Skewness vs. Kurtosis: Implications for Pricing and Hedging Options

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Abstract

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1. Introduction

The Black and Scholes (1973) option pricing model (BS model) has been used to price and hedge various derivatives products because it offers the advantages of a closed-form solution with only a single parameter to be estimated. In addition to its several strengths, however, the BS model suffers several empirical deficiencies. For example, it misprices the out-of-the-money (OTM) and in-the-money (ITM) options; a phenomenon known as the volatility smile. This result arises from the faulty assumption about the risk-neutral distribution of the underlying asset return.[1] Cross sections of option prices have generally been used to estimate the implied risk-neutral distribution. Because this risk-neutral distribution represents the forward-looking view of the distribution of prices of the underlying asset, traders and policy makers have been using the risk-neutral distribution to assess market beliefs on future movements of the underlying asset. The BS model assumes that the risk-neutral distribution of the underlying asset return follows the normal distribution. However, several previous studies have found that the skewness and kurtosis of the risk-neutral distribution of underlying asset return are not zero and differ from those of normal distribution. The empirical deficiencies of the BS model result from the skewed and leptokurtic risk-neutral distribution.

The risk-neutral distribution of the return of the stock index is known to follow the negatively skewed and leptokurtic distribution.[2] This is consistent with the postulate of Black (1976) and the leverage effect documented by Christie (1982) and others. To explain this issue, option pricing models with the stochastic volatility or jumps have been considered.[3] The stochastic volatility, which has a negative correlation with the underlying asset return or jumps with a negative mean, can induce risk-neutral distributions with fat-tails and negative

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[1] Also, the volatility smile can be explained by the market microstructure theory, for example, Bollen and Whaley (2004), Kim, Kim and Ziskind (1994) and Hentschel (2003).

[2] See Bakshi, Cao and Chen (1997, 2000), Bakshi, Kapadia and Madan (2001), Dennis and Mayhew (2002), Derman (1999) and Rubinstein (1994) for further discussion.

[3] Examples are Bakshi, Cao and Chan (1997, 2000), Bates (1991, 2000), Heston (1993), Hull and White (1987), Johnson and Shanno (1987), Kim and Kim (2004, 2005), Scott (1987), Stein and Stein (1991) and Wiggins (1987)

skewness. Duffie, Pan and Singleton (2003) and Bakshi and Cao (2003) consider the jumps of the volatility and explain the excess kurtosis of the risk-neutral distribution. To resolve this argument, Bakshi, Cao and Chen (1997, 2000) conducted a comprehensive empirical study on the relative advantages of competing option pricing models. They found that taking the stochastic volatility into account is of the greatest importance in improving upon the BS formula.

Non-zero skewness and excess kurtosis have to be considered to mitigate the empirical deficiencies arising from the BS model for pricing and hedging stock index options. Skewness determines the asymmetry of the risk-neutral return distribution. Negative skewness indicates that the tail on the left side of the probability density function is longer or fatter than that on the right side. Kurtosis measures the peakedness of the probability distribution. The leptokurtic distribution has a more acute peak around the mean and fatter tails. Both skewness and kurtosis determine negatively skewed leptokurtic distribution. If so, which one is more important when considered pricing and hedging options, skewness or kurtosis? This topic has engaged academics and practitioners' attention for the following reasons. First, skewness is an important factor for asset pricing. Harvey and Siddique (2000) claim that unconditional return distributions cannot be fully explained by the mean and the variance alone and that the investors' preference toward skewness suggests skewness should also be considered to capture the asymmetric properties in realized returns. The result of Golec and Tamarkin (1998) supports risk aversion and skewness preference for race bets. In their analysis of the data from horse race betting in the U.S, Garret and Sobel (1999), find theoretical and empirical evidence that skewness of prize distributions explains why risk averse individuals may play the lottery. These previous research results confirm that skewness is an important factor for asset pricing. In this paper, we investigate the importance of skewness in the options market. Second, the implied risk-neutral distribution from option prices is a modified form of physical distribution by the pricing kernel. Since the pricing kernel can reflect the degree of the investor's risk aversion, the risk-neutral distribution can also contain the investor's attitude of the risk. Cremers and Weinbaum (2010), Doran, Peterson and Tarrant (2007), Doran and Kreiger (2010), Kim and Lee (2010), Xing, Zang and Zhao (2010) and Yan (2011) examine the risk-neutral skewness from the option prices to forecast the underlying asset returns. They find that skewness plays an important role in predicting the movement of the underlying asset of the option products. On the other hand, it is well known that kurtosis does not play a forecasting role for the rate of return of the underlying asset. Can the superiority of skewness over kurtosis in forecasting the underlying asset movement be maintained in the options market? Finally, it is important to compare the effects of skewness and kurtosis in order to determine the best option pricing model and estimate the parameters of the chosen model. We concentrate on the option pricing model which focuses on the risk-neutral skewness or kurtosis for pricing and hedging options. For example, if kurtosis is a more important factor than skewness, the option pricing model, which assumes the double jumps in both the rate of return and the volatility suggested by Duffie, Pan and Singleton (2003) and Bakshi and Cao (2003), or the option pricing model, which considers the risk-neutral distribution with excess kurtosis suggested by Ki, Lee and Choi (2003), has to be considered to price and hedge options. There is no right answer to estimate the structural parameters of the option pricing model. If the superiority between skewness and kurtosis is determined, we can estimate the structural parameters which determine the skewness or the kurtosis of the risk-neutral distribution. For example, if skewness is more important than kurtosis, we have to estimate the correlation parameter between the rate of return and the volatility compared to other structural parameters of Heston's (1993) model. It will be interesting to examine the ascendancy of skewness over kurtosis for pricing and hedging options.

In this paper, we compare the relative importance of the skewness and kurtosis of the risk-neutral distribution for pricing and hedging S&P 500 options. First, we estimate skewness and kurtosis using both the nonparametric method suggested by Bakshi, Kapadia and Madan

(2003) and the parametric method suggested by Corrado and Su (1996). Second, we regress the pricing and hedging errors from the BS model on skewness and kurtosis. We examine the relative impact of skewness and kurtosis on pricing and hedging errors according to the degree of the significance of the regression coefficients. Third, using the option pricing model, which utilizes the skewness and kurtosis parameters separately suggested by Corrado and Su (1996), we directly examine the effect of skewness and kurtosis for pricing and hedging options. If the option pricing model that only considers skewness shows better performance than the model that only considers kurtosis, then skewness is more important than kurtosis in the options market.

To the best of our knowledge, this is the first study to examine the relative strength of the skewness and kurtosis of the risk-neutral distribution to price and hedge options. The study results will allow us to determine the comparative predominance of the non-zero skewness or excess kurtosis property of the risk-neutral distribution.

We find that the risk-neutral skewness and kurtosis are estimated to be negative and greater than three, respectively. The impact of the risk-neutral skewness on pricing and hedging errors is consistently and significantly greater than that of kurtosis. In addition, the option pricing model that only considers skewness performs better in pricing and hedging S&P 500 options than does the model that only considers kurtosis. These results are statistically significant and robust to all sub-periods.

The paper is organized as follows. The estimation methods for skewness and kurtosis are reviewed in Section 2. The data used for the analysis are described in Section 3. In Section 4, the skewness and kurtosis estimates are described, the impact of the parameters on pricing and hedging errors is evaluated and the pricing and hedging performances of alternative option pricing models are compared. Section 5 concludes our study by summarizing the results.

2. Risk-neutral Skewness and Kurtosis

2.1. Nonparametric Estimation

The implied risk-neutral skewness and kurtosis are calculated under the assumption that there are no arbitrage or information differences between the options market and the stock market. These differences are estimated using various option series with different strike prices and maturities.

We calculate the implied risk-neutral skewness and kurtosis using Bakshi, Kapadia and Madan (2003) model in which skewness and kurtosis are estimated non-parametrically. Bakshi, Kapadia and Madan (2003) show that more negative risk-neutral skewness leads to a steeper slope of implied volatilities, under the presumption that everything is equal. They show that the risk-neutral skewness can be expressed by the function of the prices of quadratic, cubic, and quartic contracts' payoff, which are calculated by a linear combination of OTM call and put option prices.

The skewness and kurtosis of Bakshi, Kapadia and Madan (2003) can be measured as follows.

$$\begin{aligned} Skew(t,\tau) &= \frac{E_t^Q\left[(R_{t,t+\tau}-\mu(t,\tau))^3\right]}{\left\{E_t^Q\left[(R_{t,t+\tau}-\mu(t,\tau))^2\right]\right\}^{3/2}} \\ &= \frac{e^{r\tau}W(t,\tau)-3\mu(t,\tau)e^{r\tau}V(t,\tau)+2\mu(t,\tau)^3}{\left[e^{r\tau}V(t,\tau)-\mu(t,\tau)^2\right]^{3/2}} \end{aligned} \tag{1}$$

$$\begin{aligned} Kurt(t,\tau) &= \frac{E_t^Q\left[(R_{t,t+\tau}-\mu(t,\tau))^4\right]}{\left\{E_t^Q\left[(R_{t,t+\tau}-\mu(t,\tau))^2\right]\right\}^{2}} \\ &= \frac{e^{r\tau}X(t,\tau)-4\mu(t,\tau)e^{r\tau}W(t,\tau)+6e^{r\tau}\mu(t,\tau)^2V(t,\tau)-3\mu(t,\tau)^4}{\left[e^{r\tau}V(t,\tau)-\mu(t,\tau)^2\right]^{2}} \end{aligned} \tag{2}$$

where $R_{t,t+\tau}=\ln[S(t+\tau)/S(t)]$ is the rate of return for τ periods from time t, Q is the risk-neutral probability and $\mu(t,\tau)$ is as follows.

$$\mu(t,\tau) \approx e^{r\tau}-1-\frac{e^{r\tau}}{2}V(t,\tau)-\frac{e^{r\tau}}{6}W(t,\tau)-\frac{e^{r\tau}}{24}X(t,\tau) \tag{3}$$

The price of a volatility contract is

$$V(t,\tau)=\int_{S(t)}^{\infty}\frac{2(1-\ln[K/S(t)])}{K^2}C(t,\tau;K)dK+\int_{0}^{S(t)}\frac{2(1+\ln[S(t)/K])}{K^2}P(t,\tau;K)dK \tag{4}$$

where $C(t,\tau;K)$ and $P(t,\tau;K)$ are the European call and put written on the stock with strike price K and expiring in τ periods from time t. The prices of cubic and quadratic contracts are defined as $W(t,\tau)$ and $X(t,\tau)$, respectively.

$$\begin{aligned}W(t,\tau)=&\int_{S(t)}^{\infty}\frac{6\ln[K/S(t)]-3(\ln[K/S(t)])^2}{K^2}C(t,\tau;K)dK\\&-\int_{0}^{S(t)}\frac{6\ln[S(t)/K]+3(\ln[S(t)/K])^2}{K^2}P(t,\tau;K)dK\end{aligned} \tag{5}$$

$$\begin{aligned}X(t,\tau)=&\int_{S(t)}^{\infty}\frac{12\ln[K/S(t)]-4(\ln[K/S(t)])^2}{K^2}C(t,\tau;K)dK\\&+\int_{0}^{S(t)}\frac{12\ln[S(t)/K]+4(\ln[S(t)/K])^2}{K^2}P(t,\tau;K)dK\end{aligned} \tag{6}$$

2.2. Parametric Estimation

Corrado and Su (1996) expand the BS model to account for non-normal skewness and kurtosis in stock return distributions. Corrado and Su's (1996) model (CS model) uses the approximation based on a Gram-Charlier expansion of the normal density function and gives a skewness and kurtosis adjustment to the BS model.

