow$ $\sum_{m=1}^{4} h_{11,m}\, \epsilon(k_m)\, k_m X_{D,1}^{k_m} - \sum_{m=1}^{2} s_{21,m}\, j_m X_{D,1}^{j_m} = 0$
8. $q_{U,21}$ at $X_{U,1}$ $\Rightarrow$ $\sum_{m=1}^{2} g_{21,m} X_{U,1}^{j_m} - \sum_{m=1}^{4} h_{11,m}\, \epsilon(k_m) X_{U,1}^{k_m} = -\frac{\hat{\lambda}_2}{r_2+\hat{\lambda}_2}$
9. $q'_{U,21}$ at $X_{U,1}$ $\Rightarrow$ $\sum_{m=1}^{2} g_{21,m}\, j_m X_{U,1}^{j_m} - \sum_{m=1}^{4} h_{11,m}\, \epsilon(k_m)\, k_m X_{U,1}^{k_m} = 0$
10. $q_{U,21}$ at $X_{U,2}$ $\Rightarrow$ $\sum_{m=1}^{2} g_{21,m} X_{U,2}^{j_m} = -\frac{\hat{\lambda}_2}{r_2+\hat{\lambda}_2}$
11. $q_{U,22}$ at $X_{D,2}$ $\Rightarrow$ $\sum_{m=1}^{2} s_{22,m} X_{D,2}^{j_m} = 0$
12. $q_{U,22}$ at $X_{D,1}$ $\Rightarrow$ $\sum_{m=1}^{4} h_{12,m}\, \epsilon(k_m) X_{D,1}^{k_m} - \sum_{m=1}^{2} s_{22,m} X_{D,1}^{j_m} = 0$
13. $q'_{U,22}$ at $X_{D,1}$ $\Rightarrow$ $\sum_{m=1}^{4} h_{12,m}\, \epsilon(k_m)\, k_m X_{D,1}^{k_m} - \sum_{m=1}^{2} s_{22,m}\, j_m X_{D,1}^{j_m} = 0$
14. $q_{U,22}$ at $X_{U,1}$ $\Rightarrow$ $\sum_{m=1}^{2} g_{22,m} X_{U,1}^{j_m} - \sum_{m=1}^{4} h_{12,m}\, \epsilon(k_m) X_{U,1}^{k_m} = 0$
15. $q'_{U,22}$ at $X_{U,1}$ $\Rightarrow$ $\sum_{m=1}^{2} g_{22,m}\, j_m X_{U,1}^{j_m} - \sum_{m=1}^{4} h_{12,m}\, \epsilon(k_m)\, k_m X_{U,1}^{k_m} = 0$
16. $q_{U,22}$ at $X_{U,2}$ $\Rightarrow$ $\sum_{m=1}^{2} g_{22,m} X_{U,2}^{j_m} = 1$

B.4 Arrow-Debreu default claim if $X_{U,1} < X_{U,2}$

	$X \leq X_{D,2}$	$X_{D,2} < X \leq X_{D,1}$	$X_{D,1} < X \leq X_{U,1}$	$X_{U,1} < X \leq X_{U,2}$	$X_{U,2} < X$
$q_{D,11}$	1	1	$\sum_{m=1}^{4} h_{11,m} X^{k_m}$	0	0
$q_{D,12}$	0	0	$\sum_{m=1}^{4} h_{12,m} X^{k_m}$	0	0
$q_{D,21}$	0	$\sum_{m=1}^{2} s_{21,m} X^{j_m} + \frac{\hat{\lambda}_2}{r_2+\hat{\lambda}_2}$	$\sum_{m=1}^{4} h_{11,m}\, \epsilon(k_m) X^{k_m}$	$\sum_{m=1}^{2} g_{21,m} X^{j_m}$	0
$q_{D,22}$	1	$\sum_{m=1}^{2} s_{22,m} X^{j_m}$	$\sum_{m=1}^{4} h_{12,m}\, \epsilon(k_m) X^{k_m}$	$\sum_{m=1}^{2} g_{22,m} X^{j_m}$	0

