The TIPS puzzle: Evidence from Korea

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Abstract

Using the model in the affine term structure class taking forecasts of inflation rates and nominal rates into account, we show that the premium of inflation risk is sizeable and the TIPS yields generated in the model are generally lower than the TIPS yield observed in the market. In regression analyses, we find that the spread between the TIPS model yields and market yields (TIPS yield spread) is affected by not only tax exemption and liquidity risk that are not considered in the model, but also the aggregate TIPS supply and lending volume of nominal bonds. Therefore, the TIPS yield spread seems to be driven by not only the model misspecification but also by some factors that affect TIPS to be mispriced.

Keywords: treasury inflation-protected securities (TIPS), break-even inflation (BEI), inflation risk premium, liquidity risk

JEL classification: G12, E43, E44

1. Introduction

Among government-issued securities, treasury inflation-protected securities (TIPS) have unique features that are different from other securities such as nominal bonds. TIPS are guaranteed to be redeemed at maturity by the government and further provide protection against inflation. In case of a plain treasury bond, known as a nominal bond, the principal and coupons of the bond stay fixed for the life of the bond. In contrast, the principal and coupons of a TIPS change depending on changes in the price level. Therefore, investors holding TIPS are protected against inflation risk, which is not the case for investors holding nominal bonds.

TIPS yields provide policymakers and finance professionals with valuable information on real interest rates, because the inflation risk in TIPS is hedged out. In other words, the yield spread between government bonds and TIPS provides information on the inflation rate that bond investors expect for certain horizons. Often, the yield difference between the nominal bond and TIPS for the same maturity is regarded as the real interest rate for that maturity. However, the information embedded in the yield spread between nominal bonds and TIPS (break-even inflation rate, BEI rate) comprises more than the expected inflation. It contains the inflation risk premium as well as the liquidity premium due to the liquidity difference between TIPS and nominal bonds. In addition, there are concerns about whether TIPS are fairly priced relative to nominal government bonds, because TIPS are new securities compared to nominal bonds and bond investors may not be familiar with them.

Numerous studies have examined these questions on TIPS in the empirical literature, and they may be summarized under the following two strands. The first strand of studies focuses on the estimation of risk premia, assuming that TIPS and nominal government bonds are fairly priced and that the difference between the expected inflation and BEI rate is due to risk premia. Thus, this strand of literature examines the risk components priced in bonds and the size of the risk premia under a pricing formula based on the prices of TIPS and nominal government bonds. Jarrow and Yildirim (2003) use the Heath–Jarrow–Morton (HJM) model to fit the time-series evolution of inflation and the real and nominal rates, and show the model's validity through its hedging performance. Chen et al. (2010) estimate the inflation risk premium using the modified quadratic term structure model and show that this premium is related to the shape of the term structure of nominal interest rates. Christensen et al. (2010) also estimate the inflation risk premium using the affine, no-arbitrage Nelsen–Siegel model and show that the expected inflation calculated by subtracting the inflation risk premium from the BEI rate can predict future inflation better than BEI itself. In other words, although the market commonly regards BEI as a proxy for expected inflation, it is not an appropriate indicator of expected inflation. In addition to expected inflation, the risk of inflation is also embedded in the yield spread between TIPS and nominal bonds, and it should not be ignored when estimating the expected inflation in the bond market. On the other hand, Gürkaynak et al. (2010) and Kajuth and Watzka (2011) find that the yield spread between nominal bonds and TIPS is closely related to liquidity variables, such as the on-the-run/off-the-run spread, trading volume, and the spread between Refcorp and treasury strips. They regard these findings as evidence that liquidity risk, in addition to inflation risk, is an important factor in determining TIPS prices or the yield spread between nominal bonds and TIPS.

The second strand of studies examines whether TIPS are fairly priced. These studies examine the information content on TIPS or calculate TIPS prices under a pricing model to compare the estimated prices with their market prices, rather than accept the TIPS prices and extract the expected inflation from their market prices. Bardong and Lehnert (2008) suggest that TIPS prices may not reflect the information available in the market. They examine the returns of strategies that speculate TIPS yields using movements of yield spreads between nominal bonds and TIPS. Haubrich et al. (2012) estimate pricing kernels using a survey of inflation forecasts, inflation swaps, nominal spot rates, and monthly inflation rates using a model in the completely affine class. They show that TIPS are underpriced in general. Fleckenstein et al. (2014) also suggest that TIPS are underpriced, showing that the price of the portfolio replicating the cash flows of TIPS using nominal government bonds and inflation swaps is sizably lower than TIPS prices. In regression analysis, they show that TIPS are underpriced due to market inefficiency as the price difference between the replicated portfolio and TIPS is explained by factors related to the availability of funding to exploit mispricing as well as liquidity risk.

In March 2007, the Korean government announced the issue of TIPS to diversify government bonds and secure the demand for government bonds even in periods of high inflation. TIPS comprised approximately 2% of Korean government bonds as of December 2014. In this study, we investigate whether TIPS are fairly priced relative to nominal government bonds in the Korean bond market. For this examination, we estimate the pricing kernels under Haubrich et al.’s (2012) model using the yields of nominal government bonds, monthly inflation rates, and a survey of inflation forecasts. Then, we compare the TIPS prices calculated using the pricing kernel with the observed TIPS market prices, and analyze the factors contributing to the difference between the TIPS model prices and market prices.

This is the first paper focusing on pricing issues in the Korean TIPS market to the best of our knowledge. Since all research so far mainly focused on the U.S. TIPS market, this study provides an out-of-sample test for prior studies by examining the Korean bond market. In addition, the Korean TIPS market is still in its developing stage, and so this paper sheds light on how the prices of TIPS are formed and what factors affect them in the emerging market.

Our main findings are as follows. First, the Haubrich et al. (2012) model has a good fit to the observed nominal spot rates in the Korean bond markets. The maximum mean error and root mean squared error (RMSE) for nominal bonds with different times to maturity are less than 7 basis points and 20 basis points in absolute terms, respectively. However, the model fits the inflation forecasts worse than the case for nominal spot rates. The maximum mean error and RMSE for inflation forecasts are up to 32 basis points and 71 basis points, respectively. In addition, the inflation risk premium and real rate risk premium embedded in the nominal spot rates for seven-year maturity bonds are estimated as 74 and 17 basis points on average, respectively, confirming the previous empirical results that BEI is not an appropriate proxy for expected inflation.

Second, TIPS are underpriced in the Korean market in general. For example, the average model yield of the first issued TIPS is estimated as 1.50% under the model without restrictions, which is 50 basis points lower than the average value of the observed market yields of TIPS. Other TIPS also show similar results.

Third, the spread between the TIPS market yields and model yields (TIPS yield spread) is driven by not only model misspecification but also market conditions. In the regression of the yield spread, we find that TIPS issuance and the lending amount of nominal government bonds as well as liquidity and tax-exemption-related variables have predictable power. Regression results show that TIPS investors require sizeable economic compensation for the illiquidity of TIPS and understand they can get additional benefits by tax exemption for the interest amount driven by inflation, which are not considered in Haubrich et al.'s (2012) model. Therefore, at least a part of the TIPS yield spread may be driven by factors that are not considered in the model. However, the empirical results show that the yield spread is also partially explained by the aggregate supply of TIPS and the lending amount of nominal government bonds, which measure the degree of short-sale restrictions. That is, the supply-demand pressure and severity of the limit to arbitrage seem to affect TIPS prices as well.

This paper proceeds as follows. Section 2 introduces the model used in this study. Section 3 describes the data and explains the estimation method. Section 4 shows the estimation results and analyses the spread between the TIPS model yields and market yields. Section 5 concludes.