The skewness and kurtosis adjusted price of a call option is of the following form:

$$C=C_{BS}+\mu_3Q_3+(\mu_4-3)Q_4 \tag{7}$$

where C_{BS} is the price in the BS model,

$$Q_3 = \frac{1}{3!} S\sigma\sqrt{\tau}\left[\left(2\sigma\sqrt{\tau} - d\right)n(d) - \sigma^2\tau N(d)\right],$$

$$Q_4 = \frac{1}{4!} S\sigma\sqrt{\tau}\left[\left(d^2 - 1 - 3\sigma\sqrt{\tau}\left(d - \sigma\sqrt{\tau}\right)\right)n(d) + \sigma^3\tau^{3/2}N(d)\right],$$

$$d = \frac{\ln(S/K) + \left(r + \sigma^2/2\right)\tau}{\sigma\sqrt{\tau}},$$

$n(d)$ and $N(d)$ are the probability density function and the cumulative probability distribution function for a standardized normal distribution.

Based on these formulae, the adjusted price of a put option is derived from the put-call parity. In this model, Q_3 and Q_4 represent the marginal effect of skewness and kurtosis on the adjusted option price, C_{BS}, respectively. The skewness parameter, μ_3, is estimated to be equal to the skewness of the implied risk-neutral distribution. Similarly, the kurtosis parameter, μ_4, is estimated to be equal to the kurtosis of the implied distribution. Considering the many previous studies reporting that the risk-neutral distribution is a negatively skewed and leptokurtic distribution in global markets, we expect the estimations of μ_3 to be negative and of μ_4 to be larger than three, which is the kurtosis of a normal distribution.

The application of the option pricing models always suffers from the difficulty of unobservable spot volatilities and structural parameters. Since the closed-form solution is available for an option price, a natural candidate for estimating the parameters in the formula is a non-linear least squares procedure, involving a minimization of the sum of squared percentage errors between the model and the market prices.

$$\min_{\phi_t} \sum_{i=1}^{N_t} \left[\frac{O_{i,t} - O^*_{i,t}}{O_{i,t}}\right]^2 \quad (t = 1, \ldots, T) \tag{8}$$

where $O^*_{i,t}$ denotes the model price of option i on day t, $O_{i,t}$ the market price of option i on day t, N_t the number of options on day t, and T the number of days in the sample.

Estimating the parameters from the asset returns can be an alternative method. However, historical data reflect only what happened in the past. Furthermore, the procedure using historical data is not capable of identifying risk premiums, which must be estimated from the options data conditional on the estimates of other parameters. However, using option prices to estimate parameters offers the important advantage of enabling the use of the forward-looking information contained in the option prices.

3. Data

The S&P 500 index option data used in this paper come from Option Metrics LLC. The data include the end-of day bid and ask quotes, implied volatilities, open interest and daily trading volume for the S&P 500 options traded on the Chicago Board Options Exchange from January 4, 1996 through December 31, 2008. The exercise style of the S&P 500 options is European, so our results are not affected by the complication that arises owing to the early exercise feature of American options. The data also include daily index values and estimates of dividend yields, as well as the term structures of zero-coupon interest rates constructed from LIBOR quotes and Eurodollar futures prices. We use the final bid-ask average as our measure of the option price.

The following rules are applied in order to filter the data needed for the empirical test. We use OTM options for calls and puts. First, since there is only a very thin trading volume for the ITM option, the credibility of price information is not entirely satisfactory. Therefore, we use the price data with regards to both put and call options that are OTM. Second, if both call and put option prices are used, ITM calls and OTM puts, which are equivalent according to the put-call parity, are used to estimate the parameters. As options with less than 7 days to expiration may induce biases due to low prices and bid-ask spreads, they are excluded from the sample. Because the liquidity is concentrated in the nearest expiration contract, we only consider options with the nearest maturity. To mitigate the impact of price discreteness on option valuation, prices lower than 0.4 are not included. Lastly, prices not satisfying the arbitrage restriction are excluded.

We divide the option data into several categories, according to the moneyness, which is defined as S/K, where S denotes the spot price of the underlying asset and K denotes the strike price. Table 1 describes certain sample properties of the S&P 500 option prices used in this study. Summary statistics are reported for the option price, as well as for the total number of observations, according to each moneyness-option type category. There are 42,396 call- and 64,316 put-option observations, with deep OTM[4] options comprising 24% of the calls and 49% of the puts, respectively. Figure 1 presents the "volatility smile (or sneer)" effects for 26 consecutive six-month sub-periods. We use six fixed intervals for the degree of moneyness, and compute the mean over alternative sub-periods of the implied volatility. The X-axis represents the moneyness, S/K, and the Y-axis represents the average value of the implied volatilities for the corresponding moneyness from the BS model. The S&P 500 options market seems to be "sneer" independent of the sub-periods used in the estimation. As S/K increases, the implied volatilities decrease to near-the-money (NTM) options; however, after that, they increase to OTM put options. The implied volatilities of deep OTM puts are larger than those of deep OTM calls. That is, a volatility smile is skewed towards one side. The skewed volatility smile is sometimes called a "volatility sneer" because it looks more like a sardonic smile than a sincere smile. In the stock index options market, the volatility sneer is often negatively skewed, where lower strike prices for the OTM puts have higher implied volatilities and, thus, higher valuations. As the sneer evidence is indicative of a negatively skewed implicit return distribution with excess kurtosis, a better model must be based on a distributional assumption that allows for negative skewness and excess kurtosis.

[4] For the call option, deep OTM options are options with $S/K < 0.94$. For the put option, deep OTM options are options with $S/K > 1.06$.

4. Empirical Results

4.1. Implied Volatility Sneer Pattern

Figure 1 reports the implied volatility sneer curves for 26 consecutive six-month sub-periods. Irrespective of the sub-period, the implied volatility sneer or smile patterns are clearly evident, which demonstrates the departure from the BS model and reveals that the skewness and kurtosis of the risk-neutral distribution from option prices are not equal to those that are assumed in the BS model.

Table 2 reports the relationship between the moneyness and the implied volatilities using the slope of the following regression suggested by Bakshi, Kapadia and Madan (2003).

$$IV_i = c_0 + c_{slope}(S/K_i) + \varepsilon_i \tag{9}$$

where IV_i and S/K_i are the implied volatility and the option moneyness of option i, respectively. In equation (9), the slope, c_{slope}, measures the degree of the volatility sneer. The higher c_{slope}, the more severe the degree of the volatility sneer pattern. As the moneyness increases or the strike price decreases, we expect the implied volatilities from option prices to increase. The slope, c_{slope}, is expected to be significantly positive. We examine the degree of the volatility sneer phenomenon for sub-periods using the t-values.

In Table 2, c_{slope} for the whole sample period is 0.4675 and significantly positive. In addition, for all sub-periods, the slope coefficients are estimated to be significantly positive. Irrespective of the sub-period, volatility sneer patterns are observed. However, the R^2 values are relatively small for some sub-periods, which can be explained by the shape of the implied volatilities curve. In Figure 1, for most of the sub-periods, the conventional implied volatility sneer patterns increase monotonically with respect to the moneyness. However, for some sub-periods, implied volatility smile patterns are observed. To explain the smile patterns properly, we need to add the squared term of the moneyness into equation (9). Only the moneyness is assumed as the independent variable in our regression and the squared moneyness is not considered for those sub-periods. Therefore, the R^2 values are small for some sub-periods.

Summarizing these results, we can expect the observed risk-neutral skewness and kurtosis to be negative and greater than three, respectively, because the implied volatility sneer patterns are observed for most of the sub-periods.

4.2. Skewness and Kurtosis

In this section, the estimates of the risk-neutral skewness and kurtosis from the option prices are discussed. Table 3 reports the averages and the standard deviations of skewness and kurtosis that are estimated on a daily basis. Both the nonparametric method suggested by Bakshi, Kapadia and Madan (2003) and the parametric method suggested by Corrado and Su (1996) are used to estimate parameters. Irrespective of the sub-period and the estimation method, skewness is estimated to be significantly negative and kurtosis greater than three. Consistent with the existing studies, the negatively skewed and leptokurtic risk-neutral distribution is verified. These results are consistent with the implied volatility sneer patterns that are observed in Figure 1 and Table 2. The high implied volatilities of OTM put options with low strike price are coherent with the left heavy-tail distribution.

The estimates of skewness are stable irrespective of the estimation method and sub-period compared to those of kurtosis. However, the standard deviations of kurtosis are dependent on the estimation method. The standard deviations of the kurtosis estimates from the nonparametric method are greater than those from the parametric method are. The estimates from the nonparametric method can be contaminated by the outliers because the individual options data are used to estimate skewness and kurtosis without the specific functional form. However, it is obvious that the outliers can influence not only the skewness estimates but also the kurtosis estimates. We conjecture that the kurtosis parameters are readily affected by the type of estimation method. The correlations between the estimates of the parametric and nonparametric methods are 0.4901 and 0.2630 for skewness and kurtosis, respectively. The skewness estimates from the two estimation methods exhibit a close relationship but the kurtosis estimates do not.