Boundary conditions

1. $q_{D,11}$ at $X_{D,1}$ $\Rightarrow$ $\sum_{m=1}^{4} h_{11,m} X_{D,1}^{k_m} = 1$
2. $q_{D,11}$ at $X_{U,1}$ $\Rightarrow$ $\sum_{m=1}^{4} h_{11,m} X_{U,1}^{k_m} = 0$
3. $q_{D,12}$ at $X_{D,1}$ $\Rightarrow$ $\sum_{m=1}^{4} h_{12,m} X_{D,1}^{k_m} = 0$
4. $q_{D,12}$ at $X_{U,1}$ $\Rightarrow$ $\sum_{m=1}^{4} h_{12,m} X_{U,1}^{k_m} = 0$
5. $q_{D,21}$ at $X_{D,2}$ $\Rightarrow$ $\sum_{m=1}^{2} s_{21,m} X_{D,2}^{j_m} = -\frac{\hat{\lambda}_2}{r_2+\hat{\lambda}_2}$
6. $q_{D,21}$ at $X_{D,1}$ $\Rightarrow$ $\sum_{m=1}^{4} h_{11,m}\, \epsilon(k_m) X_{D,1}^{k_m} - \sum_{m=1}^{2} s_{21,m} X_{D,1}^{j_m} = \frac{\hat{\lambda}_2}{r_2+\hat{\lambda}_2}$
7. $q'_{D,21}$ at $X_{D,1}$ $\Rightarrow$ $\sum_{m=1}^{4} h_{11,m}\, \epsilon(k_m)\, k_m X_{D,1}^{k_m} - \sum_{m=1}^{2} s_{21,m}\, j_m X_{D,1}^{j_m} = 0$
8. $q_{D,21}$ at $X_{U,1}$ $\Rightarrow$ $\sum_{m=1}^{2} g_{21,m} X_{U,1}^{j_m} - \sum_{m=1}^{4} h_{11,m}\, \epsilon(k_m) X_{U,1}^{k_m} = 0$
9. $q'_{D,21}$ at $X_{U,1}$ $\Rightarrow$ $\sum_{m=1}^{2} g_{21,m}\, j_m X_{U,1}^{j_m} - \sum_{m=1}^{4} h_{11,m}\, \epsilon(k_m)\, k_m X_{U,1}^{k_m} = 0$
10. $q_{D,21}$ at $X_{U,2}$ $\Rightarrow$ $\sum_{m=1}^{2} g_{21,m} X_{U,2}^{j_m} = 0$
11. $q_{D,22}$ at $X_{D,2}$ $\Rightarrow$ $\sum_{m=1}^{2} s_{22,m} X_{D,2}^{j_m} = 1$
12. $q_{D,22}$ at $X_{D,1}$ $\Rightarrow$ $\sum_{m=1}^{4} h_{12,m}\, \epsilon(k_m) X_{D,1}^{k_m} - \sum_{m=1}^{2} s_{22,m} X_{D,1}^{j_m} = 0$
13. $q'_{D,22}$ at $X_{D,1}$ $\Rightarrow$ $\sum_{m=1}^{4} h_{12,m}\, \epsilon(k_m)\, k_m X_{D,1}^{k_m} - \sum_{m=1}^{2} s_{22,m}\, j_m X_{D,1}^{j_m} = 0$
14. $q_{D,22}$ at $X_{U,1}$ $\Rightarrow$ $\sum_{m=1}^{2} g_{22,m} X_{U,1}^{j_m} - \sum_{m=1}^{4} h_{12,m}\, \epsilon(k_m) X_{U,1}^{k_m} = 0$
15. $q'_{D,22}$ at $X_{U,1}$ $\Rightarrow$ $\sum_{m=1}^{2} g_{22,m}\, j_m X_{U,1}^{j_m} - \sum_{m=1}^{4} h_{12,m}\, \epsilon(k_m)\, k_m X_{U,1}^{k_m} = 0$
16. $q_{D,22}$ at $X_{U,2}$ $\Rightarrow$ $\sum_{m=1}^{2} g_{22,m} X_{U,2}^{j_m} = 0$

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