2. Model

Haubrich et al. (2012) suggest a term structure model that is in the completely affine class (Dai and Singleton, 2000; Duffee, 2002). In the model, the dynamics of nominal pricing kernel, M_t, is defined as

$$\frac{M_{t+\Delta t}}{M_t} = e^{-i_t\Delta t-\frac{1}{2}\sum_{j=1}^{4}\phi_j^2 h_{j,t}^2\Delta t-\sum_{j=1}^{4}\phi_j h_{j,t}\sqrt{\Delta t}\epsilon_{j,t+\Delta t}}, \tag{1}$$

where $\varepsilon_{j,t+\Delta t}$, j=1, 2, 3, 4 are independent standard normal random variables, $\phi_j h_{j,t}$, j = 1, 2, 3, 4 are the market prices of risk associated with four sources of uncertainty, ϕ_j, j = 1, 2, 3, 4 are constants, $h_{j,t}$, j=1, 2, 3, 4 are annualized time-varying volatilities, i_t represents the annualized one-period nominal interest rate, and Δt represents the length of one period in years.

The dynamics of the real pricing kernel, m_t, is defined as the nominal pricing kernel multiplied by the inflation rate with assumptions about the dynamics of consumer price index, I_t, and the rate of expected inflation from t to $t+\Delta t$, π_t. This can be represented as follows:

$$\frac{m_{t+\Delta t}}{m_t} = \frac{M_{t+\Delta t}}{M_t}\frac{I_{t+\Delta t}}{I_t} = e^{\left(\pi_i-i_t-\frac{1}{2}h_{1,t}^2\right)\Delta t-\frac{1}{2}\sum_{j=1}^{4}\phi_j^2 h_{j,t}^2\Delta t-\sum_{j=1}^{4}\phi_j h_{j,t}\sqrt{\Delta t}\epsilon_{j,t+\Delta t}+h_{1,t}\sqrt{\Delta t}\varepsilon_{1,t+\Delta t}}, \tag{2}$$

where

$$\frac{I_{t+\Delta t}}{I_t} = e^{\pi_t\Delta t-\frac{1}{2}h_{1,t}^2\Delta t+h_{1,t}\sqrt{\Delta t}\varepsilon_{1,t+\Delta t}} \tag{3}$$

$$\pi_t = \frac{1}{\Delta t}\ln\left(E_t\left[\frac{I_{t+\Delta t}}{I_t}\right]\right). \tag{4}$$

In this setting, from our expectations of equation (2), we can define the one-period real interest rate, r_t. In addition, the dynamics of state variables are specified as follows:

$$\pi_{t+\Delta t} - \pi_t = [\alpha_t + a_1 r_t + a_2\pi_t]\Delta t + \sqrt{\Delta t}\sum_{j=1}^{2}\beta_j h_{j,t}\,\epsilon_{j,t+\Delta t}$$

$$r_{t+\Delta t} - r_t = [b_0 + b_1 r_t + b_2\pi_t]\Delta t + \sqrt{\Delta t}\sum_{j=1}^{3}\gamma_j h_{j,t}\,\epsilon_{j,t+\Delta t}$$

$$\alpha_{t+\Delta t} - \alpha_t = (c_0 + c_1\alpha_t)\Delta t + \sqrt{\Delta t}\sum_{j=1}^{4}\rho_j h_{j,t}\,\epsilon_{j,t+\Delta t}$$

$$h_{j,t+\Delta t}^2 - h_{j,t}^2 = \left[d_{j0} + d_{j1}h_{j,t}^2\right]\Delta t + d_{j2}\Delta t\left(\epsilon_{j,t+\Delta t} - d_{j3}h_{j,t}\right)^2, j = 1,2,3,4. \tag{5}$$

Because this model is in the completely affine class, the log prices of bonds can be represented as a linear function of state variables, as shown in Duffie and Kan (1996). The relation between log bond prices and state variables in this setting are as follows. Let $P_{N,t}^{(n)}, P_{R,t}^{(n,d)}$ be the respective time t prices of a nominal and a real bond that pays 1 at maturity $t + n\Delta t$, where d is the indexation lag for the real bond, TIPS. They can be represented as

$$P_{N,t}^{(n)} = \exp\left(-K_n - A_n\pi_t - B_n r_t - C_n\alpha_t - \sum_{j=1}^{4} D_{j,n} h_{j,t}^2 \right) \text{ for } n \geq 1 \quad (6)$$

$$P_{R,t}^{(n,d)} = \exp\left(-\widetilde{K_n} - \widetilde{A_n}\pi_t - \widetilde{B_n} r_t - \widetilde{C_n}\alpha_t - \sum_{j=1}^{4} \widetilde{D_{j,n}} h_{j,t}^2 \right) \text{ for } n \geq d, \quad (7)$$

where $A_1 = B_1 = \Delta t, K_1 = C_1 = D_{j,1} = 0,\ D_{1,1} = -\phi_1 \Delta t,\ D_{j,1} = 0$ for j = 2, 3, 4 and $\widetilde{K_d} = K_{d,}\ \widetilde{A_d} = A_d$, $\widetilde{B_d} = B_d,\ \widetilde{C_d} = C_d,\ \widetilde{D_{j,n}} = D_{j,d}$ for j = 1, 2, 3, 4. Appendix A provides the details.

In line with bond prices, the expected inflation rate for a horizon from t to t + n + Δt can be represented by a linear function of state variables as

$$E_t\left[\ln\left(\frac{I_{t+n\Delta t}}{I_t}\right)\right] = K_n^* + A_n^*\pi_t + B_n^* r_t + C_n^*\alpha_t + \sum_{j=1}^{4} D_{j,n}^* h_{j,t}^2 \text{ for } n \geq 1, \quad (8)$$

where $K_1^* = B_1^* = C_1^* = D_{j,1}^* = 0,\ A_1^* = \Delta t,\ D_{1,1}^* = -\frac{1}{2}\Delta t,$ and $D_{j,1}^* = 0$ for $j = 2, 3, 4$. Appendix A provides the details.

Besides the typical features of affine term structures, this model shows some features distinct from other completely affine class models used in several studies. First, the volatility factors related to the market price of risk are time varying but not linear functions of state variables. The volatilities of state variables and the market price of risk can be separated from the state variables related to the macro economy, and the volatilities are the state variables to determine the spot rates and the inflation rates in the model. Second, because the model is constructed in a discrete time environment with multiple periods, the state variables that are observable or extracted from observable data are used, instead of the unobservable instantaneous state variables. Third, because the state variables in the model include the real interest rates and inflation rates, the model is more intuitive than other ones in the affine term structure class that derives state variables as a linear combination of nominal rates. Additionally, because the model uses macro-economic information, such as inflation expectations and nominal spot rates, the model's fit to the data can improve, as prior studies (Ang et al., 2007; Kim and Orphanides, 2007, 2012) have shown. Fourth, the model considers the effect of indexation lag, as shown in equation (7). Finally, the model can deal with the market price of inflation risk. The traditional Fisher equation implies that nominal interest is the sum of the real interest rate and expected inflation. However, in Haubrich et al.'s (2012) model, the nominal interest rate contains the term of the market price of risk related to inflation as well as the sum of the real interest rate and expected inflation, as shown below.

$$i_t = \pi_t + r_t - \phi_1 h_{1,t}^2 \quad (9)$$

This feature indicates the possibility that the deviation in spread between the nominal bond and TIPS yields from the expected inflation could be due to this additional term of the market price of inflation risk.

3. Data and Estimation

3. 1 Data description

We estimate our model using the Korean nominal spot rates, survey of inflation forecasts, and monthly inflation rates data, and then compare the TIPS yields observed in the market with the yields estimated from the model. Specifically, we use the nominal spot rates data constructed by the Korea Asset Pricing Corporation (www.koreaap.com). The data contains the nominal spot rates for maturities of one, three, and six months, and one, two, three, seven, and ten years from May 2002 to December 2014, and five years from September 2011 to December 2014. The TIPS yields data from March 2007 to December 2014 are also obtained from the Korea Asset Pricing Corporation. All the nominal rates are those observed at the end of the trading day for each month.