The slope coefficients in Table 2 are closely related to the estimates of skewness and kurtosis in Table 3. The implied volatility sneer patterns have a close relationship with the risk-neutral skewness and kurtosis from option prices. As the degree of the volatility sneer patterns is intensified, skewness and kurtosis are decreased and increased, respectively. That is, the less steep the implied volatilities curve, the less negative skewness and the lower kurtosis of the risk-neutral distribution. Among all the sub-periods, those with the smaller slopes have less negative skewness and lower kurtosis estimates than the other sub-periods.

In Table 4, we examine the effects of skewness and kurtosis on the slope coefficients of the implied volatiles sneer curves. Table 4 reports the regression coefficients of the slope of the implied volatilities curve on the risk-neutral skewness and kurtosis.

$$c_{slope,t} = c_0 + c_{SKEW} SKEW_t + c_{KURT} KURT_t + \varepsilon_t \tag{10}$$

where $c_{slope,t}$ is the slope coefficient of the regression of the implied volatility on the moneyness in equation (9) at time t. $SKEW_t$ and $KURT_t$ are the risk-neutral skewness and kurtosis at time t, respectively. $c_{slope,t}$, $SKEW_t$ and $KURT_t$ are estimated on a daily basis. The slope of the regression of the implied volatility on the moneyness can be affected by both the skewness and the kurtosis of the risk-neutral distribution. We expect c_{SKEW} and c_{KURT} to be estimated as negative and positive, respectively. As the risk-neutral skewness becomes more negative and the risk-neutral kurtosis increases, we expect the slope to be increased. According to the degree of the significance of c_{SKEW} and c_{KURT}, we can determine the relative impact between skewness and kurtosis on the slope of the implied volatility sneer pattern. If the significance of c_{SKEW} is greater than that of c_{KURT}, skewness is a more important factor than kurtosis for making the implied volatility slope.

Irrespective of the estimation method, c_{SKEW} and c_{KURT} are estimated to be negative and positive, respectively. This result is consistent with our expectation. The higher slope, the more negative the skewness and the more positive the kurtosis. The significances of the skewness coefficients are greater than those of the kurtosis coefficients. Skewness is a more important factor than kurtosis for determining the slope of the implied volatility sneer pattern. In addition, the R^2 values of the regression considering only skewness as the independent variable are greater than those considering only kurtosis. The slope of the implied volatility sneer curve is closely related to the pricing and hedging errors from the BS model. The steep implied volatility curve enlarges the errors. We can presume that skewness, which has the close relationship with the slope of the volatility sneer pattern, can explain the options pricing and hedging errors better than kurtosis can.

4.3. Impact of Skewness and Kurtosis on Pricing and Hedging Errors

4.3.1. Methodology

To examine the impact of skewness and kurtosis on pricing and hedging errors, we regress the pricing and hedging errors from the BS model on the risk-neutral skewness and kurtosis. In Equation (11), we examine the relative influence of the risk-neutral skewness and kurtosis on pricing and hedging errors.

$$E_t = c_0 + c_{SKEW} SKEW_t + c_{KURT} KURT_t + \varepsilon_t \tag{11}$$

where E_t is the pricing and hedging errors at time t, $SKEW_t$ is the risk-neutral skewness at time t and $KURT_t$ is the risk-neutral kurtosis at time t. The risk-neutral skewness and kurtosis, $SKEW_t$ and $KURT_t$, are estimated using both the parametric method of Corrado and Su (1996) and the non-parametric method of Bakshi, Kapadia and Madan (2003) from daily OTM options. Pricing errors are defined as the difference between the model and market option prices. We define the hedging error as the difference between the change in the reported market price and the change in the model's theoretical price in line with Dumas, Fleming, and Whaley (1998) and Gemmill and Saflekos (2000).

Pricing and hedging errors are based on two measures: mean absolute percentage errors (MAPE) and root mean squared percentage error, (RMSPE) as follows.

$$MAPE_t = \sum_{i=1}^{N_t} \left(\frac{|\varepsilon_t|}{O_{i,t}} \right) / N_t \ (t = 1, \cdots, T) \tag{12}$$

$$RMSPE_t = \sqrt{\sum_{i=1}^{N_t} \left(\frac{\varepsilon_t}{O_{i,t}} \right)^2 / N_t} \ (t = 1, \cdots, T) \tag{13}$$

$$\varepsilon_t = O_{i,t} - O_{i,t}^* \text{ for the pricing errors}$$

$$\varepsilon_t = \Delta O - \Delta O^* \text{ for the hedging errors}$$

where $O_{i,t}^*$ denotes the model price of option i on day t, $O_{i,t}$ the market price of option i on day t, ΔO the change in the reported market price, ΔO^* the change in the model's theoretical price, N_t the number of options on day t, and T the number of days in the sample. MAPE measures the absolute magnitude of pricing errors, while RMSPE measures the volatility of errors. MAPE and RMSPE are used together to diagnose the variation in the errors. RMSPE will always be larger or equal to MAPE. The greater difference between them, the greater the variance in the individual errors in the sample. If MAPE is the same with RMSPE, then all the errors are of the same magnitude. For the robustness check, we consider both MAPE and RMSPE for the dependent variable of the regression.

In Table 3, skewness, $SKEW_t$, is estimated to be negative and kurtosis, $KURT_t$, is estimated to be positive and greater than three. As skewness becomes more negative and kurtosis increases, pricing and hedging errors are expected to increase. We forecast c_{SKEW} and c_{KURT}

to be estimated as negative and positive, respectively. According to the degree of the significance of c_{SKEW} and c_{KURT}, we can determine the relative impact between skewness and kurtosis on pricing and hedging errors. If the significance of c_{SKEW} is greater than that of c_{KURT}, skewness is a more important factor than kurtosis for pricing and hedging options.

4.3.2. Regression Results

Table 5 reports the coefficients of the regression of in-sample pricing errors from the BS model on the risk-neutral skewness and kurtosis. We evaluate the in-sample pricing errors by comparing the market prices with the model's prices computed by using the parameter estimates from the current day. In Panels A and B, MAPE and RMSPE are used to measure the pricing errors, respectively. In the first regression, the intercept and skewness are considered as the independent variables. In the second regression, the intercept and kurtosis are considered as the independent variables. In the third regression, the intercept, skewness and kurtosis are all considered as the independent variables.

When only skewness or kurtosis is used as the independent variable in the first and second regressions, irrespective of the pricing errors regressions measure and the estimation method, c_{SKEW} and c_{KURT} are estimated to be negative and positive, respectively. This result is consistent with our expectation. When we consider both skewness and kurtosis as independent variables, the signs of the kurtosis coefficients are negative, contrary to our expectation. However, the significances of the skewness coefficients are greater than those of the kurtosis coefficients. The explanatory power of skewness overwhelms that of kurtosis. We can conclude that skewness is a more important factor for price options or explaining the current option prices.

When we consider the estimates from the nonparametric method of Bakshi, Kapadia and Madan (2003) as dependent variables, the significances of all the coefficients are decreased compared to those from the parametric method. As mentioned before, the skewness and kurtosis estimates from the nonparametric method can be contaminated by outliers. These results can arise from the endemic characteristics of nonparametric methods. In Table 3, the large standard deviations of skewness and kurtosis from the nonparametric method are evidence of these results. Because the stability of the estimates from the nonparametric method can be problematic, the parametric method suggested by Corrado and Su (1996) is used for the remaining analysis.

Next, the R^2 values of the regressions results are examined. When we consider only skewness as the independent variable, the R^2 values from the regressions are greater than those from kurtosis. That is, the R^2 values in (1) are always greater than those in (2) for all cases. We conjecture that the explanatory power of skewness for option prices is greater than that of kurtosis. In addition, the R^2 values in the parametric method are greater than those in the nonparametric method. The R^2 values using MAPE as the dependent variable are greater than those using RMSPE. In the following analyses, we use the estimates from the parametric method as the independent variables and MAPE as the dependent variables.[5]

Table 6 reports the regression results of MAPE on skewness and kurtosis with respect to type of options and the moneyness. In Panel A, the in-sample pricing errors of call and put options are considered separately. When only skewness or kurtosis is used as the independent variable, the signs of the skewness or kurtosis coefficient are negative and positive, respectively, consistent with our expectation. For both the call and the put options, the pricing errors can be better explained by skewness than by kurtosis. For the put options, the significance of kurtosis is greater than that for the call options. Kurtosis can be the most important factor to price put options. The R^2 values for the call options are greater than that for the put options. In our sample, the pricing errors from the call options are well explained by the shape of the risk-neutral distribution from option prices.[6] If we use the option pricing

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[5] When the estimates from the non-parametric method as the independent variables and RMSPE as the dependent variables are used for the analysis, the overall results are maintained.
[6] For all sub-periods, these results remain unchanged.

model that considers skewness and kurtosis, the pricing and hedging performance of the call options can be improved more than that of the put options. The impact of skewness and kurtosis on pricing and hedging performance will be directly discussed in the next section using the Corrado and Su (1996)'s option pricing model. The characteristics of in-sample pricing errors from call and put options are observed to be somewhat different. However, this does not change the result that skewness is a more important factor than kurtosis for in-sample pricing errors, irrespective of the type of options.

In Panel B, we examine the impact of options moneyness to the relationship between pricing errors and skewness or kurtosis. We classify the options data into two types of the moneyness. Deep OTM options include the call options with S/K < 0.97 and the put options with S/K > 1.03. NTM options represent the call options with 0.97 < S/K < 1.00 and the put options with 1.00 < S/K < 1.03. Consistent with the previous results, the skewness and kurtosis coefficients have a significant negative and positive sign, respectively. Both the degree of the significance and the R^2 values for deep OTM options are greater than those for NTM options. Skewness and kurtosis explain the deep OTM option prices better than the NTM options. The middle and tail areas of the risk-neutral distribution correspond to NTM and OTM option prices, respectively. The leptokurtic characteristic mainly determines the middle area and the skewed property the tail of the distribution. The superiority of the explanatory power in OTM options over NTM options indicates that skewness is expected to be more important than kurtosis for option prices. As Huang and Wu (2004) mention, "the Black-Scholes model has been known to systematically misprice equity index options, especially those that are out-of-the-money (OTM)." We recognize the need for an alternative option pricing model in order to mitigate this effect.

Table 7 reports the regression result of out-of-sample pricing and hedging errors on skewness and kurtosis. To estimate the out-of-sample pricing and hedging errors, we use the current day's estimated parameters in order to price and hedge options for the following day (or week). Consistent with the results for in-sample pricing errors, the skewness and kurtosis coefficients are significantly negative and positive.[7] The t-values of the skewness coefficients are greater than those of the kurtosis coefficients. Skewness is a more important factor than kurtosis for out-of-sample pricing and hedging options. The degree of the significance of the coefficients and the R^2 values of out-of-sample pricing and hedging errors are less than those of the in-sample pricing errors. As the term of the out-of-sample pricing (hedging) lengthens, the influence of skewness and kurtosis on out-of-sample pricing and hedging errors is weakened. The determining structure of skewness and kurtosis seems to be changed on a daily basis.

4.4. Pricing and Hedging Options using Skewness and Kurtosis

In this section, we directly examine the impact of skewness and kurtosis for pricing and hedging S&P 500 options. We compare the performance of the BS model that does not allow for the property of the negatively skewed and leptokurtic risk-neutral distribution with that of the CS model that considers both skewness and kurtosis. In addition, we examine the marginal effect of skewness and kurtosis by comparing the CS model that assumes that the skewness parameter is zero (CS_{kurt} model) and the CS model that assumes that the kurtosis parameter is zero (CS_{skew} model). The CS_{skew} and CS_{kurt} models can capture the risk-neutral skewness and kurtosis, respectively. If the CS_{skew} model shows better performance than the CS_{kurt} model, skewness can be regarded as a significant factor for pricing and hedging options. It is closely related to the shape of the risk-neutral distribution. If either the skewness parameter or the kurtosis parameter is significant, then the distribution implicit in the option pricing equations is more skewed and leptokurtic than the distribution implicit in the BS model.

4.4.1. In-sample Pricing Performance

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[7] An exception is the kurtosis coefficient for one-week hedging errors.

We evaluate the in-sample pricing performance of each model by comparing market prices with the model's prices computed by using the parameter estimates from the current day. From the in-sample pricing performance, we can determine the option pricing model to best explain the current day's option prices.

Table 8 reports in-sample valuation errors for the alternative models computed over the whole sample of options. First, with respect to all measures, the CS model shows the best performance followed by the CS_{skew} model. This is a rather obvious result when the use of more parameters in the CS model is considered. Nevertheless, it is interesting that the CS_{skew} model shows better performance than the CS_{kurt} model, despite the two models having the same number of parameters. Many empirical papers show that the risk-neutral distribution of the stock index options is negatively skewed and leptokurtic.[8] It is proved that skewness is a more significant factor than kurtosis to explain the behavior of the current option prices. For the in-sample pricing, the better performance of the CS_{skew} model than the CS_{kurt} model can be explained by the superiority of skewness over kurtosis. This is consistent with the regression result in Table 4.

Second, for MAPE measures, the CS_{kurt} model does not show better performance than the BS model when $S/K < 1.00$, i.e., for the call options. The CS_{kurt} model has to adjust the BS model for biases related with both the call and put options. Among them, the CS_{kurt} model, which only captures the kurtosis factor, does not adjust the biases related with the call options. However, kurtosis can mitigate the pricing errors from puts. These are consistent with the regression results presented in the previous section. For the put options, the in-sample pricing errors are proved to be better explained by kurtosis than by skewness. On the other hand, the CS_{skew} model shows better performance than the BS model for all moneyness although the CS_{skew} model adjusts the BS model biases related with OTM put options by introducing a

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[8] See Shiratsuka (2001), Weinberg (2001), Anagnou, Bedendo, Hodges and Thompkins (2002), Bliss and Panigirtzoglou (2000, 2002), Bakshi, Kapadia, and Madan (2003) and Kim and Kim (2003).

negative skewness parameter into the option valuation formula.

Third, the pricing errors of the call options with S/K < 1.00 are greater than those of put options with S/K > 1.00 for all the option pricing models. On average, the errors of the calls are greater than those of puts by 8% - 66%. Why is the pricing performance of put options superior to that of call options? In Table 1, the put options form a majority in our sample over call options. Both calls and puts are used to estimate the parameters of the option pricing model. In equation (8), we adopt a non-linear least squares procedure, involving a minimization of the sum of squared percentage errors between the model and the market prices. The estimation procedure is biased to put options, which form the majority of our sample. That is, the parameters that are suited for the put option prices are estimated. It is obvious that the in-sample pricing performance of put options is better than that of call options.

Fourth, after considering the skewness and kurtosis parameters, the pricing performance of call options is more improved than is that of put options. For call options, the difference between the CS and BS models is about 54%. For put options, the improvement of the CS model over the BS model is about 34%. The pricing errors of OTM call options are well explained by the shape of the risk-neutral distribution from option prices. This is consistent with our expectation in the previous section. When we allow for both skewness and kurtosis, the pricing performance of the call options can be improved more than that of the put options.

Lastly, all models show moneyness-based valuation errors. The models exhibit the worst fit for the deep OTM options. The fit, as measured by MAPE and RMSPE, steadily improves as we move from OTM to NTM options. Overall, the CS model, which considers both the skewness and kurtosis parameters, performs the best for in-sample pricing.

4.4.2. Out-of-sample Pricing Performance

In-sample pricing performance can be biased due to the potential problem of over-fitting to the data. A good in-sample fit might be a consequence of having an increasingly larger number of structural parameters. To lower the impact of this connection to inferences, we next examine the model's out-of-sample cross-sectional pricing performance. In the out-of-sample pricing, the presence of more parameters may actually cause over-fitting and penalize the model if the extra parameters do not improve its structural fitting. This analysis also has the purpose of assessing the stability of each model's parameter over time. To control the parameters' stability over alternative time periods, we analyze out-of-sample valuation errors for the following day (week). We use the current day's estimated structural parameters to price options for the following day (week).

Table 9 reports one-day and one-week ahead, out-of-sample valuation errors for alternative models computed over the whole sample of options. For both one-day and one-week ahead, out-of-sample pricing, the CS model generally shows the best performance, closely followed by the CS_{skew} model. If both skewness and kurtosis are considered, the pricing performance can be improved. Similar to MAPE measures of in-sample pricing, the CS_{kurt} model does not show better performance than the BS model for OTM call options. The put options can benefit from kurtosis. Considering only skewness for pricing improves the performance of the call options. The improvement effectiveness of the skewness and kurtosis factors corresponds with the call and put options, respectively. With respect to moneyness-based errors, similar to the case of in-sample pricing, MAPE steadily decreases as we move from deep OTM to NTM options for all models. Generally, the CS model, which allows for both skewness and kurtosis, outperforms all the other models.

Second, we consider the relative effectiveness of the structural parameters. The CS_{skew} model, which considers only skewness, performs better than the CS_{kurt} model, which allows for only kurtosis for both one-day and one-week ahead, out-of-sample pricing performances. The CS_{kurt} model reduces MAPE of the BS model by 0.65% for one-day ahead pricing errors and even increases by 0.61% for one-week ahead pricing errors. The CS_{skew} model reduces MAPE of the

BS model by 17.37% and 13.81% for one-day and one-week ahead pricing errors, respectively. In other words, the reduction effectiveness of pricing errors for the CS_{skew} model is much better than that for the CS_{kurt} model. On the other hand, we consider the CS model that adds the CS_{skew} model to the kurtosis parameter. The CS model reduces the pricing errors of the BS model by 32.03% and 24.26% for one-day and one-week ahead pricing errors, respectively. The difference between the performances of the CS_{skew} and CS models is smaller than that between the performances of the CS_{kurt} and CS models. In view of the results so far, the kurtosis parameter has only marginal effects. Skewness is a more significant factor than kurtosis is.

Third, pricing errors increase from in-sample to out-of-sample pricing. The average of MAPE of all the models is 45.31% for the in-sample pricing, and grows to 50.80% for one-day ahead, out-of-sample pricing. One-week ahead, out-of-sample pricing, errors grow to 57.17%. Although the CS model continues to outperform other models for out-of-sample pricing, the relative margin of performance is significantly less than that of the in-sample pricing case. The difference between the BS and CS models becomes smaller in the out-of-sample pricing. The ratio of the BS model to the CS model for MAPE is 2.9627 for in-sample pricing errors. This ratio decreases to 2.0240 and then to 1.5739 for one-day ahead and one-week ahead, out-of-sample errors, respectively. As the term of the out-of-sample pricing lengthens, the difference between the two models becomes smaller. The strong pricing performance of the CS model is not maintained as the term of the out-of-sample pricing lengthens, implying that the market consensus about the skewness and kurtosis of the risk-neutral distribution is volatile and that structural parameters must be changed frequently.

Lastly, the pricing errors of the call options are greater than those of put options for all the option pricing models. On average, the errors of the calls are 15% - 81% greater than those of puts. The effect of parameters that are matched for the put option prices remains unchanged for the out-of-sample pricing performance. After considering both the skewness and kurtosis parameters, the reduction of pricing errors of call options is greater than that of put options.

The out-of-sample pricing errors of the call options are explained better than those of the put options by the shape of the risk-neutral distribution from option prices. These results agree with those from in-sample pricing performance.

Summarizing all these findings, the CS model performs better than any other model. However, our findings that the differences between the CS and CS_{skew} models are slight and that the CS_{kurt} model shows the worst performance suggest that the skewness parameter is a more important factor than the kurtosis parameter for pricing S&P 500 options.

4.4.3. Hedging Performance

Hedging performance is an important tool for gauging the forecasting power of the volatility of the underlying assets. In practice, option traders usually focus on the risk due to the underlying asset price volatility alone, and carry out a delta-neutral hedge, employing only the underlying asset as the hedging instrument. In line with Dumas, Fleming, and Whaley (1998) and Gemmill and Saflekos (2000), the hedge portfolio error is defined as the difference between the change in the reported market price and the change in the model's theoretical price

Table 10 presents one-day and one-week hedging errors over alternative moneyness categories, respectively. The CS model has the best hedging performance for one day and one week, closely followed by the CS_{skew} model, irrespective of the performance measure. The BS model is the worst performer. The ratios of the BS model to the CS model, which is the best performer, are 1.3912 and 1.3866 for one-day ahead and one-week ahead MAPE, respectively. As the term of hedging lengthens, the difference between the BS model and the best model becomes smaller. With respect to moneyness-based errors, both MAPE and RMSPE steadily decrease as we move from deep OTM to NTM options for all models. Similar to the result of the pricing performance, the hedging errors of the call options are greater than those of the put options. The effect of parameters that are matched for the put option prices is maintained for the hedging performance. After considering the skewness and kurtosis parameters, the reduction of hedging errors of call options is greater than that of put options. These results are also consistent with those from pricing performance. The superiority of the model for pricing options remains unchanged in hedging performance. Similar to the results of the pricing performance, the CS_{skew} model that considers only the skewness parameter shows better performance than the CS_{kurt} model that considers only the kurtosis parameter. For both pricing and hedging S&P 500 options, the skewness of the risk-neutral distribution is a more important factor than the kurtosis is.