The monthly inflation rates and survey of inflation forecasts data are from Bloomberg. While the monthly inflation rates and their corresponding forecasts are from May 2002 to December 2014, the survey data of inflation forecasts for horizons longer than one month are from September 2007 to December 2014.[3] Each month, economists forecast the quarterly inflation for eight quarters from the current calendar quarter, and the yearly inflation for three years from the current calendar year[4]. If the inflation forecast horizons between the quarterly and yearly time periods overlap, we use the quarterly time period forecasts. We use the mean of the inflation forecasts as the market expected inflation.

Table 1 provides the summary statistics of nominal spot rates, monthly inflation rates, and the mean of inflation forecasts.

< Table 1 here >

Panel A of Table 1 shows the means and standard deviations of the nominal spot rates. The average term structure of nominal rates for the sample period is upward sloping and its standard deviations decline slightly

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[3] The monthly inflation rates and their corresponding forecasts show statistically significant seasonality. The data are seasonally adjusted using X-12-ARIMA, a software package of the U.S. Census Bureau for seasonal adjustments.

[4] The longest forecasting horizons in the early days of the survey were six quarters for quarterly and two years for yearly time periods, and they have increased gradually.

with maturity. Panel B shows the summary statistics of monthly inflation rates and their corresponding forecasts. The table shows the standard deviation of the monthly realized inflation rates larger than that of their corresponding forecasts, which is not surprising. The average of the one-month inflation forecasts is higher than the mean of the monthly realized inflation rates, but the mean difference is statistically insignificant. Panel C provides the summary statistics of the inflation forecasts. The level of inflation forecasts shows a U-shaped curve on average, unlike the nominal spot rates, which are upward sloping. The standard deviations of inflation forecasts are decreasing with forecast horizons more steeply than those of nominal spot rates.

Table 2 provides the information on TIPS analyzed in this study. After TIPS were first issued in March 2007, three additional TIPS have been issued until December 2014. As TIPS are issued as fungible bonds to increase their liquidity, they are issued less frequently than the announcements of TIPS issuance might indicate, and the issued TIPS amount varies from time to time. TIPS pay semi-annual coupon interest with an indexation lag of three months, and are issued as ten-year maturity bonds.

< Table 2 here >

< Figure 1 here >

Figure 1 illustrates the TIPS market yields from March 2007 to December 2014. As Figure 1 shows, the yields are rather volatile. The market yield of the first TIPS, KTBi 07-2, was the highest in October 2008 and lowest in March 2013, with the difference as large as 4.25%. This difference is larger than that for nominal spot rates. The same can be stated for the other TIPS as well.

3. 2 Estimation method

In this study, one period is set as one month, then the state variables are defined as one-month real rates and one-month inflation forecasts, and their central tendency with four volatility factors. Because the state variables are not observable except for one-month inflation forecasts, we extract the unobservable state variables from the nominal rates, assuming that the one-month nominal rate, inflation forecasts, and the three-year maturity nominal spot rates are observed without error. Under these assumptions, we obtain the real interest rate from the one-month nominal rate using equation (9), $i_t = \pi_t + r_t - \phi_1 h_{1,t}^2$, given parameter ϕ and volatility factor $h_{1,t}$, related to monthly inflation rates. We obtain the central tendency of the one-month expected inflation α_t from the three-year maturity nominal spot rates, given the parameters and other state variables based on equation (6).

From the residuals $\varepsilon_{j,t+\Delta t}, j = 1, 2, 3, 4$ of the three state variables (π_t, r_t, α_t) and monthly inflation rates, we simultaneously update the volatility factors $h_{j,t}, j = 1, 2, 3, 4$. Furthermore, given the parameters and the state variables $\pi_t, r_t, \alpha_t, h_{j,t}, j = 1, 2, 3, 4$ at time t, we calculate the term structure of the nominal spot rates and the expected inflation from equations (6) and (8).

We next explain how to estimate the parameters. As previously mentioned, the time series of state variables and their term structures can be generated, given the parameters. That is, we can also determine the residual errors of the time series of state variables and their term structures from the parameters. Therefore, our objective is to find the parameters that simultaneously minimize the residual errors. To proceed with our estimation, we need to further assume that the nominal spot rates for all maturities except the one-month and three-year maturities and the inflation forecasts for periods longer than one month are observed with independent measurement errors, that is, $w_{t,i} \sim N(0, w^2)$, $v_{t,i} \sim N(0, v^2)$, respectively, where subscript i indicates the time to maturity of a bond and the inflation forecast period. We then estimate the parameters by maximizing the sum of the daily log likelihoods defined as follows.

$$f_t(\epsilon) = \left(\frac{1}{2\pi}\right)^{k/2} \det(\Sigma_t)^{-\frac{1}{2}} \exp\left(-\frac{1}{2}\, \epsilon_t{}' \Sigma_t{}^{-1} \epsilon_t\right) \tag{10}$$

where k represents the number of observations, ϵ represents the four stochastic drivers $\varepsilon_{jt+\Delta t}, j = 1, 2, 3, 4$, with measurement errors of nominal bonds and expected inflations $w_{t,i}$ and $v_{t,i}$, respectively, and Σ_t represents the covariance matrix of the observed data at time t.

The total number of parameters shown in equations (1) and (5) is from 26 to 35 depending on the assumption about the volatility process and, in principle those parameters should be estimated in one step. However, estimating such a large number of parameters in one step is not easy. Furthermore, freely estimated parameters might violate certain restrictions such as the positive definiteness of covariance matrix, or show numbers unreasonably different from the sample statistics. Therefore, following the suggestions of Haubrich et al. (2012), we estimate the parameters as follows. First, we estimate the parameters related to the volatility process of monthly inflation rates in advance because the rates are related only to the one-month inflation forecasts in equation (3), $\ln\left(\frac{I_{t+\Delta t}}{I_t}\right) = \pi_t \Delta t - \frac{1}{2} h_{1,t}^2 \Delta t + h_{1,t}\sqrt{\Delta t}\,\varepsilon_{1,t+\Delta t}$. This approach makes the other parameters more manageable. Second, if the state variables satisfy stationary conditions, the unconditional means of $\pi_t, r_t, \alpha_t, h_j^2, j = 1, 2, 3, 4$ can be represented by coefficients as shown below.

$$\bar{\pi} = -\frac{a_1 b_0 c_1 + b_1 c_0}{(a_1 b_2 - a_2 b_1)c_1}$$

$$\bar{r} = \frac{a_2 b_0 c_1 + b_2 c_0}{(a_1 b_2 - a_2 b_1)c_1}$$

$$\bar{\alpha} = -\frac{c_0}{c_1} = -(a_1 \bar{r} + a_2 \bar{\pi}),\ \bar{r} = -\bar{\alpha}/a_1 = c_0/(c_1 a_1)$$

$$\overline{h_j^2} = -\frac{d_{j0} + d_{j2}}{d_{j1} + d_{j2} d_{j3}^2}, \text{j} = 1, 2, 3, 4 \quad (11)$$

To ensure that the variances of those variables are positive and that the averages of the variables are not too different from the sample means, we estimate the parameters of the unconditional means of one month inflation ($\bar{\pi}$), the real rate ($\bar{r}$), and the unconditional mean of variances ($\overline{h_j^2}$) instead of b_0, c_0, and d_{j0} for j = 1, 2, 3, 4. Third, we restrict $\beta_2 = \gamma_3 = \rho_4 = 1$ without loss of generality.

4. Empirical Results

4. 1 Parameter estimation and performance

Table 3 reports the estimated values of parameters related to the volatility process of monthly inflation rates, $h_{1,t}$. These values are estimated using monthly inflation rates ($I_{t+\Delta t}/I_t$) and one-month inflation forecasts (π_t). The table shows that the annualized unconditional standard deviation of monthly inflation rates is estimated to be 0.92% and is statistically different from zero at any reasonably significance level. However, all the other parameters in the GARCH process are insignificantly different from zero, and, as a result, reject the hypothesis that the volatility of difference between monthly inflation rates and their corresponding forecasts is time varying.