4.4.4. Robustness Check

Although we use sufficiently long sample periods in this paper, the results may still be distorted by some specific periods. Table 11 reports the regression results of MAPE on skewness and kurtosis for 26 consecutive six-month sub-periods. Table 11 is an extension of Table 5. Irrespective of the sub-period, the coefficients of skewness are consistently negative and significant, whereas those of kurtosis are changed into negative or positive for some sub-periods. The t-values of the skewness coefficients are always greater than those of the kurtosis coefficients. For all sub-periods, the impact of skewness on pricing errors is greater than that of kurtosis. The superiority of skewness over kurtosis is maintained for all sub-periods. Table 12 reports the out-of-sample pricing and hedging errors measured by MAPE of all the option pricing models for the sub-periods. Table 12 is an extension of Tables 9 and 10 for the sub-periods. Panels A and B report the out-of-sample pricing errors and hedging errors of each model, respectively. Bold numbers represent the smallest errors of each sub-period. The results for the sub-periods are consistent with those for the full sample. The CS model shows the best performance, closely followed by the CS_{skew} model for both pricing and hedging options. The CS_{skew} model performs better than the CS_{kurt} model does. For pricing and hedging S&P 500 options, skewness is more important than kurtosis, irrespective of the sub-period.

Different from the methodology used with the two measures MAPE and RMSPE, we statistically compare the models to confirm the results. Table 13 reports the pair-wise comparison results among the models by providing the t-statistics of the probability that the absolute percentage errors of one model are larger than those of the other. Panels A and B report the t-statistics between out-of-sample pricing and hedging errors of each model, respectively. The comparison results are very clear and similar with those using MAPE and RMSPE. Almost all the differences between the errors of each model are statistically significant, except for the differences between both one-day ahead and one-week ahead, out-of-sample pricing errors of the BS and CS_{kurt} models. That is, the CS_{kurt} model that only considers the kurtosis parameter does not improve the performance of the BS model. For both out-of-sample pricing and hedging S&P 500 options, the CS model is the best. These statistical results confirm the superior performance of the CS_{skew} model to that of the CS_{kurt} and BS models.

5. Conclusion

For S&P 500 index options, we examine the relative strength between the skewness and kurtosis of the risk-neutral distribution to price and hedge options. Using both the nonparametric method suggested by Bakshi, Kapadia and Madan (2003) and the parametric method suggested by Corrado and Su (1996), we estimate the risk-neutral skewness and kurtosis. We determine the relative impact of skewness and kurtosis on pricing and hedging errors according to the degree of the significance of the regression coefficients about skewness and kurtosis. Using the CS model that considers both skewness and kurtosis, we directly examine the effect of skewness and kurtosis on price and hedge options.

We find that skewness and kurtosis are estimated to be negative and greater than three, respectively. The impact of the risk-neutral skewness on pricing and hedging errors is consistently and significantly greater than that of kurtosis. The option pricing model, which considers both skewness and kurtosis, shows the best performance to price and hedge S&P 500 options. When the impacts of skewness and kurtosis are compared, the model that only considers skewness shows better performance than that of the model that only considers kurtosis. All the results are robust to all sub-periods and are statistically significant in confirming skewness as a more important factor than kurtosis for pricing and hedging stock index options.

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Table 1: S&P 500 Options Data

This table reports the average option price and the number of options, which are shown in parentheses, for each moneyness and options type (call or put) category. The sample period is from January 4, 1996 to December 31, 2008. The last bid-ask average of each option contract is used to obtain the summary statistics. The moneyness of an option is defined as S/K, where S denotes the spot price and K denotes the strike price.

Call Options			Put Options		
Moneyness	Price	Number	Moneyness	Price	Number
S/K<0.94	2.7833	10,033	1.00<S/K<1.03	13.0104	17,966
0.94<S/K<0.96	4.8054	13,580	1.03<S/K<1.06	6.5072	15,149
0.96<S/K<1.00	12.2916	18,783	S/K>1.06	3.0967	31,201
Total	7.6435	42,396	Total	6.6693	64,316

Table 2: Volatility Sneer Pattern

This table reports the relationship between the implied volatility and the moneyness using the following regression model.

$$IV_i = c_0 + c_{slope}(S/K_i) + \varepsilon$$

where IV_i and S/K_i are the implied volatility and the moneyness of option i. The intercept and the slope of the regression model are c_0 and c_1, respectively. The values in the parentheses are t-statistics. This table reports the estimation results for each six-month sub-period. For example, 1996 01-06 is the period from January, 1996 to June, 1996.

	Intercept	Slope	R^2
1996 01-06	-0.6595 (-71.6442)	0.8049 (89.9878)	0.7750
1996 07-12	-0.6380 (-82.1403)	0.7894 (105.0141)	0.8007
1997 01-06	-0.4482 (-77.5546)	0.6313 (112.5923)	0.7985
1997 07-12	-0.5396 (-54.2746)	0.7588 (79.0441)	0.6018
1998 01-06	-0.7361 (-102.2226)	0.9185 (133.3004)	0.8163
1998 07-12	-0.4113 (-26.4694)	0.6648 (43.8286)	0.3370
1999 01-06	-0.5380 (-69.2297)	0.7585 (99.5304)	0.7287
1999 07-12	-0.5886 (-91.2083)	0.7864 (125.0176)	0.7994
2000 01-06	-0.4036 (-46.2325)	0.6241 (72.3190)	0.5996
2000 07-12	-0.2045 (-16.5926)	0.4191 (34.1670)	0.2486
2001 01-06	-0.2623 (-26.6921)	0.4912 (50.7153)	0.4600
2001 07-12	-0.5197 (-39.0849)	0.7649 (59.4262)	0.5263
2002 01-06	-0.4748 (-47.5735)	0.6748 (68.8343)	0.6494
2002 07-12	-0.2228 (-21.5991)	0.5322 (52.3073)	0.4526
2003 01-06	-0.1171 (-8.9445)	0.3611 (28.1383)	0.2111
2003 07-12	-0.5258 (-84.1482)	0.6910 (114.1092)	0.7936
2004 01-06	-0.5804 (-77.8327)	0.7277 (100.1362)	0.7396
2004 07-12	-0.5756 (-92.1463)	0.7059 (115.7861)	0.7832
2005 01-06	-0.6643 (-105.5076)	0.7829 (127.2876)	0.8193
2005 07-12	-0.7451 (-112.7614)	0.8561 (133.4385)	0.8167
2006 01-06	-0.7866 (-106.3806)	0.9062 (126.6397)	0.7764
2006 07-12	-0.7036 (-86.6714)	0.8152 (103.8956)	0.7020
2007 01-06	-0.7699 (-105.8220)	0.8921 (128.1903)	0.7400
2007 07-12	-0.4795 (-70.2796)	0.6851 (104.0370)	0.5749
2008 01-06	-0.3522 (-47.8483)	0.5711 (80.1743)	0.4613
2008 07-12	0.1251 (6.7713)	0.2810 (15.1593)	0.0275
Total	-0.2584 (-60.5469)	0.4675 (112.4375)	0.1059

Table 3: Skewness and Kurtosis

This table reports the average and the standard deviation of skewness and kurtosis that are estimated on a daily basis. The nonparametric method represents the estimates using the method suggested by Bakshi, Kapadia and Madan (2003). The parametric method represents the estimates using the method suggested by Corrado and Su (1996). This table reports the estimation results for each six-month sub-period. For example, 1996 01-06 is the period from January, 1996 to June, 1996.

Panel A: Estimates

	Nonparametric Method		Parametric Method	
	Skewness	Kurtosis	Skewness	Kurtosis
1996 01-06	-1.7767 (0.3909)	8.1799 (1.7375)	-1.2228 (0.4046)	5.3746 (0.5687)
1996 07-12	-1.7262 (0.3031)	8.0368 (1.3082)	-1.1726 (0.2661)	5.3963 (1.3545)
1997 01-06	-1.1861 (0.3241)	5.8701 (1.2313)	-0.8682 (0.3781)	4.5683 (0.6251)
1997 07-12	-1.3583 (0.3723)	5.9774 (1.6857)	-1.1026 (0.4366)	4.8159 (0.7708)
1998 01-06	-1.8462 (0.2575)	8.4970 (1.6507)	-1.2364 (0.3147)	5.3307 (0.3595)
1998 07-12	-1.6783 (0.3429)	7.4240 (2.4231)	-1.3456 (0.3407)	5.3966 (0.4431)
1999 01-06	-1.3973 (0.2101)	6.4347 (1.2985)	-1.1531 (0.3863)	4.5389 (0.3769)
1999 07-12	-1.6042 (0.2652)	7.5423 (1.6772)	-1.1706 (0.2999)	4.8235 (0.4442)
2000 01-06	-1.2703 (0.2961)	6.5529 (1.2293)	-0.9673 (0.3108)	4.3967 (0.5068)
2000 07-12	-1.1135 (0.3689)	6.3973 (1.8478)	-0.8970 (0.5929)	4.2422 (0.5282)
2001 01-06	-1.1457 (0.2490)	6.5052 (1.1807)	-0.8422 (0.2680)	4.3695 (0.4261)
2001 07-12	-1.4671 (0.3472)	7.3373 (2.1381)	-1.0340 (0.2608)	5.0735 (0.4487)
2002 01-06	-1.5143 (0.3443)	8.0966 (1.7817)	-0.9829 (0.2532)	4.8889 (0.3441)
2002 07-12	-1.0678 (0.2757)	5.2317 (1.3558)	-1.0154 (0.2657)	4.7650 (1.1761)
2003 01-06	-0.9633 (0.2948)	5.5231 (1.0161)	-0.7775 (0.2879)	4.3895 (1.1782)
2003 07-12	-1.3758 (0.4115)	7.5246 (1.9805)	-0.8560 (0.2681)	4.7974 (0.4993)
2004 01-06	-1.6447 (0.3756)	8.8218 (3.7079)	-1.0478 (0.2105)	5.0557 (0.4380)
2004 07-12	-1.4826 (0.4132)	8.1049 (2.5907)	-0.9539 (0.3052)	5.1268 (1.9509)
2005 01-06	-1.6955 (0.4246)	9.4881 (2.5203)	-0.9969 (0.2717)	5.1094 (1.4183)
2005 07-12	-1.9424 (0.4111)	10.9886 (6.3724)	-1.1606 (0.2501)	5.0515 (0.3328)
2006 01-06	-1.9156 (0.4844)	10.3004 (3.1447)	-1.1794 (0.3491)	5.1834 (0.4934)
2006 07-12	-1.8512 (0.4672)	9.8543 (3.6113)	-1.2040 (0.2918)	5.2240 (0.5407)
2007 01-06	-2.0871 (0.8010)	13.0283 (11.8294)	-1.3768 (0.2541)	5.5694 (0.5191)
2007 07-12	-1.5048 (0.3744)	7.0300 (4.1965)	-1.4268 (0.2857)	4.5185 (0.6813)
2008 01-06	-1.1501 (0.2453)	5.0356 (1.0488)	-1.1325 (0.2453)	3.8232 (0.3927)
2008 07-12	-0.7236 (0.3919)	3.7473 (1.8985)	-1.0913 (0.3602)	4.2816 (1.3009)
Full	-1.4799 (0.5026)	7.5931 (3.8942)	-1.0852 (0.3620)	4.8502 (0.9173)

Table 4: Regression of Implied Volatility Slope on Skewness and Kurtosis

This table reports the coefficients of the regression of the slope of the implied volatilities curve on skewness and kurtosis.

$$c_{slope,t} = c_0 + c_{SKEW} SKEW_t + c_{KURT} KURT_t + \varepsilon_t$$

$c_{slope,t}$ is the slope coefficient of the regression of the implied volatility on the moneyness in Table 2. $SKEW_t$ and $KURT_t$ are the risk-neutral skewness and kurtosis at time t, respectively. Intercept, Skewness, and Kurtosis are c_0, c_{SKEW} and c_{KURT} from the above regression model. The values in the parentheses are t-statistics. The nonparametric method represents the estimates using the method suggested by Bakshi, Kapadia and Madan (2003). The parametric method represents the estimates using the method suggested by Corrado and Su (1996).

	Nonparametric Method			Parametric Method		
	(1)	(2)	(3)	(1)	(2)	(3)
Intercept	0.3089	0.6474	0.3125	0.5483	0.3847	0.2923
	(27.7572)	(66.9437)	(27.6831)	(39.5965)	(16.2020)	(12.2716)
Skewness	-0.3322		-0.3400	-0.2324		-0.1834
	(-46.6534)		(-41.2244)	(-19.2031)		(-14.8044)
Kurtosis		0.0202	-0.0020		0.0857	0.0638
		(17.8013)	(-1.8741)		(17.8215)	(13.0439)
R^2	0.4008	0.0887	0.4014	0.1018	0.0889	0.1464

Table 5: Regression of Pricing Errors on Skewness and Kurtosis

This table reports the coefficients of the regression of MAPE on skewness and kurtosis.

$$PE_t = c_0 + c_{SKEW} SKEW_t + c_{KURT} KURT_t + \varepsilon_t$$

PE_t is the mean absolute percentage error (MAPE) or the root mean squared percentage error (RMSPE) of Black and Scholes's (1973) model for the in-sample pricing performance. In Panel A, the MAPE are the dependent variables. In Panel B, the absolute errors are the dependent variables. $SKEW_t$ and $KURT_t$ are the risk-neutral skewness and kurtosis at time t, respectively. Intercept, Skewness, and Kurtosis are c_0, c_{SKEW} and c_{KURT} from the above regression model. The values in the parentheses are t-statistics. The nonparametric method represents the estimates using the method suggested by Bakshi, Kapadia and Madan (2003). The parametric method represents the estimates using the method suggested by Corrado and Su (1996).

Panel A: MAPE

	Nonparametric Method			Parametric Method		
	(1)	(2)	(3)	(1)	(2)	(3)
Intercept	0.3090	0.5034	0.3160	0.1641	0.3720	0.1825
	(37.2313)	(75.8776)	(37.6442)	(26.2253)	(22.9381)	(16.5395)
Skewness	-0.1752		-0.1903	-0.3725		-0.3760
	(-32.9928)		(-31.0289)	(-68.1100)		(-65.5392)
Kurtosis		0.0086	-0.0038		0.0405	-0.0046
		(11.0070)	(-4.8636)		(12.3201)	(-2.0254)
R^2	0.2507	0.0359	0.2561	0.5877	0.0446	0.5883

Panel B: RMSPE

	Nonparametric Method			Parametric Method		
	(1)	(2)	(3)	(1)	(2)	(3)
Intercept	0.5099	0.7186	0.5302	0.1721	0.5860	0.3142
	(35.8292)	(68.9616)	(37.1031)	(15.6764)	(23.0071)	(16.4204)
Skewness	-0.1472		-0.1913	-0.5119		-0.5392
	(-16.1634)		(-18.3256)	(-53.3429)		(-54.1840)
Kurtosis		0.0012	-0.0113		0.0292	-0.0354
		(0.9782)	(-8.3715)		(5.6628)	(-9.0142)
R^2	0.0743	0.0003	0.0938	0.4665	0.0098	0.4795

Table 6: Regression of Pricing Errors on Skewness and Kurtosis with respect to Type of Options and the Moneyness

This table reports the coefficients of the regression of MAPE on skewness and kurtosis.

$$MAPE_t = c_0 + c_{SKEW} SKEW_t + c_{KURT} KURT_t + \varepsilon_t$$

$MAPE_t$ is the mean absolute percentage error of Black and Scholes's (1973) model for the in-sample pricing performance. $SKEW_t$ and $KURT_t$ are the risk-neutral skewness and kurtosis at time t, respectively. Intercept, Skewness, and Kurtosis are c_0, c_{SKEW} and c_{KURT} from the above regression model. The values in the parentheses are t-statistics. The parametric method suggested by Corrado and Su (1996) are used to estimate skewness and kurtosis. Deep out-of-the-money options represent the call options with S/K < 0.97 and the put options with S/K > 1.03. Near-the-money options represent the call options with 0.97 < S/K < 1.00 and the put options with 1.00 < S/K < 1.03.

Panel A: Type of Options

	Call Options			Put Options		
	(1)	(2)	(3)	(1)	(2)	(3)
Intercept	-0.2239	0.4794	0.0761	0.4050	0.3079	0.2548
	(-17.1499)	(14.2377	(3.4397)	(63.5082)	(28.0935)	(23.6181)
Skewness	(-0.7427		-0.8002	-0.1341		-0.1053
	(-65.0901)		(-69.5310)	(-24.0603)		(-18.7749)
Kurtosis		0.0212	-0.0747		0.0500	0.0374
		(3.1048)	(-16.4521)		(22.5273)	(16.8912)
R^2	0.5656	0.0030	0.5990	0.1510	0.1349	0.2195

Panel B: Moneyness

	Deep Out-of-The-Money Options			Near-the-Money Options		
	(1)	(2)	(3)	(1)	(2)	(3)
Intercept	0.3887	0.5244	0.3426	-0.0156	0.0181	-0.0667
	(44.0225)	(27.5575)	(22.0351)	(-2.4029)	(1.4745)	(-5.8572)
Skewness	-0.3687		-0.3599	-0.1781		-0.1683
	(-47.7800)		(-44.5200)	(-31.4225)		(-28.4153)
Kurtosis		0.0546	0.01150		0.0329	0.0127
		(14.1527)	(3.6048)		(13.2278)	(5.4491)
R^2	0.412459	0.0580	0.4148	0.2328	0.0510	0.2397

Table 7: Regression of Out-of-Sample Pricing and Hedging Errors on Skewness and Kurtosis

This table reports the coefficients of the regression of MAPE on skewness and kurtosis.

$$MAPE_t = c_0 + c_{SKEW} SKEW_t + c_{KURT} KURT_t + \varepsilon_t$$

$MAPE_t$ is the mean absolute percentage error of Black and Scholes's (1973) model. $SKEW_t$ and $KURT_t$ are the risk-neutral skewness and kurtosis at time t, respectively. Intercept, Skewness, and Kurtosis are c_0, c_{SKEW} and c_{KURT} from the above regression model. The values in the parentheses are t-statistics. The parametric method suggested by Corrado and Su (1996) is used to estimate skewness and kurtosis.

Panel A: Out-of-Sample Pricing

	One Day			One Week		
	(1)	(2)	(3)	(1)	(2)	(3)
Intercept	0.3094	0.443928	0.3151	0.3792	0.5448	0.4318
	(27.2573)	(21.0955)	(15.7293)	(21.9309)	(17.9625)	(14.1541)
Skewness	-0.2546		-0.2557	-0.2145		-0.2246
	(-25.6609)		(-24.5499)	(-14.1916)		(-14.1629)
Kurtosis		0.0292	-0.0014		0.01385	-0.0131
		(6.8574)	(-0.3439)		(2.2538)	(-2.0900)
R^2	0.1683	0.0143	0.1684	0.0584	0.0016	0.0596

Panel B: Hedging

	One Day			One Week		
	(1)	(2)	(3)	(1)	(2)	(3)
Intercept	0.1898	0.3014	0.2584	0.1975	0.4573	0.2761
	(15.5560)	(14.4144)	(12.0300)	(8.5690)	(11.1710)	(6.7929)
Skewness	-0.0722		-0.0854	-0.3448		-0.3599
	(-6.7739)		(-7.6455)	(-17.1187)		(-17.0343)
Kurtosis		-0.0069	-0.0171		0.0236	-0.0196
		(-1.6184)	(-3.8775)		(2.8432)	(-2.3477)
R^2	0.0139	0.0008	0.0184	0.0827	0.0025	0.0843

Table 8: In-sample Pricing Performance

This table reports in-sample pricing errors with respect to moneyness. The in-sample pricing performance of each model is evaluated by comparing the market prices with the model's prices computed by using the parameter estimates from the current day. The moneyness of an option is defined as S/K, where S denotes the spot price of the underlying asset and K denotes the strike price. MAPE denotes mean absolute percentage error and RMSPE denotes root mean squared percentage error. BS is the Black-Scholes (1973) option pricing model. CS_{kurt} is the Corrado and Su (1996) option pricing model that assumes that the skewness parameter is zero. CS_{skew} is the Corrado and Su (1996) option pricing model that assumes that the kurtosis parameter is zero. CS is the Corrado and Su (1996) option pricing model.