< Table 3 here >

Table 4 reports the other parameter estimates of the model. The first three columns of Table 4 report the estimates under the restriction that only one of the volatility factors (h_{2t}, h_{3t}, or h_{4t}) follows the GARCH process while the others are constant. The last column of the table reports the estimates under no restriction, allowing for all volatility factors to be time varying. For models under the restriction that only one of the volatility factors follows the GARCH process, the log likelihood value of the model having time-varying volatility with $h_{3,t}$ is the largest. The models of time-varying volatilities with $h_{2,t}$ and $h_{4,t}$ show the second largest and smallest log likelihood values, respectively. However, the model under no restriction shows the largest log likelihood value among all the models, and the increase in value is statistically significant. That is, time-varying volatility plays a significant role in improving data fitting, and therefore some of the parameters related to the GARCH process for each of the volatility factors are significant.

< Table 4 here >

The implications of the estimates reported in Table 4 can be summarized as follows. First, the time-varying feature of volatility factors plays a significant role in improving data fitting. Although not reported here, the constant volatility (volatilities) assumption is rejected at the 1% significance level for all the models. Second, the unconditional mean of volatility of one-month inflation forecasts, $\overline{h_2^2}$, is estimated in the range of 3.8% to 4.4%, depending on the model, and is higher than 2.2%, the sample volatility of one-month inflation forecasts observed in the data. Third, the measurement error of nominal spot rates proxied by their standard deviation is estimated to be smaller than the measurement error of inflation forecasts. Fourth, the market prices of risk are statistically significant. The sum of the risk premia related to inflation with ϕ_1, ϕ_2, and ϕ_4 is estimated on average at 74 basis points for a seven-year maturity in the model under no restriction[5]. The real risk premium related to ϕ_3 is estimated at 17 basis points for the same maturity, smaller than the inflation risk premium. The significant market prices of risks indicate that the yield spread between nominal bonds and TIPS, the BEI rate, could be sizably different from the expected inflation of market participants.

Table 5 reports the model's fit to nominal spot rates, monthly inflation rates, and inflation forecasts based on estimates under no restriction. Panel A of Table 5 shows the model's fit to nominal spot rates. When an error is defined as a market spot rate minus its model spot rate, the average value of the mean errors for all maturities is 3 basis points, the average RMSEs for all maturities is 12 basis points, and the maximum RMSE is 18 basis points. Panel B shows the model's fit to inflation forecasts. In general, the model's fit to inflation forecasts is worse than that to nominal rates. If we compare the estimated model inflation forecasts with the observed market inflation forecasts, the model underpredicts the market inflation rates for one- to three-quarter horizons by more than 20 basis points on average but overpredicts those for the five- and six-quarter horizons by

[5] Risk premium is calculated as the difference between the model yield with estimated parameters and the model yield with zero market price(s) of risk(s), other things being equal.

approximately 10 basis points. In addition, the maximum RMSE of inflation forecasts among different horizons is 69 basis points, much larger than that for nominal spot rates. This result is consistent with the estimation result that the standard deviation of measurement errors for inflation forecasts, v, is larger than that for nominal spot rates, w.

< Table 5 here >

4. 2 Analysis of difference between the TIPS model yields and TIPS market yields

So far, we estimated and evaluated the performance of Haubrich et al.’s (2012) model using the TIPS and nominal bonds in the Korean bond market. From now one, we analyze the difference between the observed market yields and the TIPS model yields generated using model parameters under no restriction.

Figure 2 shows the TIPS yield spread defined as the TIPS market yield minus TIPS model yield. The TIPS yield spread is volatile, as in the case of the TIPS market yields shown in Figure 1. The spread is positive on average, as in the case of the U.S. TIPS (Haubrich et al., 2012; Fleckenstein et al., 2014). The time-series means of the yield spreads of KTBi 07-2, KTBi 10-4, KTBi 11-4, and KTBi 13-4 are 50, 34, 36, and 88 basis points, respectively. These positive spreads are sizable compared to the mean errors of the nominal spot rates and inflation forecasts in Table 5, and the spreads are economically significant even after considering transaction costs such as the bid–ask spread, which is 0.5–1 basis point.

< Figure 2 here>

In our model, the TIPS yield spreads should be zero. If spreads exist, they result from the mispricing of TIPS in the market. However, in reality, certain factors that our model ignore such as the taxes and illiquidity problems as well as the model misspecification regarding pricing kernel dynamics that we assume can affect the TIPS yield spread. In this section, we examine how such factors ignored in the model affect the TIPS yield spread and whether there are mispricings in the spread.

Four factors not taken into account in our model may affect the TIPS yield spread. First, some features of TIPS make the cash flows of TIPS different from those of nominal bonds, other than inflation-compensation, although the maturity and coupon rates are the same. Tax exemption and protection on the nominal principal amount are typical. A certain amount of income from TIPS is not taxable for investors. The interest income due to inflation is tax exempt in Korea, enabling TIPS investors to receive more interest income after tax compared to investors holding nominal government bonds, other things being equal. That is, for taxable investors, holding TIPS that pay interest is equivalent to holding nominal government bonds that pay higher interest. Thus, this tax exemption clause makes TIPS prices higher than the nominal government bond prices under similar conditions. This indicates that, given the tax rate, the additional benefit for taxable investors holding TIPS compared to holding nominal government bonds increases with the expected inflation rate: a higher expected inflation rate results in larger benefits to taxable investors from holding TIPS than holding nominal bonds. Therefore, the TIPS yield spread is expected to be negatively related to the level of expected inflation. In addition to tax exemption on the interest due to inflation, the protection on the nominal amount also makes TIPS costlier than nominal government bonds. The principal amount of TIPS is protected for redemption at maturity even in periods of deflation[6]. That is, TIPS holders have a put option with an exercise price of the face value of TIPS. This clause would make TIPS costlier compared to the nominal government bonds that have no protection in case of deflation. That is, a higher possibility of deflation results in higher TIPS prices than nominal government bond prices. Thus, this protection clause implies that expected inflation is negatively related to TIPS prices or positively related to the TIPS yield spread. Depending on which effect dominates, that is, the tax exemption effect or protection clause effect, expected inflation may have a negative or positive effect on the TIPS yield spread. In general, we expect that the tax exemption effect dominates the protection clause effect, because a negative inflation rate is rare in Korea. Another factor determining the value of the put option is the volatility of inflation rates. When the volatility of inflation rates is high, it becomes difficult to forecast inflation, and so the standard deviation of inflation rates becomes large. Thus, the standard deviation of inflation rates can be negatively related to the TIPS yield spread. In this study, we use the standard deviation of inflation forecasts as a proxy for the standard deviation of inflation rates.

Second, TIPS yield spreads might be affected by liquidity. The existing literature has found that liquidity risk plays a key role in determining the yield spreads among bonds in fixed income markets, because investors require compensation for the risk of failing to construct desired portfolios due to illiquidity (Longstaff et al.,

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[6] The protection clause is included for TIPS issued after 2010.

2005; Chen et al., 2007; Lin et al., 2011). The U.S. TIPS are no exception. Kajuth and Watzka (2011) and D'Amico et al. (2014) set up models with liquidity factors in TIPS pricing and show a sizeable liquidity risk premium embedded in TIPS. Gürkaynak et al. (2010) and Fleckenstein et al. (2014) also document the importance of liquidity risk in TIPS pricing by examining the relation between liquidity proxies and the movement of TIPS prices. If liquidity risk is also an important factor for TIPS pricing in Korea, consistent with the previous studies described, we expect that liquidity-related factors explain the TIPS yield spread. the TIPS issued amount and trading volume are negatively related to TIPS yield spreads, whereas the on-the run/off-the-run spread defined as the yield of the off-the-run bonds minus the yield of the on-the-run bonds is positively related to TIPS yield spreads.[7]

Third, heterogeneity in beliefs may affect TIPS prices. We use the mean value of inflation forecasts as the market expectation of future inflation. However, the standard deviation of inflation forecasts, which can be regarded as the disagreement of forecasts, in addition to the mean value of forecasts, might contain important information for bond investors, because the heterogeneity in forecasts of the macro economy can affect bond prices through trading based on their own expectations. Wright (2011) shows that disagreement in the forecasts of inflation and GDP growth can explain the term premium. Xiong and Yan (2010), Buraschi and Whelan (2012), and Ehling et al. (2013) also show that disagreement in beliefs plays a key role in explaining bond excess returns, asset volatility, and time-varying risk premia. Thus, the standard deviation of inflation rates as a proxy for investors' disagreement on the future inflation may affect the TIPS yield spread.

Fourth, limits to arbitrage, as suggested by Shleifer and Vishny (1997), may lead to mispricing of TIPS and to the TIPS yield spreads observed in this paper. If there are no limits to arbitrage and the market is efficient without model misspecification, there should be no TIPS mispricing, and we should observe zero TIPS yields spreads. If positive TIPS yield spreads are observed in the market, they should disappear quickly through arbitrage in the market. However, in reality there are limits to arbitrage, as noted by Shleifer and Vishny.

[7] Numerous variables are used as liquidity proxies in previous studies. The issued amount, trading volume, on-the-run/off-the-run spread, and bid–ask spread are used by Houweling et al. (2005), Rösch and Kaserer (2013), Longstaff et al. (2005), Chen et al. (2007), Goyenko et al. (2011), and Fleckenstein et al. (2014); the cash flows into bond funds or money market funds are used by Longstaff et al. (2005) and Chordia et al. (2005); the Pastor and Stambaugh liquidity measure and Amihud illiquidity measure are used by Li et al. (2009) and Lin et al. (2011), and the latent bond amount estimated as the amount that brokers hold is used by Mahanti et al. (2008).

Especially, as Stambaugh et al. (2012, 2014) document, most of the anomalies observed in the market are from the short side of arbitrage portfolios of concern and can be at least partially explained by the existence of short-sale restrictions. Following Stambaugh et al., we hypothesize that TIPS yield spreads could be driven by short-sale restrictions in the government bond market, both the nominal government bond and TIPS markets. If positive TIPS yield spreads are driven by underpriced TIPS or overpriced nominal government bonds, the spread could be exploited by the short sale of nominal government bonds. On the other hand, if TIPS are overpriced and positive TIPS yield spreads are observed, it could be resolved by the short sale of TIPS. Therefore, we expect TIPS yield spreads to be positively related to TIPS lending and negatively related to nominal government bond lending. Another factor that could affect the mispricing of TIPS is the extent of competition in the market. If more informed traders exist in the market and competition is high, the likelihood of mispricing can become smaller. In addition, we expect that cash inflows to bond funds will increase market efficiency. As more money flows into bond funds, the power of bond fund managers—likely informed investors—is more likely to become stronger in the market and the bond market could become more competitive, implying that arbitrage opportunities related to TIPS decline or disappear, as noted by Fleckenstein et al. (2014). Therefore, cash inflows to bond funds can be negatively related to TIPS yield spreads if the spreads are due to TIPS mispricing.

From the variables discussed so far, we investigate the TIPS yield spreads through regression analyses. The dependent variable in our analyses is the cross-sectional average yield spread of four TIPS calculated from the model assuming that all the four volatility factors follow the GARCH process. The explanatory variables are the estimated annualized expected inflation for a five-year horizon period, the on-the-run/off-the-run spread of five-year maturity bonds, the TIPS trading volume, the TIPS issued amount, the standard deviation of one-month inflation forecasts, the TIPS and nominal government bond lending volumes, and bond fund cash inflows. The on-the run/off-the-run spread is defined as the yield of the off-the-run nominal bond minus that of the on-the-run nominal bond. Each month, the off-the-run bond is chosen as the bond whose time to maturity is closest to the time to maturity of the benchmark bond among non-on-the-run bonds. The data of market yields of bonds, trading volume, issued amount, and security lending volume are obtained from KOSCOM (www.koscom.co.kr). Bond fund cash flows are obtained from FnGuide (www.fnguide.com). Table 6 reports the regression results.

< Table 6 here >

The results reported in Table 6 provide some interesting insights into the determinants of the TIPS yield spread, which are summarized as follows. First, liquidity-related variables are related to the TIPS yield spread. In regression (ii), the yield spread is positively related to the on-the-run/off-the-run spread, consistent with our hypothesis. Its coefficient is approximately 3.6 and significant at the 1% significance level. Then, TIPS seem to require a larger compensation when the off-the-run bond requires a larger yield relative to the on-the-run bond, meaning that the TIPS yield becomes larger when the bond market is less liquid. The TIPS trading volume which is also related to liquidity shows a significant negative relation with the TIPS yield spread in regression (iii). These results—a positive relation with the on-the-run/off-the-run spread and negative relation with TIPS trading volume—show that liquidity in both the TIPS and nominal government bond markets is an important determinant of the TIPS yield spread. As regressions (ix) and (x) show, both the on-the-run/off-the-run spread and TIPS trading volume explain the yield spread of TIPS, and the coefficients are statistically significant at the 1% significance level, even after controlling for each other and other variables. However, the TIPS issued amount, which can be regarded as a proxy for liquidity, does not show the expected relation. Given our hypothesis and the results from other liquidity-related variables, its coefficient should be negative in our regressions because the liquidity for TIPS is expected to be larger when the issued amount of TIPS is larger and the TIPS market is larger. However, the coefficient is positively significant at the 10% significance level in regression (iv) and at the 1% significance level in regressions (ix) and (x) that control other variables. This unexpected result may be attributed to the supply effect documented by Krishnamurthy and Vissing-Jorgensen (2012). Because TIPS are issued without any significant redemption during the sample period, the aggregate TIPS amount in the market increases, causing a downward price pressure. Thus, an increase in issued amount of TIPS may decrease the TIPS yield, thus resulting in a negative coefficient of the issued TIPS amount.

Second, the tax exemption effect is another important factor in pricing TIPS. The coefficient of expected inflation is negative and statistically significant at the 1% significance level and continues to be significantly negative after controlling for other variables. This negative coefficient shows that the tax exemption effect dominates the protection clause effect. Although this negative coefficient of expected inflation is consistent with our tax exemption hypothesis, its absolute magnitude is much larger than that expected from the tax exemption effect. The tax exemption effect—frequently called the phantom income effect—implies that the coefficient should be less than the tax rate of marginal taxable investors multiplied by the inflation compensation part of coupon rates. Since the maximum tax rate for investors in Korea and the maximum TIPS coupon rate are 41.8%

and 2.75%, respectively, the maximum value for this coefficient is $2.75 \times 0.418 = 1.14$. However, the absolute values of the coefficients in this table are from 1.39 to 1.66, all greater than 1.14. This result is puzzling, for which we do not have a satisfactory explanation and further research is warranted.

Third, the lending volumes of TIPS and nominal government bonds seem unrelated to the TIPS yield spread in regressions (vi) and (vii). In contrast, after controlling for the other variables in regression (x), the coefficient of the volume of nominal government bond lending is negative, which is consistent with our hypothesis, and statistically significant at the 10% significance level. This implies that the TIPS yield spread is a decreasing function of the volume of nominal government bond lending, implying that underpricing of TIPS becomes larger when the short-sale restriction in the nominal government bond market becomes more severe. However, the volume of TIPS lending still has no power to explain the TIPS yield spread in regression (x), possibly because of the low possibility of TIPS overpricing.