	Moneyness	BS	CS_{kurt}	CS_{skew}	CS
MAPE	S/K<0.94	1.3779	1.5984	1.3561	0.5277
	0.94<S/K<0.96	0.7573	1.2059	0.4770	0.1491
	0.96<S/K<1.00	0.2267	0.2531	0.0948	0.0527
	1.00<S/K<1.03	0.1640	0.1531	0.0381	0.0476
	1.03<S/K<1.06	0.5121	0.3000	0.1610	0.0895
	S/K>1.06	0.8381	0.6176	0.5588	0.3686
	Total	0.6112	0.5972	0.3975	0.2063
RMSPE	S/K<0.94	1.8321	2.3991	2.0179	0.9116
	0.94<S/K<0.96	1.0144	1.7778	0.8624	0.3216
	0.96<S/K<1.00	0.3623	0.4888	0.2181	0.1447
	1.00<S/K<1.03	0.2176	0.1800	0.0715	0.0740
	1.03<S/K<1.06	0.5674	0.326	0.2714	0.1604
	S/K>1.06	0.8597	0.6683	0.6568	0.5051
	Total	0.8599	1.0661	0.7895	0.4173

Table 9: Out-of-Sample Pricing Performance

This table reports one-day and one-week ahead, out-of-sample pricing errors with respect to moneyness. Each model is estimated every day during the sample period; one-day and one-week ahead, out-of-sample pricing errors are computed using the estimated parameters from the previous trading day (week). The moneyness of an option is defined as S/K, where S denotes the spot price of the underlying asset and K denotes the strike price. MAPE denotes mean absolute percentage error and RMSPE denotes root mean squared percentage error. BS is the Black-Scholes (1973) option pricing model. CS_{kurt} is the Corrado and Su (1996) option pricing model that assumes that the skewness parameter is zero. CS_{skew} is the Corrado and Su (1996) option pricing model that assumes that the kurtosis parameter is zero. CS is the Corrado and Su (1996) option pricing model.

		One Day				One Week			
	Moneyness	BS	CS_{kurt}	CS_{skew}	CS	BS	CS_{kurt}	CS_{skew}	CS
MAPE	S/K<0.94	1.4727	1.7521	1.4527	0.7691	1.5856	1.9535	1.5425	1.0085
	0.94<S/K<0.96	0.8005	1.2391	0.5736	0.3117	0.8785	1.2888	0.6716	0.4430
	0.96<S/K<1.00	0.2700	0.2991	0.1682	0.1432	0.3168	0.3472	0.2307	0.2057
	1.00<S/K<1.03	0.1787	0.1665	0.1129	0.1153	0.2122	0.1990	0.1836	0.1872
	1.03<S/K<1.06	0.5019	0.3007	0.2312	0.2006	0.5009	0.3197	0.3177	0.3208
	S/K>1.06	0.8340	0.6182	0.5758	0.4368	0.8265	0.6276	0.6153	0.5408
	Total	0.6331	0.6266	0.4594	0.3128	0.6653	0.6714	0.5272	0.4227
RMSPE	S/K<0.94	2.2877	2.7034	2.1029	1.3080	3.2334	3.4736	2.6164	2.1934
	0.94<S/K<0.96	1.1821	1.8714	0.8881	0.4655	1.4602	2.0408	0.9911	0.6957
	0.96<S/K<1.00	0.4587	0.5702	0.2932	0.2518	0.5398	0.6377	0.3788	0.3385
	1.00<S/K<1.03	0.2370	0.2072	0.1681	0.1707	0.2737	0.2496	0.2577	0.2640
	1.03<S/K<1.06	0.5652	0.3439	0.3244	0.3000	0.5704	0.3790	0.4165	0.4538
	S/K>1.06	0.8583	0.6743	0.6675	0.5578	0.8556	0.6889	0.6982	0.7470
	Total	0.9885	1.1607	0.8254	0.5552	1.2566	1.3808	0.9862	0.8594

Table 10: Hedging Performance

This table reports one-day and one-week ahead hedging error with respect to moneyness. For each option, its hedging error is the difference between the change in the reported market price and the change in the model's theoretical price from day t until day t+1 (t+7). MAPE denotes mean absolute percentage error and RMSPE denotes root mean squared percentage error. The moneyness of an option is defined as S/K, where S denotes the spot price of the underlying asset and K denotes the strike price. BS is the Black-Scholes (1973) option pricing model. CS_{kurt} is the Corrado and Su (1996) option pricing model that assumes that the skewness parameter is zero. CS_{skew} is the Corrado and Su (1996) option pricing model that assumes that the kurtosis parameter is zero. CS is the Corrado and Su (1996) option pricing model.

		One Day				One Week			
	Moneyness	BS	CS_{kurt}	CS_{skew}	CS	BS	CS_{kurt}	CS_{skew}	CS
MAPE	S/K<0.94	0.8464	0.8389	0.5548	0.4843	1.6904	1.6135	1.0623	0.8430
	0.94<S/K<0.96	0.3713	0.3036	0.3385	0.2797	0.6463	0.5240	0.6272	0.4715
	0.96<S/K<1.00	0.1446	0.1214	0.1746	0.1493	0.2157	0.1838	0.2744	0.2227
	1.00<S/K<1.03	0.1047	0.1155	0.1136	0.1263	0.1800	0.2087	0.1790	0.2119
	1.03<S/K<1.06	0.1772	0.1778	0.1458	0.1643	0.3893	0.3825	0.2579	0.3124
	S/K>1.06	0.2767	0.2321	0.1916	0.1693	0.8172	0.6241	0.4041	0.3369
	Total	0.2440	0.2220	0.1961	0.1784	0.4354	0.3757	0.2954	0.2579
RMSPE	S/K<0.94	1.3159	1.3122	0.8474	0.7615	2.4725	2.3531	1.5251	1.2409
	0.94<S/K<0.96	0.6047	0.4906	0.5153	0.4279	1.0556	0.8514	0.9654	0.7415
	0.96<S/K<1.00	0.2746	0.2324	0.3094	0.2681	0.4051	0.3412	0.4770	0.3853
	1.00<S/K<1.03	0.1623	0.1787	0.1696	0.1866	0.2664	0.3173	0.2595	0.3103
	1.03<S/K<1.06	0.2733	0.2709	0.2174	0.2472	0.5721	0.5740	0.3691	0.4629
	S/K>1.06	0.4067	0.3458	0.2750	0.2422	1.2133	0.9925	0.5723	0.5520
	Total	0.4981	0.4670	0.3638	0.3265	0.9271	0.8248	0.5900	0.5149

Table 11: Regression of Pricing Errors on Skewness and Kurtosis for the Sub-periods

This table reports the coefficients of the regression of MAPE on skewness and kurtosis.

$$MAPE_t = c_0 + c_{SKEW} SKEW_t + c_{KURT} KURT_t + \varepsilon_t$$

$MAPE_t$ is the mean absolute percentage error of Black and Scholes's (1973) model for the in-sample pricing performance. $SKEW_t$ and $KURT_t$ are the risk-neutral skewness and kurtosis at time t, respectively. Intercept, Skewness, and Kurtosis are c_0, c_{SKEW} and c_{KURT} from the above regression model. The values in the parentheses are t-statistics. The parametric method suggested by Corrado and Su (1996) is used to estimate skewness and kurtosis. This table reports the estimation results for each six-month sub-period. For example, 1996 01-06 is the period from January, 1996 to June, 1996.

	Intercept	Skewness	Kurtosis	R^2
1996 01-06	-0.0078 (-0.1935)	-0.2277 (-22.9168)	0.0557 (7.8851)	0.8276
1996 07-12	0.2731 (7.2641)	-0.2824 (-13.0759)	-0.0064 (-1.5167)	0.5995
1997 01-06	-0.0747 (-2.1745)	-0.2006 (-17.2353)	0.0736 (10.4511)	0.7632
1997 07-12	0.0477 (1.3334)	-0.2165 (-14.2355)	0.0512 (5.9398)	0.7908
1998 01-06	0.1545 (1.8345)	-0.1729 (-9.9128)	0.0467 (3.0591)	0.4721
1998 07-12	0.2797 (2.2035)	-0.3256 (-10.3673)	-0.0041 (-0.1697)	0.4785
1999 01-06	0.1080 (1.2145)	-0.1900 (-10.2161)	0.0415 (2.1776)	0.4769
1999 07-12	0.2521 (2.4389)	-0.2255 (-6.9892)	0.0153 (0.7020)	0.3043
2000 01-06	0.1606 (1.8883)	-0.2885 (-7.3683)	0.0192 (0.8011)	0.4868
2000 07-12	0.2460 (3.6377)	-0.3676 (-24.9295)	-0.0290 (-1.7536)	0.8441
2001 01-06	0.0419 (0.6084)	-0.2052 (-6.6961)	0.0480 (2.4926)	0.5460
2001 07-12	-0.0019 (-0.0251)	-0.1851 (-6.5390)	0.0658 (3.9999)	0.4654
2002 01-06	-0.0692 (-0.7333)	-0.2800 (-9.7313)	0.0510 (2.4089)	0.5591
2002 07-12	0.2229 (6.4895)	-0.3526 (-9.4662)	-0.0207 (-2.4589)	0.4727
2003 01-06	0.2393 (9.7762)	-0.2550 (-9.9816)	-0.0174 (-2.7858)	0.4738
2003 07-12	-0.1202 (-2.7854)	-0.1888 (-10.3197)	0.0824 (8.3919)	0.7122
2004 01-06	0.0952 (1.5263)	-0.2711 (-11.4693)	0.0309 (2.7177)	0.5362
2004 07-12	0.2555 (14.8490)	-0.3079 (-22.9949)	-0.0083 (-3.9644)	0.8159
2005 01-06	0.1953 (7.7304)	-0.3798 (-20.5181)	-0.0066 (-1.8524)	0.7753
2005 07-12	0.1194 (2.0432)	-0.2886 (-17.9221)	0.0365 (3.0200)	0.7654
2006 01-06	-0.0599 (-1.2621)	-0.3717 (-24.0457)	0.0559 (5.1103)	0.9125
2006 07-12	0.1982 (3.1184)	-0.4509 (-15.9140)	-0.0130 (-0.8498)	0.7705
2007 01-06	0.1016 (2.2087)	-0.3498 (-20.7835)	0.0352 (4.2774)	0.8143
2007 07-12	0.3015 (3.8286)	-0.4234 (-8.4458)	-0.0031 (-0.1450)	0.4696
2008 01-06	0.3006 (4.7281)	-0.5997 (-18.4512)	-0.0636 (-3.1333)	0.7804
2008 07-12	0.3243 (5.5051)	-0.3289 (-5.6639)	-0.0151 (-0.9411)	0.2631