Fourth, all the other effects investigated in Table 6 do not seem relevant to the TIPS yield spread. The standard deviation of inflation forecasts as a proxy for heterogeneity in beliefs and a factor affecting the value of the put option from the protection clause, and bond fund cash flows as a proxy for market competition, do not seem to affect the TIPS yield spreads; their coefficients are not statistically significant even at the 10% significance level.

In conclusion, the TIPS yield spread seems to be driven by not only our model misspecification that ignores the liquidity risk and tax exemption effect of TIPS, but also factors related to the market condition. This implies that the market yields of TIPS reflect the tax exemption effects embedded in TIPS and the liquidity of TIPS as well as bond market liquidity, but some mispricings exist due to the limits to arbitrage such as short-sale restrictions in the nominal bond market and market pressure of the aggregate TIPS supply.

5. Conclusion

TIPS were first issued in Korea in March 2007, and since then they have grown to comprise approximately 2% of Korean government bonds as of December 2014. TIPS are very useful financial instruments because (i) they can be used to hedge against inflation risks through their inflation-linked coupons and redemption at maturity, and (ii) they provide information on real interest rates to market participants and policy makers. This paper evaluates whether these two virtues of TIPS are well functioning in Korea by examining the relation between nominal government bonds and TIPS. More concretely, we estimate the pricing kernels of nominal government bonds and TIPS using Haubrich et al.'s (2012) model and compare the estimated TIPS prices with the observed TIPS market prices to examine whether TIPS are fairly priced and how inflation risk is priced out. The results are summarized as follows.

First, our estimation results show that the premium of inflation risk is economically sizable. Thus, the BEI rates defined as the yield spreads between nominal government bonds and TIPS are not the same as expected inflation, implying that BEI rates are not an unbiased estimator of future inflation.

Second, the TIPS yields generated in our model are generally lower than the observed TIPS market yields, and the spread between the two yields are time varying. This means that TIPS are on average underpriced, if our model is correctly specified.

Third, our regression analysis of the cross-sectional average TIPS yield spreads, defined as the TIPS yields observed in the market minus yields estimated by the model and, show that the spreads can be partially explained by the liquidity risk and tax exemption effects that are not taken into account in our model. In addition, the TIPS yield spreads are also affected by the price pressure of aggregate TIPS supply and the limit to arbitrate in the nominal bond market. Therefore, the TIPS yield spread seems to be driven by not only the misspecification of our model but also by some factors that affect TIPS to be mispriced.

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Appendix

A. The nominal bond price under the dynamics in the paper is given by

$$P_{n,t} = \exp\left(-K_n - A_n\pi_t - B_n r_t - C_n\alpha_t - \sum_{j=1}^{4} D_{j,n} h_{j,t}^2 \right), \text{for } n \geq 1$$

where $A_1 = B_1 = \Delta t, K_1 = C_1 = D_{j,1} = 0, D_{1,1} = -\phi_1 \Delta t$ and

$$A_{n+1} = \Delta t + (1 + a_2\Delta t)A_n + b_2 B_n \Delta t$$

$$B_{n+1} = \Delta t + a_1 \Delta t A_n + (1 + b_1\Delta t)B_n$$

$$C_{n+1} = A_n \Delta t + (1 + c_1\Delta t)C_n$$

$$K_{n+1} = K_n + \left(b_0 B_n + c_0 C_n + \sum_{j=1}^{4} d_{j0} D_{j,n} \right)\Delta t + \frac{1}{2}\sum_{j=1}^{4} \ln(1 + 2D_{j,n} d_{j2}\Delta t)$$

$$D_{j,n+1} = \left(-\phi_1 I_{\{j=1\}} + \frac{\phi_j^2}{2} \right)\Delta t + D_{j,n}\left(1 + \left(d_{j1} + d_{j2} d_{j3}^2\right)\Delta t\right) - \frac{Q_{j,n}^2}{2\left(1 + 2D_{j,n} d_{j2}\Delta t\right)}$$

where I is the indicator function that is equal to 1 if j = 1 and 0 otherwise, and

$$Q_{1,n} = \left(\phi_1 + A_n\beta_1 + B_n\gamma_1 + C_n\rho_1 - 2D_{1,n} d_{12} d_{13}\sqrt{\Delta t}\right)\sqrt{\Delta t}$$

$$Q_{2,n} = \left(\phi_2 + A_n\beta_2 + B_n\gamma_2 + C_n\rho_2 - 2D_{2,n} d_{22} d_{23}\sqrt{\Delta t}\right)\sqrt{\Delta t}$$

$$Q_{3,n} = \left(\phi_3 + B_n\gamma_3 + C_n\rho_3 - 2D_{3,n} d_{32} d_{33}\sqrt{\Delta t}\right)\sqrt{\Delta t}$$

$$Q_{4,n} = \left(\phi_4 + C_n\rho_4 - 2D_{4,n} d_{42} d_{43}\sqrt{\Delta t}\right)\sqrt{\Delta t}$$

B. The TIPS price under the dynamics in the paper is given by

$$P_{n,t} = \exp\left(-\widetilde{K_n} - \widetilde{A_n}\pi_t - \widetilde{B_n} r_t - \widetilde{C_n}\alpha_t - \sum_{j=1}^{4} \widetilde{D_{j,n}} h_{j,t}^2 \right), \text{for } n \geq d$$

where d indicates the indexation lag of TIPS and

$$\widetilde{K_d} = K_d, \widetilde{A_d} = A_d, \widetilde{B_d} = B_d, \widetilde{C_d} = C_d, \widetilde{D_{j,n}} = D_{j,d}$$

$$\widetilde{A_{n+1}} = (1 + a_2\Delta t)\widetilde{A_n} + b_2\widetilde{B_n}\Delta t$$

$$\widetilde{B_{n+1}} = \Delta t(1 + a_1\widetilde{A_n}) + (1 + b_1\Delta t)\widetilde{B_n}$$

$$\widetilde{C_{n+1}} = \widetilde{A_n}\Delta t + (1 + c_1\Delta t)\widetilde{C_n}$$

$$\widetilde{K_{n+1}} = \widetilde{K_n} + \left(b_0\widetilde{B_n} + c_0\widetilde{C_n} + \sum_{j=1}^{4} d_{j0}\widetilde{D_{j,n}} \right)\Delta t + \frac{1}{2}\sum_{j=1}^{4} \ln(1 + 2\widetilde{D_{j,n}}d_{j2}\Delta t)$$

$$\widetilde{D_{j,n+1}} = \left((\frac{1}{2} - \phi_1)I_{\{j=1\}} + \frac{\phi_j^2}{2} \right)\Delta t + \widetilde{D_{j,n}}\left(1 + \left(d_{j1} + d_{j2}d_{j3}^2\right)\Delta t\right) - \frac{\widetilde{Q_{j,n}^2}}{2\left(1 + 2\widetilde{D_{j,n}}d_{j2}\Delta t\right)}$$

where I is the indicator function that is equal to 1 if j = 1 and 0 otherwise, and

$$\widetilde{Q_{1,n}} = \left(1 - \phi_1 - \widetilde{A_n}\beta_1 - \widetilde{B_n}\gamma_1 - \widetilde{C_n}\rho_1 + 2\widetilde{D_{1,n}}d_{12}d_{13}\sqrt{\Delta t}\right)\sqrt{\Delta t}$$

$$\widetilde{Q_{2,n}} = \left(-\phi_2 - \widetilde{A_n}\beta_2 - \widetilde{B_n}\gamma_2 - \widetilde{C_n}\rho_2 + 2\widetilde{D_{2,n}}d_{22}d_{23}\sqrt{\Delta t}\right)\sqrt{\Delta t}$$

$$\widetilde{Q_{3,n}} = \left(-\phi_3 - \widetilde{B_n}\gamma_3 - \widetilde{C_n}\rho_3 + 2\widetilde{D_{3,n}}d_{32}d_{33}\sqrt{\Delta t}\right)\sqrt{\Delta t}$$

$$\widetilde{Q_{4,n}} = \left(-\phi_4 - \widetilde{C_n}\rho_4 + 2\widetilde{D_{4,n}}d_{42}d_{43}\sqrt{\Delta t}\right)\sqrt{\Delta t}$$

C. Expected inflation from t to t + nΔt under the dynamics in the paper is given by

$$E_t\left[\ln\left(\frac{I_{t+n\Delta t}}{I_t}\right)\right] = K_n^* + A_n^*\pi_t + B_n^*r_t + C_n^*\alpha_t + \sum_{j=1}^{4} D_{j,n}^*h_{j,t}^2, \text{for } n \geq 1$$

where $K_1^* = B_1^* = C_1^* = D_{j,1}^* = 0\ for\ j = 2,3,4\ \ A_1^* = \Delta t, D_{1,1}^* = -\frac{1}{2}\Delta t$

$$A_{n+1}^* = \Delta t + (1 + a_2\Delta t)A_n^* + b_2B_n^*\Delta t$$

$$B_{n+1}^* = a_1\Delta t A_n^* + (1 + b_1\Delta t)B_n^*$$

$$C_{n+1}^* = A_n^*\Delta t + (1 + c_1\Delta t)C_n^*$$

$$K_{n+1}^* = K_n^* + \left(b_0B_n^* + c_0C_n^* + \sum_{j=1}^{4}\left(d_{j0} + d_{j2}\right)D_{j,n}^* \right)\Delta t$$

$$D_{j,n+1}^* = -\frac{1}{2}\Delta t I_{\{j=1\}} + D_{j,n}^*\left(1 + \left(d_{j1} + d_{j2}d_{j3}^2\right)\Delta t\right)$$

Table 1 Summary statistics

This table provides the summary statistics of nominal spot rates, monthly inflation rates, and the mean of inflation forecasts. The nominal spot rates from May 2002 to December 2014 are from the Korea Asset Pricing Corporation. The monthly inflation rates and survey of inflation forecasts for the horizons from one month to eight quarters are from Bloomberg. The data period for the monthly inflation rates and their corresponding forecasts are the same as the period for the spot rates. The data period for the survey of inflation forecasts for horizons longer than one month is from September 2007 to December 2014.

Panel A. Nominal spot rates

	1 month	3 months	6 months	1 year	2 years	3 years	7 years	10 years
Average (%)	3.418	3.491	3.612	3.773	3.997	4.092	4.533	4.641
Std. dev.	0.0101	0.0101	0.0101	0.0098	0.0099	0.0097	0.0097	0.0094

Panel B. Monthly inflation rates

	May 2002–Dec 2014		Mar 2007–Dec 2014	
	Actual	Forecasts	Actual	Forecasts
Average (%)	2.685	3.154	2.561	3.035
Std. dev	0.029	0.022	0.030	0.021

Panel C. Survey of inflation forecasts

	2 quarters	3 quarters	4 quarters	5 quarters	6 quarters	7 quarters	8 quarters	9 quarters	10 quarters
Average (%)	2.889	2.858	2.872	2.775	2.786	2.868	2.873	2.938	2.979
Std. dev.	0.006	0.004	0.003	0.004	0.004	0.003	0.002	0.002	0.003

Table 2 TIPS specifications

ISIN	Security name	Abbreviation	Issue date	Maturity	Coupon rate (%)
KR1035027T36	KTBi 02750-1703(07-2)	KTBi 07-2	2007-03-10	2017-03-10	2.750
KR1035037061	KTBi 02750-2006(10-4)	KTBi 10-4	2010-06-10	2020-06-10	2.750
KR1035037160	KTBi 01500-2106(11-4)	KTBi 11-4	2011-06-10	2021-06-10	1.500
KR103502G362	KTBi 01125-2306(13-4)	KTBi 13-4	2013-06-10	2023-06-10	1.125

Table 3 Estimates of inflation volatility process

The inflation volatility process is estimated using monthly inflation rates and their corresponding forecasts from May 2002 to December 2014. The dynamics of inflation and its volatility process are

$$\frac{I_{t+\Delta t}}{I_t} = e^{\pi_t \Delta t - \frac{1}{2} h_{1,t}^2 \Delta t + h_{1,t} \sqrt{\Delta t} \varepsilon_{1,t+\Delta t}}$$

$$\mathrm{h}_{1,t+\Delta t}^2 - \mathrm{h}_{1,t}^2 = [\mathrm{d}_{10} + \mathrm{d}_{11} \mathrm{h}_{1,t}^2] \Delta t + \mathrm{d}_{12} \Delta t (\epsilon_{1,t+\Delta t} - \mathrm{d}_{13} \mathrm{h}_{1,t})^2$$

$$\overline{h_1^2} = -\frac{d_{10} + d_{12}}{d_{11} + d_{12} d_{13}^2}.$$

	$\overline{h_1^2}$	d_{11}	d_{12}	d_{13}
Estimates	0.00008	-0.74984	0.00001	0.16757
t-Statistic	8.18	-0.12	0.20	0.01

Table 4 Model parameter estimates except for inflation volatility process

The parameters are estimated using monthly inflation rates, nominal spot rates, and inflation forecasts under the assumption that the measurement errors of inflation forecasts for periods longer than one month and the nominal spot rates of all maturities except for one month and three years are independent. For all models, the parameters of the GARCH process for inflation (h_1) are fixed at the point estimates reported in Table 3. The first three columns show estimates under the restriction that one of the volatility factors (h_{2t}, h_{3t}, or h_{4t}) follows the GARCH process, and the other volatility factors are constant. The last column shows the estimates under no restriction. The data period for both monthly inflation rates and nominal spot rates is from May 2002 to December 2014, and the period for survey of inflation forecasts is from September 2007 to December 2014. The asterisk indicates that the number is significantly different from zero at the 5% significance level. The model dynamics are assumed as follows:

$$\frac{M_{t+\Delta t}}{M_t} = e^{-i_t\Delta t - \frac{1}{2}\sum_{j=1}^{4}\phi_j^2 h_{j,t}^2\Delta t - \sum_{j=1}^{4}\phi_j h_{j,t}\sqrt{\Delta t}\epsilon_{j,t+\Delta t}}$$

$$\pi_{t+\Delta t} - \pi_t = [\alpha_t + a_1 r_t + a_2\pi_t]\Delta t + \sqrt{\Delta t}\sum_{j=1}^{2}\beta_j h_{j,t}\,\epsilon_{j,t+\Delta t}$$

$$r_{t+\Delta t} - r_t = [b_0 + b_1 r_t + b_2\pi_t]\Delta t + \sqrt{\Delta t}\sum_{j=1}^{3}\gamma_j h_{j,t}\,\epsilon_{j,t+\Delta t}$$

$$\alpha_{t+\Delta t} - \alpha_t = (c_0 + c_1\alpha_t)\Delta t + \sqrt{\Delta t}\sum_{j=1}^{4}\rho_j h_{j,t}\,\epsilon_{j,t+\Delta t}$$

$$h_{j,t+\Delta t}^2 - h_{j,t}^2 = \left[d_{j0} + d_{j1}h_{j,t}^2\right]\Delta t + d_{j2}\Delta t\left(\epsilon_{j,t+\Delta t} - d_{j3}h_{j,t}\right)^2, j = 2,3,4$$

$$\bar{\pi} = -\frac{a_1 b_0 c_1 + b_1 c_0}{(a_1 b_2 - a_2 b_1)c_1}, \bar{r} = \frac{a_2 b_0 c_1 + b_2 c_0}{(a_1 b_2 - a_2 b_1)c_1}$$

$$\bar{\alpha} = -\frac{c_0}{c_1} = -(a_1\bar{r} + a_2\bar{\pi}),\ \bar{r} = -\bar{\alpha}/a_1 = c_0/(c_1 a_1)$$

$$\overline{h_j^2} = -\frac{d_{j0} + d_{j2}}{d_{j1} + d_{j2}d_{j3}^2}, j = 2,3,4$$