Table 12: Errors of Each Model for the Sub-periods

This table reports the mean absolute percentage error (MAPE) of each model for the sub-periods. Panel A reports out-of-sample pricing errors of each model. Panel B reports hedging errors of each model. BS is the Black-Scholes (1973) option pricing model. CS_{kurt} is the Corrado and Su (1996) option pricing model that assumes that the skewness parameter is zero. CS_{skew} is the Corrado and Su (1996) option pricing model that assumes that the kurtosis parameter is zero. CS is the Corrado and Su (1996) option pricing model. Bold numbers represent the smallest errors of each sub-period. This table reports the estimation results for each six-month sub-period. For example, 1996 01-06 is the period from January, 1996 to June, 1996.

Panel A: Out-of-Sample Pricing

	One Day				One Week			
	BS	CS_{kurt}	CS_{skew}	CS	BS	CS_{kurt}	CS_{skew}	CS
1996 01-06	0.5894	0.5030	0.3822	**0.2468**	0.6018	0.5355	0.4248	**0.3236**
1996 07-12	0.5881	0.5084	0.4012	**0.2708**	0.5908	0.5289	0.4482	**0.3384**
1997 01-06	0.4565	0.3888	0.3149	**0.2217**	0.4653	0.4194	0.3811	**0.3064**
1997 07-12	0.5538	0.5230	0.3597	**0.2398**	0.5657	0.5541	0.4343	**0.3654**
1998 01-06	0.6296	0.5618	0.4223	**0.2853**	0.6409	0.5897	0.4818	**0.3814**
1998 07-12	0.6956	0.6992	0.5599	**0.3406**	0.7059	0.7437	0.6479	**0.4715**
1999 01-06	0.5383	0.5600	0.3814	**0.2722**	0.5626	0.6205	0.4437	**0.3654**
1999 07-12	0.6013	0.5864	0.4322	**0.2886**	0.5870	0.6181	0.4908	**0.3750**
2000 01-06	0.5352	0.5273	0.4107	**0.2899**	0.5737	0.5997	0.4888	**0.3761**
2000 07-12	0.4714	0.4741	0.4290	**0.3069**	0.4861	0.5113	0.4940	**0.4017**
2001 01-06	0.4342	0.4469	0.3922	**0.2473**	0.4727	0.4849	0.4346	**0.3420**
2001 07-12	0.5350	0.5708	0.4303	**0.2399**	0.5774	0.6231	0.4664	**0.3443**
2002 01-06	0.4626	0.4939	0.3917	**0.2311**	0.4703	0.5218	0.4310	**0.2962**
2002 07-12	0.5046	0.5307	0.4126	**0.2279**	0.5986	0.6388	0.4990	**0.3953**
2003 01-06	0.3677	0.3624	0.2828	**0.1699**	0.3918	0.3979	0.3347	**0.2696**
2003 07-12	0.4585	0.4508	0.3568	**0.2247**	0.4750	0.4687	0.3947	**0.2958**
2004 01-06	0.5583	0.5883	0.4406	**0.2753**	0.6081	0.6315	0.5051	**0.3970**
2004 07-12	0.5200	0.5252	0.4054	**0.2555**	0.5314	0.5439	0.4390	**0.3623**
2005 01-06	0.5662	0.5932	0.4592	**0.2976**	0.5972	0.6038	0.4948	**0.3932**
2005 07-12	0.6493	0.6484	0.4676	**0.3174**	0.6565	0.6490	0.4899	**0.3648**
2006 01-06	0.7095	0.7364	0.5354	**0.3592**	0.7178	0.7319	0.5535	**0.4194**
2006 07-12	0.7200	0.7051	0.5171	**0.3535**	0.7397	0.7094	0.5361	**0.4315**
2007 01-06	0.8099	0.8305	0.6177	**0.4412**	0.8352	0.8630	0.6832	**0.5473**
2007 07-12	0.9332	1.0223	0.6360	**0.4416**	0.9486	1.0612	0.7325	**0.5875**
2008 01-06	0.7839	0.7983	0.4223	**0.3374**	0.7842	0.8099	0.4943	**0.4307**
2008 07-12	0.7548	0.6107	0.5482	**0.4212**	0.9451	0.8179	0.7452	**0.6689**

Panel B: Hedging

	One Day				One Week			
	BS	CS_{kurt}	CS_{skew}	CS	BS	CS_{kurt}	CS_{skew}	CS
1996 01-06	0.1821	0.1602	0.1671	**0.1577**	0.3604	0.3096	0.2925	**0.2834**
1996 07-12	0.1841	0.1640	0.1612	**0.1529**	0.3619	0.3016	0.2626	**0.2585**
1997 01-06	0.1941	0.1820	0.1642	**0.1591**	0.3984	0.3558	0.2843	**0.2767**
1997 07-12	0.2392	0.2143	0.2003	**0.1839**	0.5405	0.4626	0.3766	**0.3466**
1998 01-06	0.2294	0.1922	0.1936	**0.1757**	0.5901	0.4464	0.3739	**0.3163**
1998 07-12	0.3413	0.3196	0.2845	**0.2444**	0.8563	0.7535	0.6012	**0.4743**
1999 01-06	0.2564	0.2456	0.2105	**0.1965**	0.6026	0.5649	0.4123	**0.3666**
1999 07-12	0.2723	0.2529	0.2224	**0.2057**	0.6295	0.5500	0.4267	**0.3553**
2000 01-06	0.3099	0.3069	0.2518	**0.2244**	0.6574	0.6307	0.4577	**0.3796**
2000 07-12	0.2846	0.2844	0.2413	**0.2240**	0.5391	0.5381	0.3924	**0.3437**
2001 01-06	0.2566	0.2491	0.2029	**0.1822**	0.5856	0.5554	0.3925	**0.3194**
2001 07-12	0.2300	0.2097	0.1827	**0.1598**	0.5385	0.4689	0.3556	**0.3026**
2002 01-06	0.2265	0.2147	0.1806	**0.1587**	0.4614	0.4148	0.3244	**0.2438**
2002 07-12	0.2742	0.2679	0.2088	**0.1900**	0.5799	0.5628	0.3976	**0.3492**
2003 01-06	0.1699	0.1609	0.1346	**0.1267**	0.4325	0.3921	0.2587	**0.2485**
2003 07-12	0.1662	0.1455	0.1403	**0.1284**	0.3913	0.3214	0.2553	**0.2373**
2004 01-06	0.2279	0.2025	0.1895	**0.1692**	0.5610	0.4900	0.3957	**0.3507**
2004 07-12	0.1955	0.1685	0.1621	**0.1490**	0.4245	0.3536	0.3007	**0.2719**
2005 01-06	0.2355	0.2012	0.1921	**0.1719**	0.5034	0.3988	0.3487	**0.2971**
2005 07-12	0.2070	0.1668	0.1690	**0.1528**	0.4341	0.3082	0.2860	**0.2507**
2006 01-06	0.2531	0.2027	0.2084	**0.1834**	0.5255	0.3817	0.3820	**0.3142**
2006 07-12	0.2218	0.1685	0.1809	**0.1565**	0.5753	0.3961	0.3646	**0.2906**
2007 01-06	0.2968	0.2302	0.2464	**0.2130**	0.6714	0.4646	0.4643	**0.3786**
2007 07-12	0.4111	0.3642	0.3225	**0.2823**	0.8752	0.7471	0.5741	**0.4859**
2008 01-06	0.3390	0.3254	0.2425	**0.2335**	0.6811	0.6315	0.4212	**0.3819**
2008 07-12	0.4342	0.4380	0.3332	**0.3198**	0.7735	0.7681	0.5441	**0.5290**

Table 13: Differences between the Errors of Each Model

This table reports the t-statistics of the difference between each model's mean absolute percentage error (MAPE) for pricing and hedging S&P 500 options are shown. Panel A reports t-statistics between out-of-sample pricing errors of each model. Panel B reports t-statistics between hedging errors of each model. BS is the Black-Scholes (1973) option pricing model. CS_{kurt} is the Corrado and Su (1996) option pricing model that assumes that the skewness parameter is zero. CS_{skew} is the Corrado and Su (1996) option pricing model that assumes that the kurtosis parameter is zero. CS is the Corrado and Su (1996) option pricing model. ** and * indicate test statistic values that are significantly different from 1% and 5%, respectively.

Panel A: Out-of-Sample Pricing

	One Day			One Week		
	CS_{kurt}	CS_{skew}	CS	CS_{kurt}	CS_{skew}	CS
BS	1.7255	55.4581**	117.9479**	-1.2390	33.3285**	60.8291**
CS_{kurt}		45.7433**	94.9542**		32.1154**	57.2090**
CS_{skew}			58.0429**			30.4670**

Panel B: Hedging

	One Day			One Week		
	CS_{kurt}	CS_{skew}	CS	CS_{kurt}	CS_{skew}	CS
BS	12.3030**	30.1241**	42.7085**	18.6932**	50.2122**	65.9103**
CS_{kurt}		16.8293**	29.4305**		30.9891**	47.3453**
CS_{skew}			14.4458**			19.3510**

Figure 1: Implied Volatility

This figure represents the implied volatility sneer patterns for 26 consecutive six-month sub-periods. The X-axis represents the moneyness, S/K, and the Y-axis the average value of the implied volatilities for the corresponding moneyness from Black and Scholes (1973) option pricing model. This figure reports the estimation results for each six-month sub-period. For example, 1996 01-06 is the period from January, 1996 to June, 1996.