	h_2 GARCH	h_3 GARCH	h_4 GARCH	h_2, h_3, h_4 GARCH
ϕ_1	-3.5028*	-3.89114*	-3.02698*	-2.80296*
ϕ_2	28.7033*	17.1883*	29.08201*	23.47174*
ϕ_3	0.01498	-0.00092*	0.05757	0.02499*
ϕ_4	-0.52641	-0.72174*	-0.4185	-0.18956*
$\bar{\pi}$	0.02919*	0.02934*	0.0292*	0.02966*
a_1	0.13854*	0.06583*	0.10046*	0.09325*
a_2	-9.82209*	-9.82743*	-9.76814*	-9.86749*
β_1	-0.03885	-0.06006*	-0.06204	-0.03988*
$\bar{r}$	0.00001	0.00001*	0.00001	0.00001*
b_1	-0.72099*	-0.65502*	-0.66091*	-0.67688*
b_2	9.3932*	9.52177*	9.34995*	9.48285*
γ_1	0.03014	0.04197*	0.01117	0.01486*
γ_2	-1.00386*	-0.98838*	-0.99253*	-1.00307*
c1	-0.07923*	-0.07699*	-0.09042*	-0.07708*
ρ_1	0.01899	-0.08366*	-0.05104	-0.01188*
ρ_2	-0.02825*	-0.05023*	-0.03256*	-0.04234*
ρ_3	0.03005*	0.01457*	0.06101*	0.04422*
$\overline{h_2^2}$	0.00199*	0.00198*	0.00199*	0.00149*
d_{21}	-0.34026*			-0.36739*
d_{22}	0.00006*			0.00006*
d_{23}	0.00001			0.00001
$\overline{h_3^2}$	0.00058*	0.00058*	0.00057*	0.00057*
d_{31}		-0.00001*		-0.00002*
d_{32}		0.00011*		0.00011*
d_{33}		0.00001		0.00001
$\overline{h_4^2}$	0.00005*	0.00005*	0.00005*	0.00005*
d_{41}			-4.61229	-4.45554*
d_{42}			0.00002	0.00001*
d_{43}			0.00002	0.00002
w^2	0.00001*	0.00001*	0.00001*	0.00001*
v^2	0.00002*	0.00003*	0.00002*	0.00002*
log Likelihood	10392.22	10448.28	10385.78	10465.59

Table 5 Model performance: Mean errors and root mean squared errors

The mean errors and RMSE of the nominal spot rates and inflation forecasts using the parameter estimates under no restriction are reported. Error is defined as the actual value minus the model value. The data periods for the nominal spot rates and the surveys of inflation forecasts are from May 2002 to December 2014 and from September 2007 to December 2014, respectively.

Panel A. Nominal spot rates

	3 months	6 months	1 year	2 years	5 years	7 years	10 years
Mean error	0.03%	–0.02%	–0.05%	–0.07%	–0.01%	–0.03%	0.02%
RMSE	0.07%	0.10%	0.12%	0.12%	0.06%	0.18%	0.18%

Panel B. Inflation forecasts

	1 quarter	2 quarters	3 quarters	4 quarters	5 quarters	6 quarters
Mean error	0.31%	0.31%	0.22%	0.03%	–0.16%	–0.15%
RMSE	0.62%	0.69%	0.57%	0.40%	0.36%	0.37%

Table 6 Regression results of TIPS yield spread

Table 6 shows the regression results of the TIPS yield spreads, which is the difference between the observed TIPS market yields and the TIPS model yields generated using the model parameters under no restriction. In the regressions, the dependent variable is the cross-sectional average yield spread of four TIPS estimated. The explanatory variables are the estimated annualized expected inflation for a five-year horizon period, the on-the-run/off-the-run spread of five-year maturity bonds, TIPS trading volume, TIPS issued amount, the standard deviation of one-month inflation forecasts, TIPS and nominal government bond lending volume, and bond fund flows. The constant coefficients of regressions are not reported. t-values are in parentheses. Superscript ** and * denote significance at the 5% and 10% levels, respectively.

	Expected inflation	On-the-run/off-the-run spread	TIPS trading volume (100 bn won)	TIPS issued amount (100 bn won)	Standard deviation of inflation forecasts	Nominal bonds lending volume (100 bn won)	TIPS lending volume (100 bn won)	Bond fund cash flows (100 bn won)	R^2
(i)	–1.5674								0.07
	(–2.69**)								
(ii)		3.5626							0.28
		(5.85**)							
(iii)			–0.0221						0.07
			(–2.56**)						
(iv)				0.0046					0.04
				(1.93*)					
(v)					–0.1212				0.01
					(–0.23)				
(vi)						–0.0006			0.01
						(–0.69)			
(vii)							–0.1397		0.02
							(–1.43)		
(viii)								0.0001	0.02
								(1.33)	
(ix)	–1.6608	3.6298	–0.0480	0.0083					0.58
	(–2.49**)	(7.22**)	(–6.41**)	(3.38**)					
(x)	–1.3887	3.8979	–0.0443	0.0097	0.1907	–0.0016	0.0860	0.0001	0.60
	(–2.03**)	(7.26**)	(–4.08**)	(3.16**)	(0.53)	(–1.69*)	(0.99)	(1.14)	

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Figure 1 Market yields on TIPS

Figure 1 shows the market yields of four TIPS from March 2007 to December 2014.

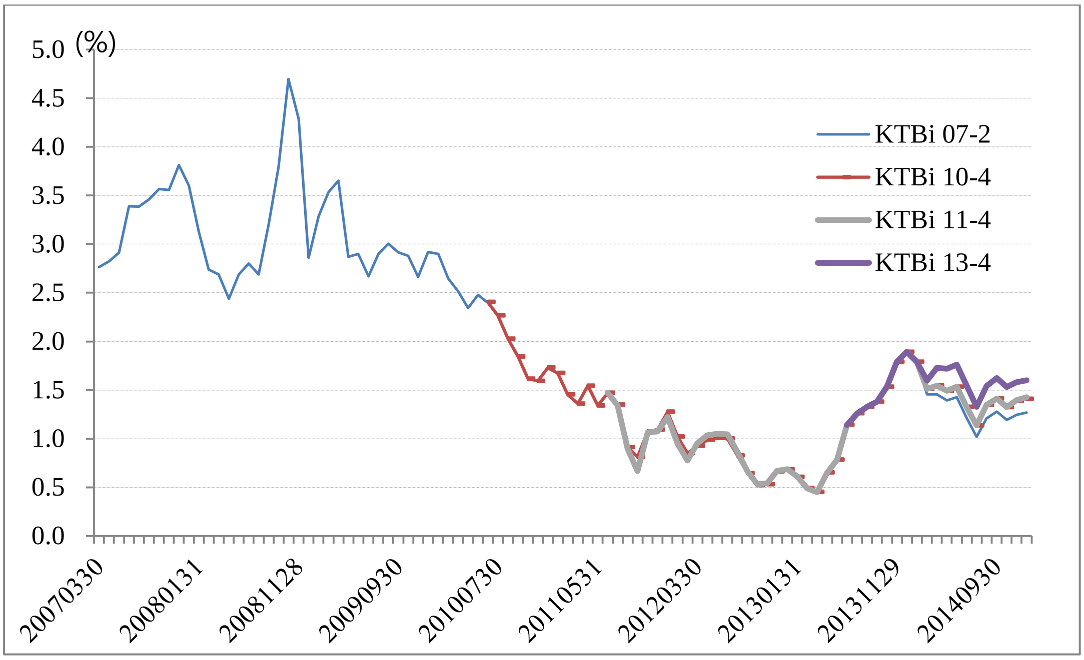

Figure 2 Spread between the estimated TIPS model yields and observed TIPS market yields

Figure 2 shows the time series of four TIPS yield spreads defined as the market yields minus the model yields from March 2007 to December 2014.